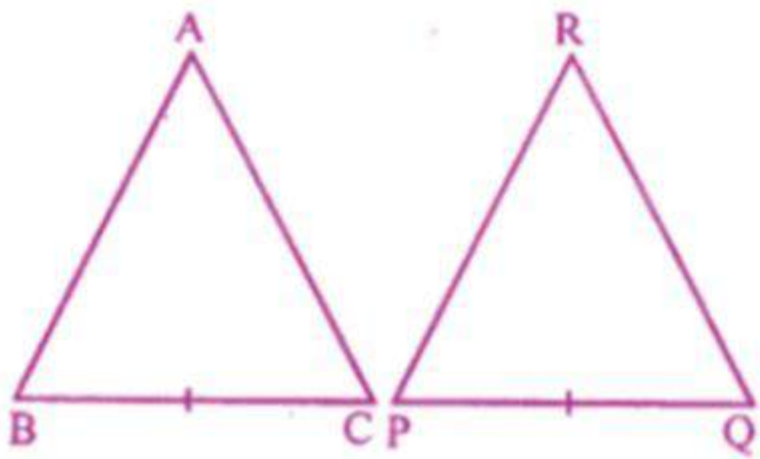
EXERCISE 10.1

1. It is given that ΔABC ≅ ΔRPQ. Is it true to say that BC = QR ? Why?

Solution:

Given ΔABC ≅ ΔRPQ

Therefore, their corresponding sides and angles are equal.



Therefore BC = PQ

Hence it is not true to say that BC = QR

2. "If two sides and an angle of one triangle are equal to two sides and an angle of another triangle, then the two triangles must be congruent." Is the statement true? Why?

Solution:

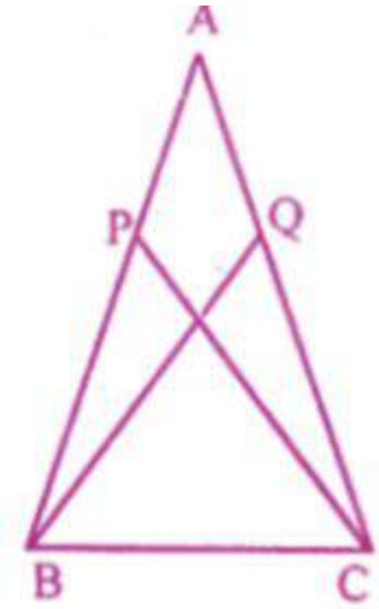
No, it is not true statement as the angles should be included angle of there two given sides.

3. In the given figure, AB=AC and AP=AQ. Prove that

(i) ΔAPC ≅ ΔAQB

(ii) CP = BQ

(iii) ∠APC = ∠AQB.



Solution:

(i) In △ APC and △AQB

AB=AC and AP=AQ [given]

From the given figure, ∠A = ∠A [common in both the triangles]

Therefore, using SAS axiom we have ΔAPC ≅ ΔAQB

(ii) In △ APC and △AQB

AB=AC and AP=AQ [given]

From the given figure, ∠A = ∠A [common in both the triangles]

By using corresponding parts of congruent triangles concept we have

BQ = CP

(iii) In △ APC and △AQB

AB=AC and AP=AQ [given]

From the given figure, ∠A = ∠A [common in both the triangles]

By using corresponding parts of congruent triangles concept we have

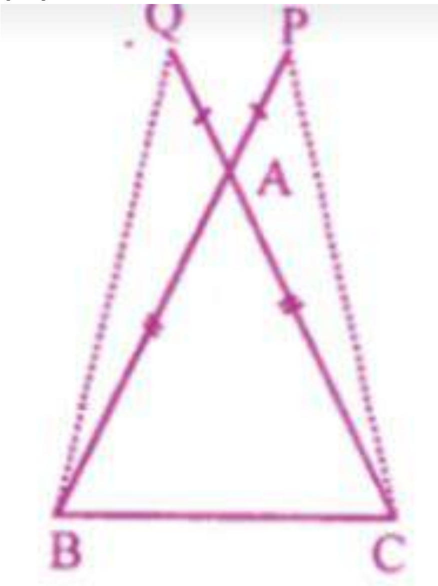
∠APC = ∠AQB.

4. In the given figure, AB = AC, P and Q are points on BA and CA respectively such that AP = AQ. Prove that

(i) ΔAPC ≅ ΔAQB

(ii) CP = BQ

(iii) ∠ACP = ∠ABQ.



Solution:

(i) In the given figure AB = AC

P and Q are point on BA and CA produced respectively such that AP = AQ

Now we have to prove ΔAPC ≅ ΔAQB

By using corresponding parts of congruent triangles concept we have

CP = BQ

∠ACP = ∠ABQ

(ii) CP = BQ

(iii) ∠ACP = ∠ABQ

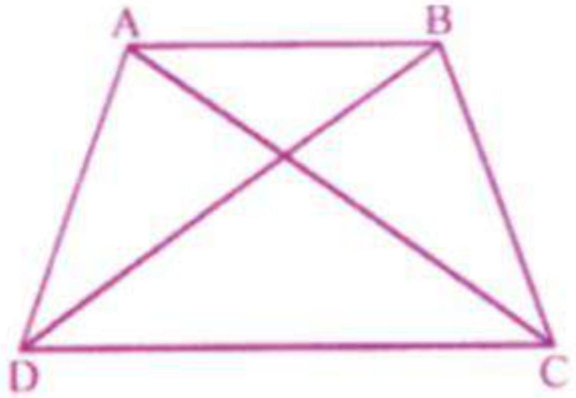
In Δ APC and ΔAQB

AC = AB (Given)

AP = AQ (Given)

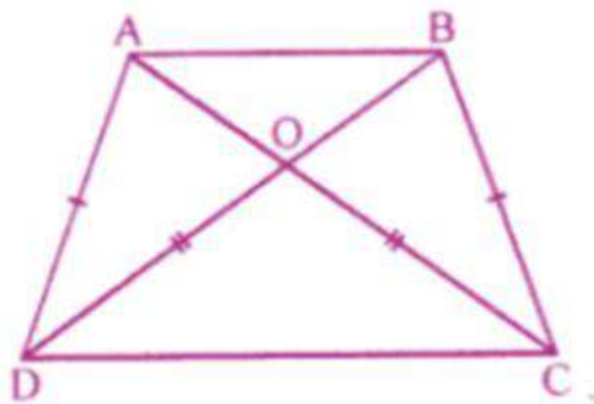
∠PAC =∠QAB (Vertically opposite angle)

5. In the given figure, AD = BC and BD = AC. Prove that :
∠ADB = ∠BCA and ∠DAB = ∠CBA.



Solution:

Given: in the figure, AD = BC, BD = AC



To prove :

(i) ∠ADB = ∠BCA

(ii) ∠DAB = ∠CBA

Proof : in ΔADB and ΔACB

AB = AB (Common)

AD = BC (given)

DB = AC (Given)

ΔADB = ΔACD (SSS axiom)

∠ADB = ∠BCA

∠DAB = ∠CBA

6. In the given figure, ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA. Prove that

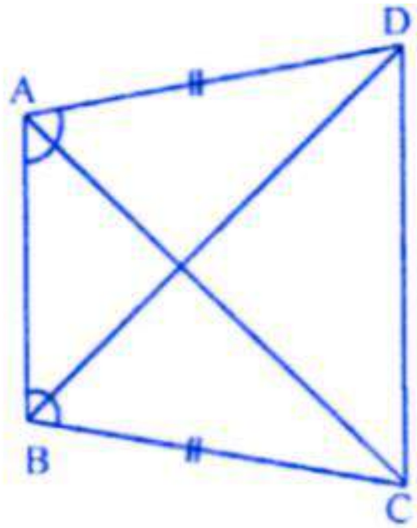
(i) ΔABD ≅ ΔBAC

(ii) BD = AC

(iii) ∠ABD = ∠BAC.

Solution:

Given : in the figure ABCD is a quadrilateral
In which AD = BC
∠DAB = ∠CBA



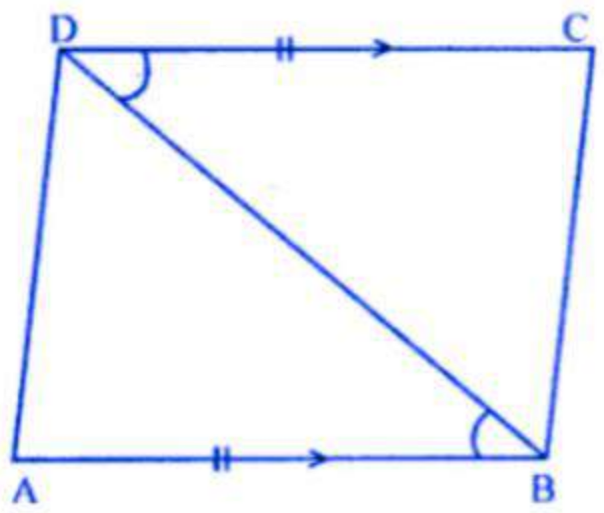
To prove :
(i) ΔABD = ΔBAC
(ii) ∠ABD = ∠BAC
Proof : in ΔABD and ΔABC
AB = AB (common)
∠DAB = ∠CBA (Given)
AD = BC
(i) ΔABD ≅ ΔABC (SAS axiom)
(ii) BD = AC
(ii) ∠ABD = ∠BAC

7.In the given figure, AB = DC and AB || DC. Prove that AD = BC.

Solution :

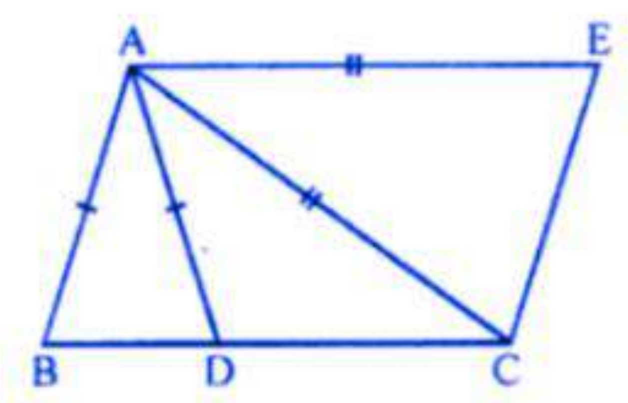
Given: in the given figure.
AB = DC, AB ‖ DC
To prove : AD = BC
Proof : AB ‖ DC

∠ABD = ∠CDB (Alternate angles)
In ΔABD and ΔCDB
AB = DC

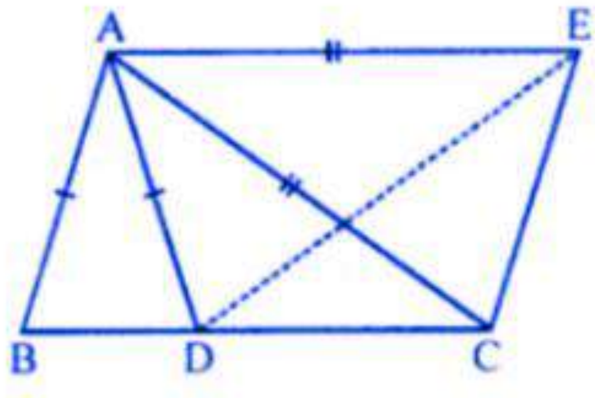


∠ABD = ∠CDB (Alternate angles)
BD = BD (common)
ΔABD ≅ CDB (SAS axiom)
AD = BC

8. In the given figure. AC = AE, AB = AD and ∠BAD = ∠CAE. Show that BC = DE.



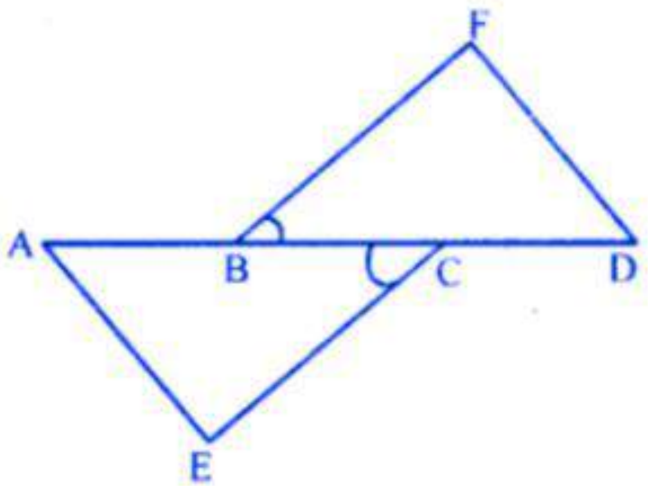
Solution:
Given: in the figure, AC = AE, AB = AD
∠BAD = ∠CAE
To prove : BC = DE



Proof : in ΔABC and ΔADE
AB = AD (given)
AC = AE (given)
∠BAD + ∠DAC + ∠CAE
∠BAC = ∠DAE
ΔABC = Δ ADE (SAS axiom)
BC = DE

9. In the adjoining figure, AB = CD, CE = BF and ∠ACE = ∠DBF. Prove that
(i) ΔACE ≅ ΔDBF
(ii) AE = DF.

Solution:
Given : in the given figure
AB = CD
CE = BF
∠ACE = ∠DBF



To prove : (i) ΔACE ≅ ΔDBF

(i) ΔACE ≅ ΔDBF (SAS axiom)

AE = DE

(ii) AE = DF

Proof : AB = CD

Adding BC to both sides

AB + BC = BC + CD

AC = BD

Now in ΔACE and ΔDBF

AC = BD (Proved)

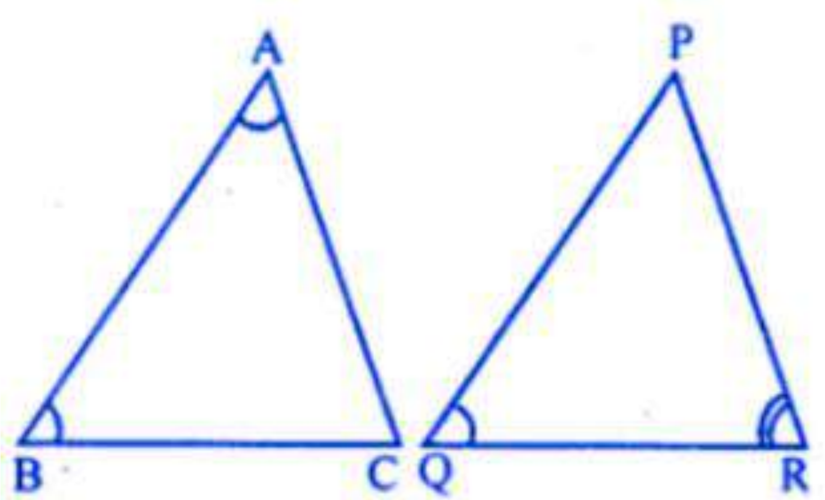
CE = BF (Given)

∠ACE = ∠DBF (SAS axiom)

EXERCISE 10.2

1. In triangles ABC and PQR, ∠A= ∠Q and ∠B = ∠R. Which side of APQR should be equal to side AB of AABC so that the two triangles are congruent? Give reason for your answer.

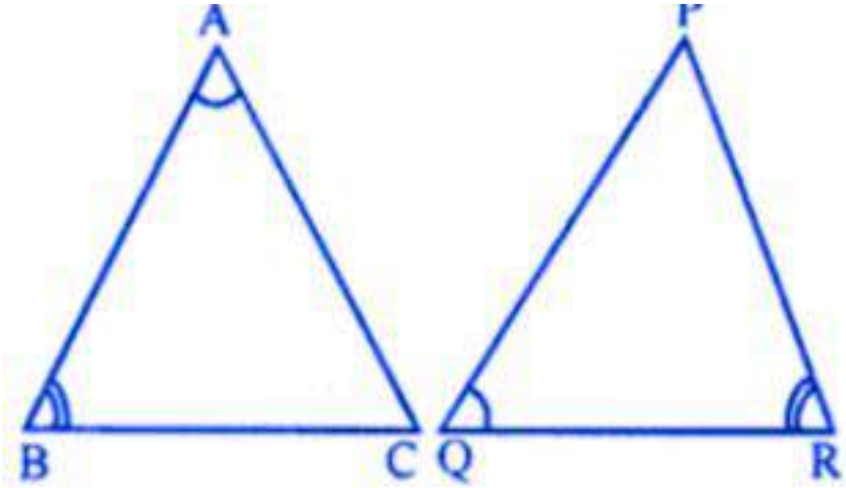
Solution:
In triangle ABC and triangle PQR
∠A = ∠Q
∠B = ∠R



AB = QP
Because triangles are congruent of their corresponding two angles and included sides are equal

2. In triangles ABC and PQR, ∠A = ∠Q and ∠B = ∠R. Which side of APQR should be equal to side BC of AABC so that the two triangles are congruent? Give reason for your answer.

Solution:
In ΔABC and ΔPQR



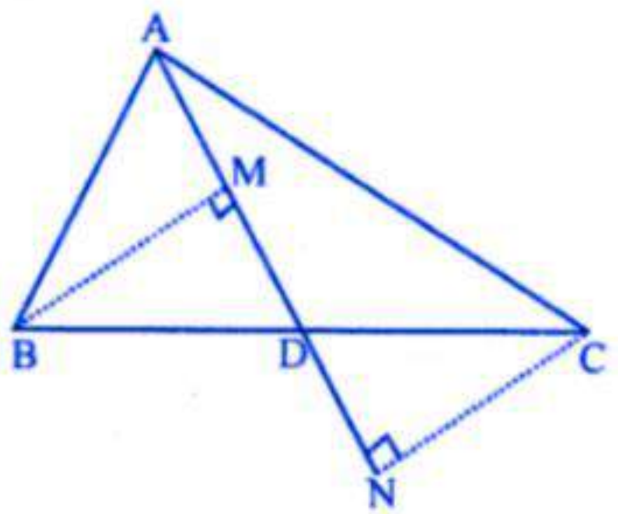
∠A = ∠Q
∠B = ∠R
Their included sides AB and QR will be equal for their congruency.
Therefore, BC = PR by corresponding parts of congruent triangles.

3. "If two angles and a side of one triangle are equal to two angles and a side of another triangle, then the two triangles must be congruent". Is the statement true? Why?

Solution:
The given statement can be true only if the corresponding (included) sides are equal otherwise not.

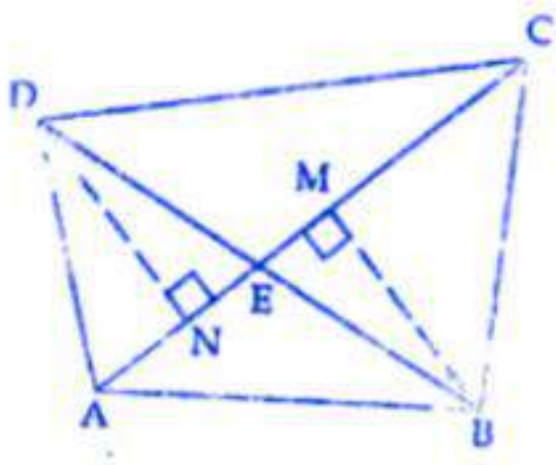
4. In the given figure, AD is median of ΔABC, BM and CN are perpendiculars drawn from B and C respectively on AD and AD produced. Prove that BM = CN.

Solution:



Given in ΔABC, AD is median BM and CN are perpendicular to AD form B and C respectively.
To prove:
BM = CN
Proof:
In ΔBMD and ΔCND
BD = CD (because AD is median)
∠M = ∠N
∠BDM = ∠CDN (vertically opposite angles)
ΔBMD ≅ ΔCND (AAS axiom)
Therefore, BM = CN.

5. In the given figure, BM and DN are perpendiculars to the line segment AC. If BM = DN, prove that AC bisects BD.



Solution:

Given in figure BM and DN are perpendicular to AC

BM = DN

To prove:

AC bisects BD that is BE = ED

Construction:

Join BD which intersects Ac at E

Proof:

In ΔBEM and ΔDEN

BM = DN

∠M = ∠N (given)

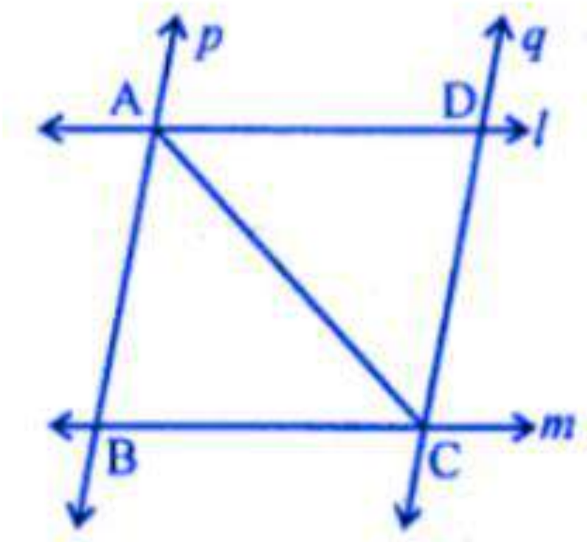
∠DEN = ∠BEM (vertically opposite angles)

ΔBEM ≅ ΔDEN

BE = ED

Which implies AC bisects BD

6. In the given figure, l and m are two parallel lines intersected by another pair of parallel lines p and q. Show that ΔABC ≅ ΔCDA.



Solution:

In the given figure, two lines l and m are parallel to each other and lines p and q are also a pair of parallel lines intersecting each other at A, B, C and D. AC is joined.

To prove:

ΔABC ≅ ΔCDA

Proof:

In ΔABC and ΔCDA

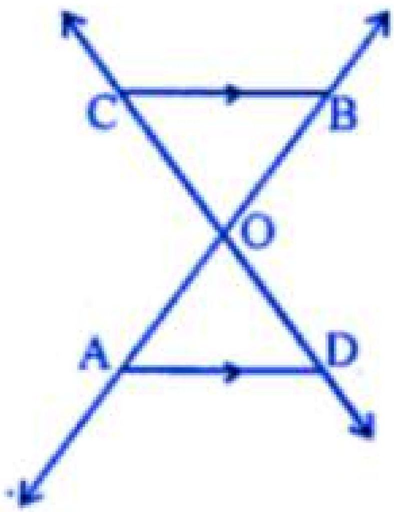
AC = AC (common)

∠ACB = ∠CAD (alternate angles)

∠BAC = ∠ACD (alternate angles)

ΔABC ≅ Δ DCA (ASA axiom)

7. In the given figure, two lines AB and CD intersect each other at the point O such that BC || DA and BC = DA. Show that O is the mid-point of both the line segments AB and CD.



Solution:

In the given figure, lines AB and CD intersect each other at O such that BC ∥ AD and BC = DA

To prove:

O is the midpoint of Ab and Cd

Proof:

Consider ΔAOD and ΔBOC

AD = BC (given)

∠OAD = ∠OBC (alternate angles)

∠ODA = ∠OCB (alternate angles)

ΔAOD ≅ ΔBOC (SAS axiom)

Therefore, OA = OB and OD = OC

Therefore O is the midpoint of AB and CD.

EXERCISE 10.3

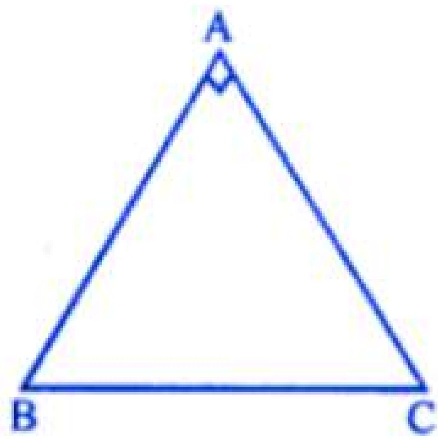
1. ABC is a right angled triangle in which ∠A = 90° and AB = AC. Find ∠B and ∠C.

Solution:

In right angled triangle ABC, ∠A = 90°

∠B + ∠C = 180° - ∠A

= 180° – 90° = 90°



Because AB = AC

∠C = ∠B (Angles opposite to equal sides)

∠B + ∠B = 90° (2∠B = 90°

∠B = 90/2° = 45°

∠B = ∠C = 45°

∠B = ∠C = 45°

2. Show that the angles of an equilateral triangle are 60° each.

Solution:

ΔABC is an equilateral triangle

AB = BC = CA

∠A = ∠B = ∠C (opposite to equal sides)

But ∠A + ∠B + ∠C = 180° (sum of angles of a triangle)

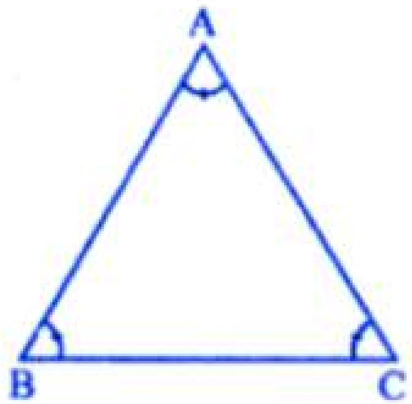
3∠A = 180° (∠A = 180°/3 = 60°)

∠A = ∠B = ∠C = 60°

3. Show that every equiangular triangle is equilateral.

Solution:

ΔABC is an equiangular



∠A = ∠B = ∠C

In ΔABC

∠B = ∠C

AC = AB (sides opposite to equal angles)

Similarly, ∠C = ∠A

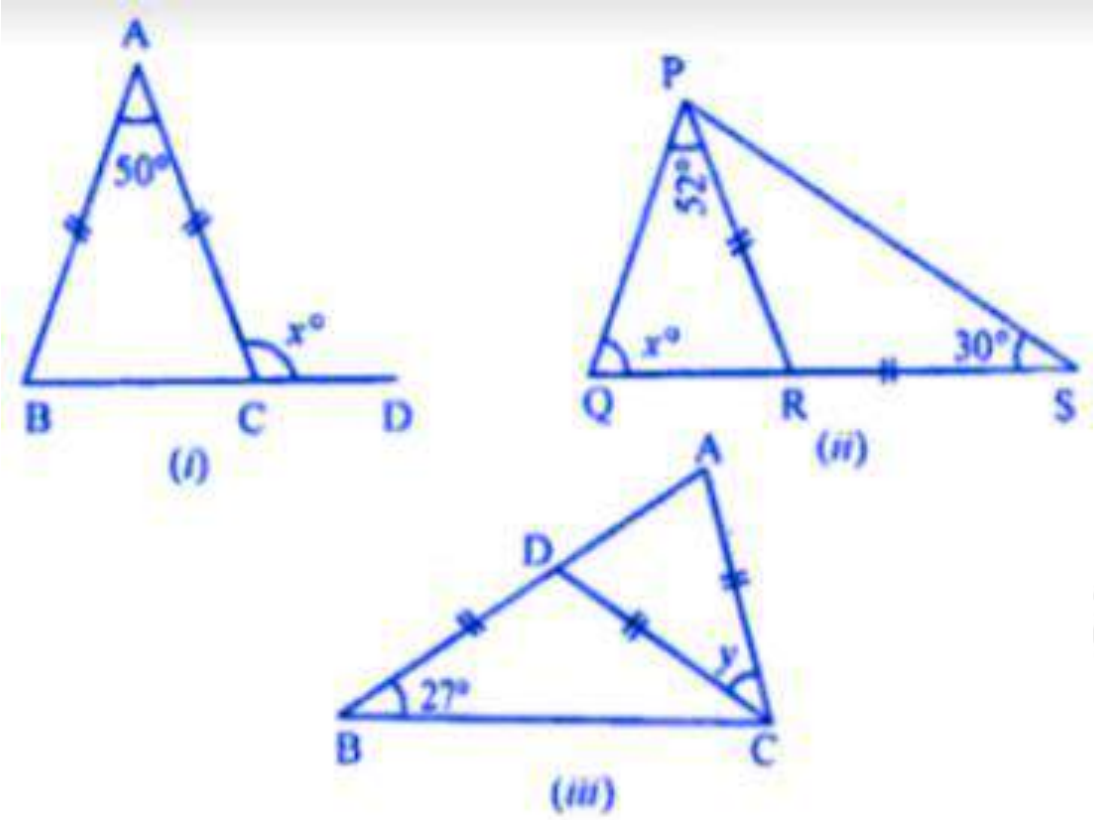
BC = AB

From (i) and (ii)

AB = BC = AC

ΔABC is an equilateral triangle

4. In the following diagrams, find the value of x:



Solution:

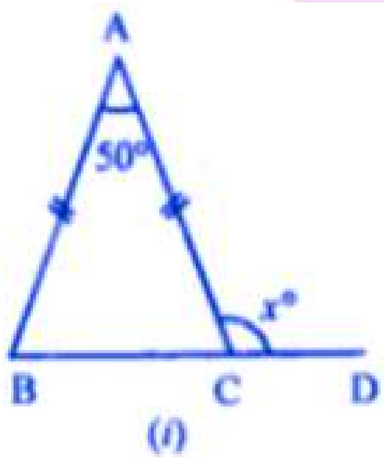
(i) in following diagram given that AB =AC

That is ∠B = ∠ ACB (angles opposite to equal sides in a triangle are equal)

In a triangle are equal)

Now, ∠A + ∠B + ∠ACB = 180°

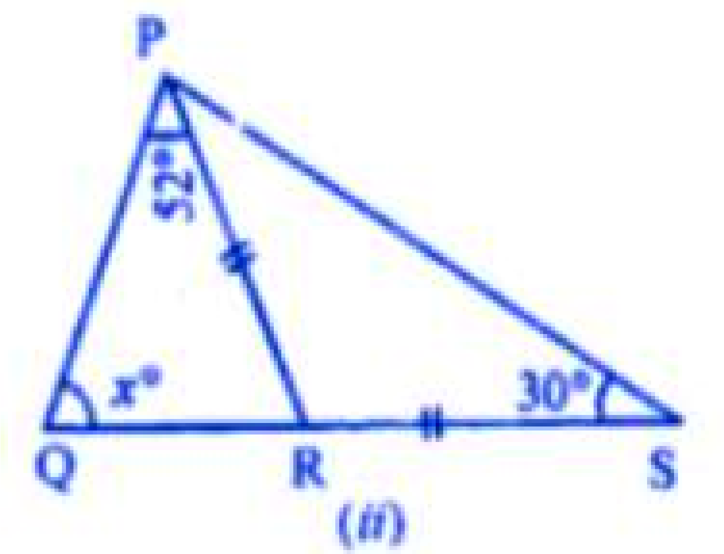
(sum of all angles in a triangle is 180°)



50 + ∠B + ∠B = 180°

(∠A = 50° (given) ∠B = ∠ACB)
50° + 2 ∠B = 180° (2 ∠B = 180° – 50°
2 ∠B = 130° (∠B = 130/2 = 65°
∠ACB = 65°
Also ∠ACB + x° = 180° (Linear pair)
65° + x° = 180° (x° = 180° – 65°
x° = 115°
Hence, Value of x = 115

(ii) in ΔPRS,
Given that PR = RS
∠PSR = ∠RPS
(Angles opposite in a triangle, equal sides are equal)

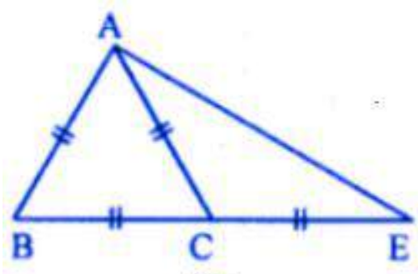


30° = ∠RPS (∠PPS = 30°……(1)
∠QPS = ∠QPR + ∠RPS
∠QPS = 52° + 30°
(Given, ∠QPR = 52° and from (i), ∠RPS = 30°
∠QPS = 82°
Now, In ΔPQS
∠QPS + ∠QSP + PQS = 180°
(sum of all angles in a triangles is 180°)
= 82° + 30° + x° = 180°
(from (2) ∠QPS = 82° and ∠QSP = 30° (given)

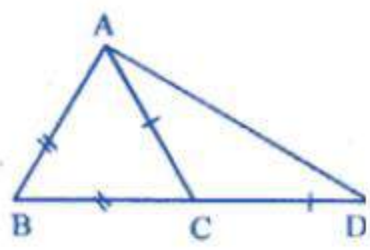
$112^o + x^o = 180^o$ ($x^o = 180^o – 112^0$
Hence, Value of x = 68

(iii) In the following figure, Given
That, BD = CD = AC and ∠DBC = 27°
Now in Δ BCD
BD = CD (Given)
∠DBC = ∠BCD …….(1)
(in a triangle sides opposite equal angles are equal)
Also,, ∠DBC = 27° (given) ………..(2)
From (1) and (2) we get
∠BCD = 27°
Now, ext ∠CDA = ∠DBC + ∠BCD
(exterior angles is equal to sum of two interior opposite angles)
Ext ∠CDA = $27^o + 27^0$ (from (2) and (3)
∠CDA = 54° (from (4)) ……..(5)
Also, in ΔACD
∠CAD + ∠CDA + ∠ACD = 180^0
(sum of all angles in a triangle is 180°)
$54^O + 54^O + Y = 180^O$
$108^O + Y = 180^O$ ($Y = 180^O – 108^O$
y = 72°

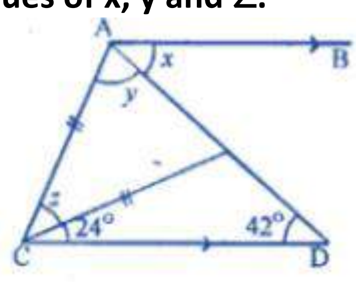
5. (a) In the figure (1) given below, ABC is an equilateral triangle. Base BC is produced to E, such that BC'= CE. Calculate ∠ACE and ∠AEC.
(b) In the figure (2) given below, prove that ∠ BAD : ∠ ADB = 3 : 1.
(c) In the figure (3) given below, AB || CD. Find the values of x, y and ∠.



(1)



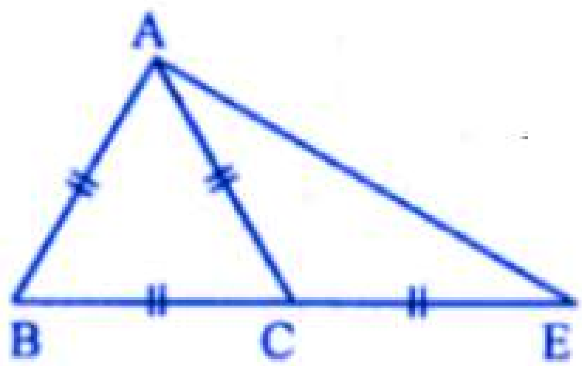
(2)



(3)

Solution:

(a) in following figure
Given. ABC is an equilateral triangle BC = CE
To find. ∠ACE and ∠AEC



(1)

As given that ABC is an equilateral triangle,
That is ∠BAC = ∠B = ∠ACB = 60^0 ……..(1)
(each angle of an equilateral triangle is 60^o)
Now, ∠ACE = ∠BAC + CB
(Exterior angle is equal to sum of two interior opposite angles)
(∠ACE = $60^o + 60^o$)
∠ACE = 120^0
Then, in ΔACE
Given, AC = CE … [because AC = BC = CE]
∠CAE = ∠AEC … (2)
We know that, in a triangle equal sides have equal angles opposite to them.
So, ∠CAE + ∠AEC + 120^o = 80^o
∠AEC + ∠AEC + 120^o = 180^o … [by equation (2) we get]
2∠AEC = $180^o – 120^o$
2∠AEC = 60^o
∠AEC = $60^o/2$
∠AEC = 30^o
Therefore, ∠ACE = 120^o and ∠AEC = 30^o.

(b) In given figure,
Given, ΔABD, AC meets BD in C. AB = BC, AC = CD.
We have to prove that, ∠BAD : ∠ADB = 3: 1

Then, consider ΔABC,

AB = BC … [given]

Therefore, ∠ACB = ∠BAC … (1)

(In a triangle, equal angles opposite to them)

In ΔACD,

AC = CD … [given]

Therefore, ∠ADC = ∠CAD

(In a triangle, equal sides have equal angles opposite to them)

∠CAD = ∠ADC … (2)

From, adding (1) and (2), we get

∠BAC + ∠CAD = ∠ACB + ∠ADC

∠BAD = ∠ACB + ∠ADC … (3)

Now, in ΔACD

Exterior ∠ACB = ∠CAD + ∠ADC … (4)

(In an triangle, exterior angle is equal to sum of two interior opposite angles)

Therefore, ∠ACB = ∠ADC + ∠ADC … [from (2) and (4)]

∠ACB = 2∠ADC … (5)

Now, ∠BAD = 2∠ADC + ∠ADC … [from (3) and (4)]

∠BAD = 3∠ADC = (∠BAD/∠ADC) = 3/1

∠BAD: ∠ADC = 3: 1

(c) In given figure,

Given, AB parallel to CD, ∠ECD = 24^o, ∠CDE = 42^o

We have to find the value of x, y and z.

Consider, ΔCDE

Exterior, ∠CEA = $24^o + 42^o$

(In a triangle exterior angle is equal to sum of two interior opposite angles)

∠CEA = 66^o … (1)

Then, in ΔACE

AC = CE … (given)

Therefore, ∠CAE = ∠CEA

(In a triangle equal side have equal angles opposite to them)

By equation (1),

Y = 66^o … (2)

Also, y + z + ∠CEA = 180^o

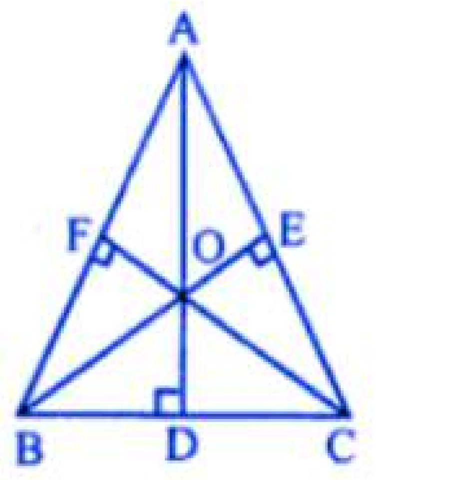
We know that, sum of all angles in a triangle is 180^o

$66^o + z + 66^o = 180^o$

z + 132° = 180°
z = 180° – 132°
z = 48° … (3)
Then it is given that, AB is parallel to CD,
∠x = ∠ADC … [alternate angles]
x = 42° … (4)
Therefore, from (2), (3) and (4) equation gives x = 42°, y = 66° and z = 48°.

6. In the given figure, AD, BE and CF arc altitudes of ΔABC. If AD = BE = CF, prove that ABC is an equilateral triangle.

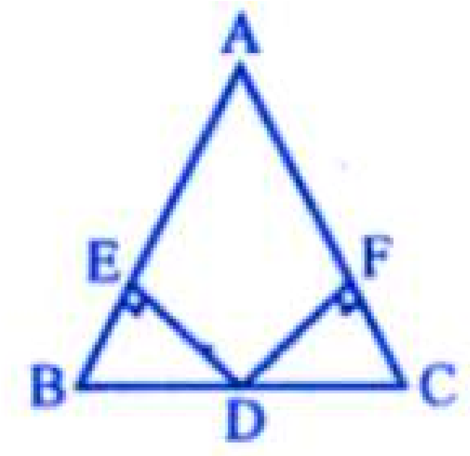
Given : in the figure given,
AD, BE and CF are altitudes of ΔABC and
AD = BE = CF



To prove : ΔABC is an equilateral triangle
Proof: in the right ΔBEC and ΔBFC
Hypotenuse BC = BC (Common)
Side BE = CF (Given)
ΔBEC ≅ ΔBFC (RHS axiom)
∠C = ∠B
AB = AC (sides opposite to equal angles)
Similarly we can prove that ΔCFA ≅ ΔADC
∠A = ∠C
AB = BC
From (i) and (ii)
AB = BC = AC

ΔABC is an equilateral triangle

7. In the given figure, D is mid-point of BC, DE and DF are perpendiculars to AB and AC respectively such that DE = DF. Prove that ABC is an isosceles triangle.



Solution:

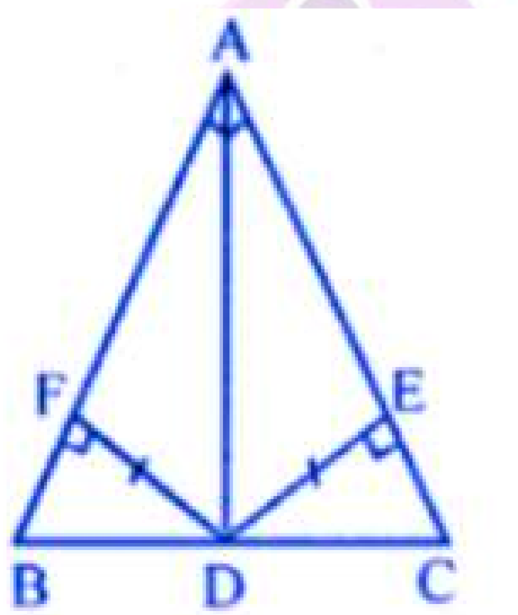
In triangle ABC

D is the midpoint of BC

DE perpendicular to AB

And DF perpendicular to AC

DE = DE



To prove:

Triangle ABC is an isosceles triangle
Proof:
In the right angled triangle BED and CDF
Hypotenuse BD = DC (because D is a midpoint)
Side DF = DE (given)
$\Delta BED \cong \Delta CDF$ (RHS axiom)
$\angle C = \angle B$
AB = AC (sides opposite to equal angles)
ΔABC is an isosceles triangle

EXERCISE 10.4

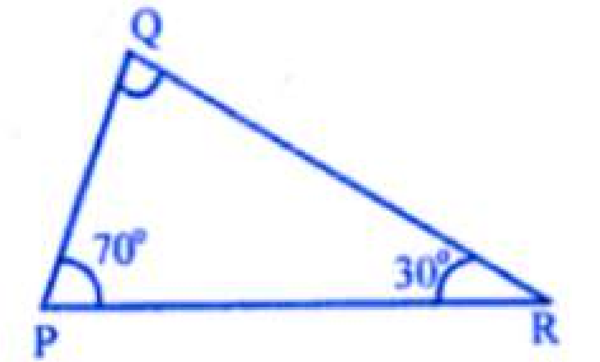
1. In ΔPQR, ∠P = 70° and ∠R = 30°. Which side of this triangle is longest? Give reason for your answer.

Solution:

In ΔPQR, $\angle P = 70^o$, $\angle R = 30^o$

But $\angle P + \angle Q + \angle R = 180^o$

$100^o + \angle Q = 180^o$



$\angle Q = 180^o – 100^o = 180^o$

$\angle Q = 80^o$ the greatest angle

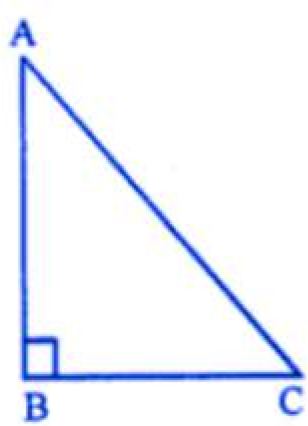
Its opposite side PR is the longest side

(side opposite to greatest angle is longest)

2. Show that in a right angled triangle, the hypotenuse is the longest side.

Solution:

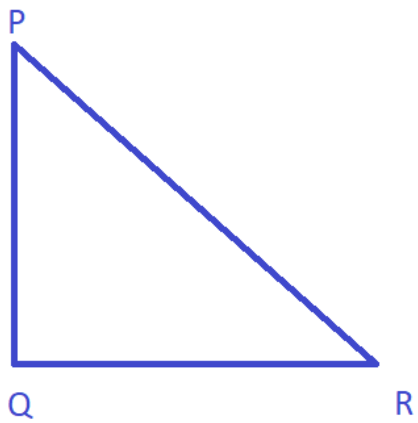
Given: in right angled ΔABC, $\angle B = 90^o$



To prove: AC is the longest side
Proof : in ΔABC,
∠B = 90°
∠A and ∠C are acute angles
That is less then 90°
∠B is the greatest angle
Or ∠B> ∠C and ∠B> ∠A
AC > AB and AC > BC
Hence AC is the longest side

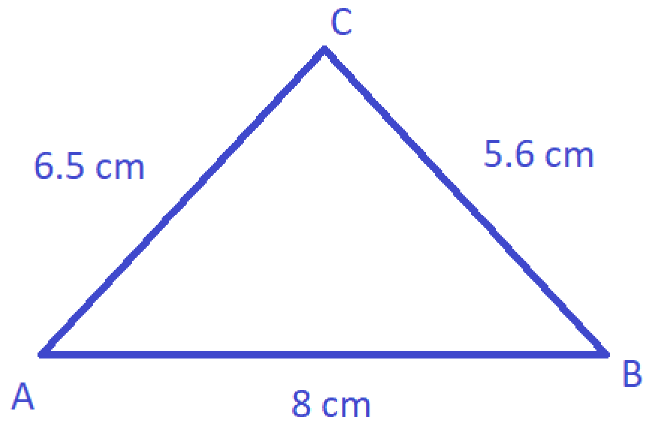
3. PQR is a right angle triangle at Q and PQ : QR = 3:2. Which is the least angle.

Solution:



Here, PQR is a right angle triangle at Q. Also given that
PQ : QR = 3:2
Let PQ = 3x, then, QR = 2x
It is clear that QR is the least side,
Then, we know that the least angle has least side
Opposite to it.
Hence ∠P is the least angle

4. **In Δ ABC, AB = 8 cm, BC = 5.6 cm and CA = 6.5 cm. Which is (i) the greatest angle ? (ii) the smallest angle ?**



Solution:

Given that AB = 8 cm, BC = 5.6 cm, CA = 6.5 cm.

Here AB is the greatest side

Then ∠C is the least angle

The greatest side has greatest angle opposite to it)

Also, BC Is the least side

Then ∠A is the least angle

(the least side has least angle opposite to it)

CHAPTER TEST

1. In triangles ABC and DEF, ∠A = ∠D, ∠B = ∠E and AB = EF. Will the two triangles be congruent? Give reasons for your answer.

Solution:

In ΔABC and ΔDEF

∠A = ∠D

∠B = ∠E

AB = EF

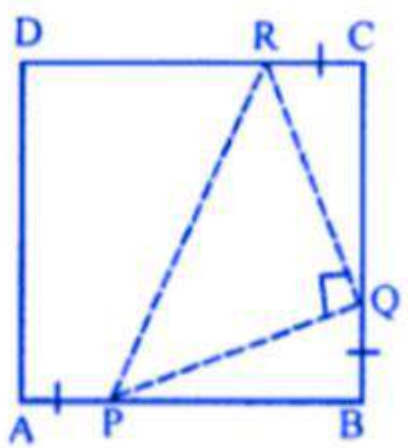
In ΔABC, two angles and included side is

Given but in ΔDEF, corresponding angles are

Equal but side is not included of there angle.

Triangles Cannot be congruent.

2. In the adjoining figure, ABCD is a square. P, Q and R are points on the sides AB, BC and CD respectively such that AP= BQ = CR and ∠PQR = 90°. Prove that (a) ΔPBQ ≅ ΔQCR (b) PQ = QR (c) ∠PRQ = 45°



Solution:

Given : in the given figure, ABCD is a square

P,Q and R are the Points on the sides AB,

BC and CD respectively such that

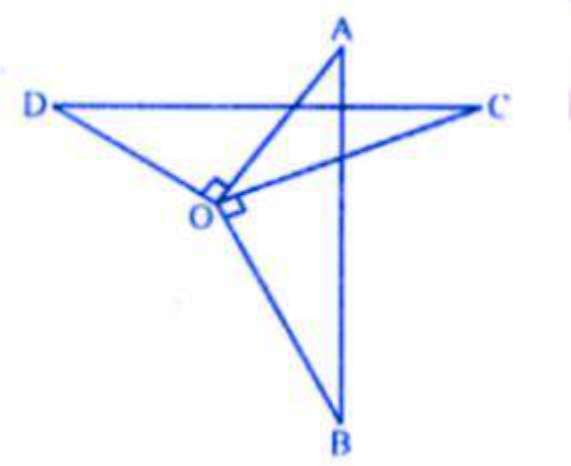
AP = BQ = CR, ∠PQR = 90°

To prove: (a) ΔPBQ = ΔQCR

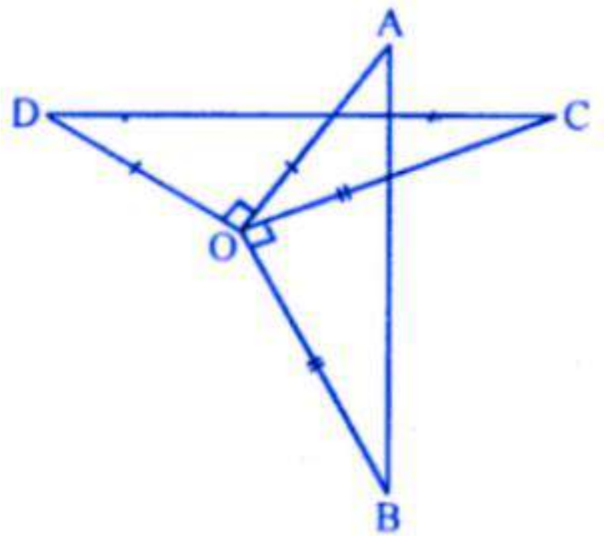
(b) PQ = QR

(c) ∠PQR = 45°
Proof : AB = BC = CD (Sides of Square)
And AP = BQ = CR (Given)
Subtracting, we get
AB – AP = BC – BQ = CD – CR
(PB = QC = RD)
Now in ΔPBQ and ∠QCR
PB = QC (Proved)
BQ = CR (Given)
∠B = ∠C (Each 90°)
ΔPBQ ≅ ΔQCR
PQ = QR
But ∠PQR = 90°
∠RPQ = ∠PQR
(Angles opposite to equal angles)
But ∠RPQ + ∠PRQ = 90°
∠RPQ = ∠PQR = 90°
∠RPQ = ∠PRQ = 90°/2 = 45°

3. In the given figure, OA ⊥ OD, OC X OB, OD = OA and OB = OC. Prove that AB = CD.

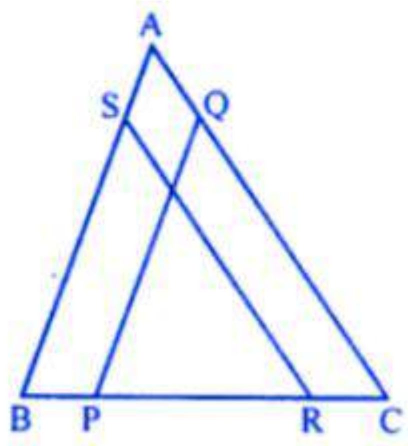


Solution :
Given : in the figure OA ⊥ OD, OC ⊥ OB.

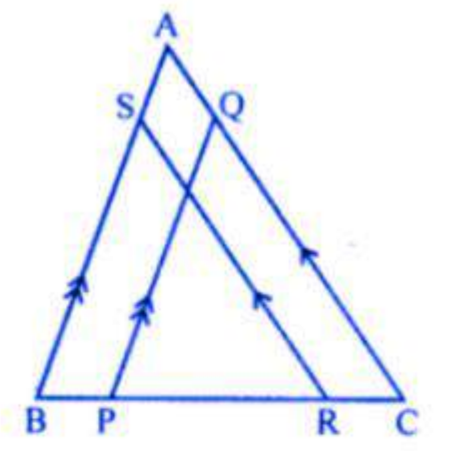


To prove : AB = CD
Proof : ∠AOD = ∠COB (each 90°)
Adding ∠AOC
∠AOD + ∠AOC = ∠AOC + ∠COB
∠COD = ∠AOB
Now, in ΔAOB and ΔDOC
OA = OD (Given)
OB = OC (Given)
∠AOB = ∠COD (Proved)
ΔAOB ≅ ΔDOC
AB = CD

4. In the given figure, PQ || BA and RS CA. If BP = RC, prove that: (i) ΔBSR ≅ ΔPQC (ii) BS = PQ (iii) RS = CQ.



Solution:
PQ ∥ BA, RS ∥ CA
BP = RC



To prove :
(i) ΔBSR ≅ ΔPQC
(ii) RS = CQ
Proof : BP = RC
BC – RC = BC – BP
BR = PC
Now, in ΔBSR and ΔPQC
∠B = ∠P (Corresponding angles)
∠R = ∠C (Corresponding angles)
BR = PC (Proved)

ΔBSR ≅ ΔPQC

BS = PQ

RS = CQ