EXERCISE 3.1

By using standard formulae, expand the following (1 to 9):

1. (i) $(2x + 7y)^2$

(ii) $(1/2\ x + 2/3\ y)^2$

Solution:

(i) $(2x + 7y)^2$

It can be written as

$= (2x)^2 + 2 \times 2x \times 7y + (7y)^2$

So we get

$= 4x^2 + 28xy + 49y^2$

(ii) $(1/2\ x + 2/3\ y)^2$

It can be written as

$= (1/2\ x)^2 + 2 \times ½x + 2/3y + (2/3\ y)^2$

So we get

$= ¼\ x^2 + 2/3\ xy + 4/9\ y^2$

2. (i) $(3x + 1/2x)^2$

(ii) $(3x^2y + 5z)^2$

Solution:

(i) $(3x + 1/2x)^2$

It can be written as

$= (3x)^2 + 2 \times 3x \times 1/2x + (1/2x)^2$

So we get

$= 9x^2 + 3 + 1/4x^2$

$= 9x^2 + 1/4x^2 + 3$

(ii) $(3x^2y + 5z)^2$

It can be written as

$= (3x^2y)^2 + 2 \times 3x^2y \times 5z + (5z)^2$

So we get

$= 9x^4y^2 + 30x^2yz + 25z^2$

3. (i) $(3x – 1/2x)^2$

(ii) $(1/2\ x – 3/2\ y)^2$

Solution:

(i) $(3x - 1/2x)^2$
It can be written as
$= (3x)^2 - 2 \times 3x \times 1/2x + (1/2x)^2$
So we get
$= 9x^2 - 3 + 1/4x^2$
$= 9x^2 + 1/4x^2 - 3$

(ii) $(1/2\ x - 3/2\ y)^2$
It can be written as
$= (1/2\ x)^2 + (3/2\ y)^2 - 2 \times ½\ x \times 3/2\ y$
So we get
$= ¼\ x^2 + 9/4\ y^2 - 3/2\ xy$
$= ¼\ x^2 - 3/2\ xy + 9/4\ y^2$

4. (i) (x + 3) (x + 5)
(ii) (x + 3) (x – 5)
(iii) (x – 7) (x + 9)
(iv) (x – 2y) (x – 3y)
Solution:

(i) $(x + 3)(x + 5)$
By further calculation
$= x^2 + (3 + 5)x + 3 \times 5$
So we get
$= x^2 + 8x + 15$

(ii) $(x + 3)(x - 5)$
By further calculation
$= x^2 + (3 - 5)x - 3 \times 5$
So we get
$= x^2 - 2x - 15$

(iii) $(x - 7)(x + 9)$
By further calculation
$= x^2 - (7 - 9)x - 7 \times 9$
So we get
$= x^2 + 2x - 63$

(iv) $(x - 2y)(x - 3y)$

By further calculation
$= x^2 - (2y + 3y)x + 2y \times 3y$
So we get
$= x^2 - 5xy + 6y^2$

5. (i) $(x - 2y - z)^2$
(ii) $(2x - 3y + 4z)^2$
Solution:

(i) $(x - 2y - z)^2$
It can be written as
$= [x + (-2y) + (-z)]^2$
By further calculation
$= (x)^2 + (-2y)^2 + (-z)^2 + 2 \times x \times (-2y) + 2 \times (-2y) \times (-z) + 2 \times (-z) \times x$
So we get
$= x^2 + 4y^2 + z^2 - 4xy + 4yz - 2zx$

(ii) $(2x - 3y + 4z)^2$
It can be written as
$= [2x + (-3y) + 4z]^2$
By further calculation
$= (2x)^2 + (-3y)^2 + (4z)^2 + 2 \times 2x \times (-3y) + 2 \times (-3y) \times 4z + 2 \times 4z \times 2x$
So we get
$= 4x^2 + 9y^2 + 16z^2 -12xy - 24yz + 16zx$

6. (i) $(2x + 3/x - 1)^2$
(ii) $(2/3\ x - 3/2x - 1)^2$
Solution:

(i) $(2x + 3/x - 1)^2$
It can be written as
$= [2x + 3/x + (-1)]^2$
By further calculation
$= (2x)^2 + (3/x)^2 + (-1)^2 + 2 \times 2x \times 3/x + 2 \times 3/x \times (-1) + 2 \times (-1) \times 2x$
So we get
$= 4x^2 + 9/x^2 + 1 + 12 - 6/x - 4x$
$= 4x^2 + 9/x^2 + 13 - 6/x - 4x$

(ii) $(2/3\ x - 3/2x - 1)^2$

It can be written as

$= [2/3\ x - 3/2x - 1]^2$

By further calculation

$= (2/3\ x)^2 + (-3/2x)^2 + (-1)^2 + 2 \times 2/3\ x \times (-3/2x) + 2 \times (-3/2x) \times (-1) + 2 \times (-1) \times (2/3\ x)$

So we get

$= 4/9\ x^2 + 9/4x^2 + 1 - 2 + 3/x - 4/3\ x$

$= 4/9\ x^2 + 9/4x^2 - 1 - 4/3\ x + 3/x$

7. (i) $(x + 2)^3$

(ii) $(2a + b)^3$

Solution:

(i) $(x + 2)^3$

It can be written as

$= x^3 + 2^3 + 3 \times x \times 2\ (x + 2)$

By further calculation

$= x^3 + 8 + 6x\ (x + 2)$

So we get

$= x^3 + 8 + 6x^2 + 12x$

$= x^3 + 6x^2 + 12x + 8$

(ii) $(2a + b)^3$

It can be written as

$= (2a)^3 + b^3 + 3 \times 2a \times b\ (2a + b)$

By further calculation

$= 8a^3 + b^3 + 6ab\ (2a + b)$

So we get

$= 8a^3 + b^3 + 12a^2b + 6ab^2$

8. (i) $(3x + 1/x)^3$

(ii) $(2x - 1)^3$

Solution:

(i) $(3x + 1/x)^3$

It can be written as

$= (3x)^3 + (1/x)^3 + 3 \times 3x \times 1/x\ (3x + 1/x)$

By further calculation

$= 27x^3 + 1/x^3 + 9\ (3x + 1/x)$

So we get

$= 27x^3 + 1/x^3 + 27x + 9/x$

(ii) $(2x - 1)^3$
It can be written as
$= (2x)^3 - 1^3 - 3 \times 2x \times 1\ (2x - 1)$
By further calculation
$= 8x^3 - 1 - 6x\ (2x - 1)$
So we get
$= 8x^3 - 1 - 12x^2 + 6x$
$= 8x^3 - 12x^2 + 6x - 1$

9. (i) $(5x - 3y)^3$
(ii) $(2x - 1/3y)^3$
Solution:

(i) $(5x - 3y)^3$
It can be written as
$= (5x)^3 - (3y)^3 - 3 \times 5x \times 3y\ (5x - 3y)$
By further calculation
$= 125x^3 - 27y^3 - 45xy\ (5x - 3y)$
So we get
$= 125x^3 - 27y^3 - 225x^2y + 135xy^2$

(ii) $(2x - 1/3y)^3$
It can be written as
$= (2x)^3 - (1/3y)^3 - 3 \times 2x \times 1/3y\ (2x - 1/3y)$
By further calculation
$= 8x^3 - 1/27y^3 - 2x/y\ (2x - 1/3y)$
So we get
$= 8x^3 - 1/27y^3 - 4x^2/y + 2x/3y^2$

Simplify the following (10 to 19):
10. (i) $(a + b)^2 + (a - b)^2$
(ii) $(a + b)^2 - (a - b)^2$
Solution:

(i) $(a + b)^2 + (a - b)^2$
It can be written as
$= (a^2 + b^2 + 2ab) + (a^2 + b^2 - 2ab)$

By further calculation

$= a^2 + b^2 + 2ab + a^2 + b^2 – 2ab$

So we get

$= 2a^2 + 2b^2$

Taking 2 as common

$= 2 (a^2 + b^2)$

(ii) $(a + b)^2 – (a – b)^2$

It can be written as

$= (a^2 + b^2 + 2ab) – (a^2 + b^2 – 2ab)$

By further calculation

$= a^2 + b^2 + 2ab – a^2 – b^2 + 2ab$

So we get

$= 4ab$

11. (i) $(a + 1/a)^2 + (a – 1/a)^2$
(ii) $(a + 1/a)^2 – (a – 1/a)^2$
Solution:

(i) $(a + 1/a)^2 + (a – 1/a)^2$

It can be written as

$= [a^2 + (1/a)^2 + 2 \times a \times 1/a] + [a^2 + (1/a)^2 – 2 \times a \times 1/a]$

By further calculation

$= [a^2 + 1/a^2 + 2] + [a^2 + 1/a^2 – 2]$

So we get

$= a^2 + 1/a^2 + 2 + a^2 + 1/a^2 – 2$

$= 2a^2 + 2/a^2$

Taking 2 as common

$= 2 (a^2 + 1/a^2)$

(ii) $(a + 1/a)^2 – (a – 1/a)^2$

It can be written as

$= [a^2 + (1/a)^2 + 2 \times a \times 1/a] - [a^2 + (1/a)^2 – 2 \times a \times 1/a]$

By further calculation

$= [a^2 + 1/a^2 + 2] - [a^2 + 1/a^2 – 2]$

So we get

$= a^2 + 1/a^2 + 2 – a^2 – 1/a^2 + 2$

$= 4$

12. (i) $(3x - 1)^2 - (3x - 2)(3x + 1)$
(ii) $(4x + 3y)^2 - (4x - 3y)^2 - 48xy$
Solution:

(i) $(3x - 1)^2 - (3x - 2)(3x + 1)$
It can be written as
$= [(3x)^2 + 1^2 - 2 \times 3x \times 1] - [(3x)^2 - (2 - 1)(3x) - 2 \times 1]$
By further calculation
$= [9x^2 + 1 - 6x] - [9x^2 - 3x - 2]$
So we get
$= 9x^2 + 1 - 6x - 9x^2 + 3x + 2$
$= -3x + 3$
$= 3 - 3x$

(ii) $(4x + 3y)^2 - (4x - 3y)^2 - 48xy$
It can be written as
$= [(4x)^2 + (3y)^2 + 2 \times 4x \times 4y] - [(4x)^2 + (3y)^2 - 2 \times 4x \times 3y] - 48xy$
By further calculation
$= [16x^2 + 9y^2 + 24xy] - [16x^2 + 9y^2 - 24xy] - 48xy$
So we get
$= 16x^2 + 9y^2 + 24xy - 16x^2 - 9y^2 + 24xy - 48xy$
$= 0$

13. (i) $(7p + 9q)(7p - 9q)$
(ii) $(2x - 3/x)(2x + 3/x)$
Solution:

(i) $(7p + 9q)(7p - 9q)$
It can be written as
$= (7p)^2 - (9q)^2$
$= 49p^2 - 81q^2$

(ii) $(2x - 3/x)(2x + 3/x)$
It can be written as
$= (2x)^2 - (3/x)^2$
$= 4x^2 - 9/x^2$

14. (i) $(2x - y + 3)(2x - y - 3)$
(ii) $(3x + y - 5)(3x - y - 5)$

Solution:

(i) $(2x – y + 3) (2x – y – 3)$
It can be written as
$= [(2x – y) + 3] [(2x – y) – 3]$
$= (2x – y)^2 - 3^2$
By further calculation
$= (2x)^2 + y^2 – 2 \times 2x \times y – 9$
So we get
$= 4x^2 + y^2 – 4xy – 9$

(ii) $(3x + y – 5) (3x – y – 5)$
It can be written as
$= [(3x – 5) + y] [(3x – 5) – y]$
$= (3x – 5^2) – y^2$
By further calculation
$= (3x)^2 + 5^2 – 2 \times 3x \times 5 – y^2$
So we get
$= 9x^2 + 25 – 30x – y^2$
$= 9x^2 – y^2 – 30x + 25$

15. (i) $(x + 2/x – 3) (x – 2/x – 3)$
(ii) $(5 – 2x) (5 + 2x) (25 + 4x^2)$
Solution:

(i) $(x + 2/x – 3) (x – 2/x – 3)$
It can be written as
$= [(x – 3) + (2/x)] [(x – 3) – (2/x)]$
$= (x – 3)^2 – (2/x)^2$
Expanding using formula
$= x^2 + 9 – 2 \times x \times 3 – 4/x^2$
By further calculation
$= x^2 + 9 – 6x – 4/x^2$
So we get
$= x^2 – 4/x^2 – 6x + 9$

(ii) $(5 – 2x) (5 + 2x) (25 + 4x^2)$
It can be written as
$= [5^2 – (2x)^2] (25 + 4x^2)$

By further calculation
$= (25 – 4x^2) (25 + 4x^2)$
So we get
$= 25^2 – (4x^2)^2$
$= 625 – 16x^4$

16. (i) (x + 2y + 3) (x + 2y + 7)
(ii) (2x + y + 5) (2x + y – 9)
(iii) (x – 2y – 5) (x – 2y + 3)
(iv) (3x – 4y – 2) (3x – 4y – 6)
Solution:

(i) $(x + 2y + 3) (x + 2y + 7)$
Consider $x + 2y = a$
$(a + 3) (a + 7) = a^2 + (3 + 7) a + 3 \times 7$
By further calculation
$= a^2 + 10a + 21$
Substituting the value of a
$= (x + 2y)^2 + 10 (x + 2y) + 21$
By expanding using formula
$= x^2 + 4y^2 + 2 \times x \times 2y + 10x + 20y + 21$
So we get
$= x^2 + 4y^2 + 4xy + 10x + 20y + 21$

(ii) $(2x + y + 5) (2x + y – 9)$
Consider $2x + y = a$
$(a + 5) (a – 9) = a^2 + (5 – 9) a + 5 \times (-9)$
By further calculation
$= a^2 – 4a – 45$
Substituting the value of a
$= (2x + y)^2 – 4 (2x + y) – 45$
By expanding using formula
$= 4x^2 + y^2 + 2 \times 2x \times y - 8x – 4y – 45$
So we get
$= 4x^2 + y^2 + 4xy – 8x – 4y – 45$

(iii) $(x – 2y – 5) (x – 2y + 3)$
Consider $x – 2y = a$
$(a – 5) (a + 3) = a^2 + (- 5 + 3) a + (-5) (3)$

By further calculation
$= a^2 – 2a – 15$
Substituting the value of a
$= (x – 2y)^2 – 2 (x – 2y) – 15$
By expanding using formula
$= x^2 + 4y^2 – 2 \times x \times 2y – 2x + 4y – 15$
So we get
$= x^2 + 4y^2 – 4xy – 2x + 4y – 15$

(iv) $(3x – 4y – 2) (3x – 4y – 6)$
Consider $3x – 4y = a$
$(a – 2) (a – 6) = a^2 (- 2 – 6)a + (-2) (-6)$
By further calculation
$= a^2 – 8a + 12$
Substituting the value of a
$= (3x – 4y)^2 – 8 (3x – 4y) + 12$
Expanding using formula
$= 9x^2 + 16y^2 – 2 \times 3x \times 4y – 24x + 32y + 12$
So we get
$= 9x^2 + 16y^2 – 24xy – 24x + 32y + 12$

17. (i) $(2p + 3q) (4p^2 – 6pq + 9q^2)$
(ii) $(x + 1/x) (x^2 - 1 + 1/x^2)$
Solution:

(i) $(2p + 3q) (4p^2 – 6pq + 9q^2)$
It can be written as
$= (2p + 3q) [(2p)^2 - 2p \times 3q + (3q)^2]$
By further simplification
$= (2p)^3 + (3q)^3$
$= 8p^3 + 27q^3$

(ii) $(x + 1/x) (x^2 - 1 + 1/x^2)$
It can be written as
$= (x + 1/x) [x^2 – x \times 1/x + (1/x)^2]$
By further simplification
$= x^3 + (1/x)^3$
$= x^3 + 1/x^3$

18. (i) $(3p - 4q)(9p^2 + 12pq + 16q^2)$
(ii) $(x - 3/x)(x^2 + 3 + 9/x^2)$
Solution:

(i) $(3p - 4q)(9p^2 + 12pq + 16q^2)$
It can be written as
$= (3p - 4q)[(3p)^2 + 3p \times 4q + (4q)^2]$
By further simplification
$= (3p)^3 - (4q)^3$
$= 27p^3 - 64q^3$

(ii) $(x - 3/x)(x^2 + 3 + 9/x^2)$
It can be written as
$= (x - 3/x)[x^2 + x \times 3/x + (3/x)^2]$
By further simplification
$= x^3 - (3/x)^3$
$= x^3 - 27/x^3$

19. $(2x + 3y + 4z)(4x^2 + 9y^2 + 16z^2 - 6xy - 12yz - 8zx)$.
Solution:

$(2x + 3y + 4z)(4x^2 + 9y^2 + 16z^2 - 6xy - 12yz - 8zx)$
It can be written as
$= (2x + 3y + 4z)((2x)^2 + (3y)^2 + (4z)^2 - 2x \times 3y - 3y \times 4z - 4z \times 2x)$
By further calculation
$= (2x)^3 + (3y)^3 + (4z)^3 - 3 \times 2x \times 3y \times 4z$
So we get
$= 8x^3 + 27y^3 + 64z^3 - 72xyz$

20. Find the product of the following:
(i) $(x + 1)(x + 2)(x + 3)$
(ii) $(x - 2)(x - 3)(x + 4)$
Solution:

(i) $(x + 1)(x + 2)(x + 3)$
It can be written as
$= x^3 + (1 + 2 + 3)x^2 + (1 \times 2 + 2 \times 3 + 3 \times 1)x + 1 \times 2 \times 3$
By further calculation
$= x^3 + 6x^2 + (2 + 6 + 3)x + 6$

So we get
$= x^3 + 6x^2 + 11x + 6$

(ii) $(x – 2) (x – 3) (x + 4)$
It can be written as
$= x^3 + (- 2 – 3 + 4) x^2 + [(-2) \times (-3) + (-3) \times 4 + 4 \times (-2)]x + (-2) (-3) (4)$
By further calculation
$= x^3 – x^2 + (6 – 12 – 8)x + 24$
$= x^3 – x^2 – 14x + 24$

21. Find the coefficient of x^2 and x in the product of (x – 3) (x + 7) (x – 4).
Solution:

It is given that
$(x – 3) (x + 7) (x – 4)$
By further calculation
$= x^3 + (- 3 + 7 – 4) x^2 + [(-3) (7) + 7 \times (-4) + (-4) (-3) + (-3) (7) (-4)]$
It can be written as
$= x^3 + 0x^2 + (- 21 – 28 + 12) x + 84$
So we get
$= x^3 + 0x^2 – 37x + 84$

Hence, coefficient of x^2 is zero and coefficient of x is – 3.

22. If $a^2 + 4a + x = (a + 2)^2$, find the value of x.
Solution:

It is given that
$a^2 + 4a + x = (a + 2)^2$
By expanding using formula
$a^2 + 4a + x = a^2 + 2^2 + 2 \times a \times 2$
By further calculation
$a^2 + 4a + x = a^2 + 4 + 4a$
So we get
$x = a^2 + 4 + 4a – a^2 – 4a$
$x = 4$

23. Use $(a + b)^2 = a^2 + 2ab + b^2$ to evaluate the following:
(i) $(101)^2$

(ii) $(1003)^2$
(iii) $(10.2)^2$
Solution:

(i) $(101)^2$
It can be written as
$= (100 + 1)^2$
Expanding using formula
$= 100^2 + 1^2 + 2 \times 100 \times 1$
By further calculation
$= 10000 + 1 + 200$
$= 10201$

(ii) $(1003)^2$
It can be written as
$= (1000 + 3)^2$
Expanding using formula
$= 1000^2 + 3^2 + 2 \times 1000 \times 3$
By further calculation
$= 1000000 + 9 + 6000$
$= 1006009$

(iii) $(10.2)^2$
It can be written as
$= (10 + 0.2)^2$
Expanding using formula
$= 10^2 + 0.2^2 + 2 \times 10 \times 0.2$
By further calculation
$= 100 + 0.04 + 4$
$= 104.04$

24. Use $(a - b)^2 = a^2 - 2ab - b^2$ to evaluate the following:
(i) $(99)^2$
(ii) $(997)^2$
(iii) $(9.8)^2$
Solution:

(i) $(99)^2$
It can be written as

$= (100 - 1)^2$
Expanding using formula
$= 100^2 - 2 \times 100 \times 1 + 1^2$
By further calculation
$= 10000 - 200 + 1$
$= 9801$

(ii) $(997)^2$
It can be written as
$= (1000 - 3)^2$
Expanding using formula
$= 1000^2 - 2 \times 1000 \times 3 + 3^2$
By further calculation
$= 1000000 - 6000 + 9$
$= 994009$

(iii) $(9.8)^2$
It can be written as
$= (10 - 0.2)^2$
Expanding using formula
$= 10^2 - 2 \times 10 \times 0.2 + 0.2^2$
By further calculation
$= 100 - 4 + 0.04$
$= 96.04$

25. By using suitable identities, evaluate the following:
(i) $(103)^3$
(ii) $(99)^3$
(iii) $(10.1)^3$
Solution:

(i) $(103)^3$
It can be written as
$= (100 + 3)^3$
Expanding using formula
$= 100^3 + 3^3 + 3 \times 100 \times 3\ (100 + 3)$
By further calculation
$= 1000000 + 27 + 900 \times 103$
So we get

= 1000000 + 27 + 92700
= 1092727

(ii) $(99)^3$
It can be written as
$= (100 - 1)^3$
Expanding using formula
$= 100^3 - 1^3 - 3 \times 100 \times 1\ (100 - 1)$
By further calculation
$= 1000000 - 1 - 300 \times 99$
So we get
= 1000000 – 1 – 29700
= 1000000 – 29701
= 970299

(iii) $(10.1)^3$
It can be written as
$= (10 + 0.1)^3$
Expanding using formula
$= 10^3 + 0.1^3 + 3 \times 10 \times 0.1\ (10 + 0.1)$
By further calculation
$= 1000 + 0.001 + 3 \times 10.1$
So we get
= 1000 + 0.001 + 30.3
= 1030.301

26. If 2a – b + c = 0, prove that $4a^2 - b^2 + c^2 + 4ac = 0$.
Solution:

It is given that
$2a - b + c = 0$
$2a + c = b$
By squaring on both sides
$(2a + c)^2 = b^2$
Expanding using formula
$(2a)^2 + 2 \times 2a \times c + c^2 = b^2$
By further calculation
$4a^2 + 4ac + c^2 = b^2$
So we get

$4a^2 - b^2 + c^2 + 4ac = 0$

Hence, it is proved.

27. If $a + b + 2c = 0$, prove that $a^3 + b^3 + 8c^3 = 6abc$.
Solution:

It is given that
$a + b + 2c = 0$
We can write it as
$a + b = - 2c$
By cubing on both sides
$(a + b)^3 = (-2c)^3$
Expanding using formula
$a^3 + b^3 + 3ab (a + b) = -8c^3$
Substituting the value of a + b
$a^3 + b^3 + 3ab (-2c) = -8c^3$
So we get
$a^3 + b^3 + 8c^3 = 6abc$

Hence, it is proved.

28. If $a + b + c = 0$, then find the value of $a^2/bc + b^2/ca + c^2/ab$.
Solution:

It is given that
$a + b + c = 0$
We can write it as
$a^3 + b^3 + c^3 - 3abc = 0$
$a^3 + b^3 + c^3 = 3abc$
Now dividing by abc on both sides
$a^3/abc + b^3/abc + c^3/abc = 3$
By further calculation
$a^2/bc + b^2/ac + c^2/ab = 3$

Therefore, the value of $a^2/bc + b^2/ca + c^2/ab$ is 3.

29. If $x + y = 4$, then find the value of $x^3 + y^3 + 12xy - 64$.
Solution:

It is given that

$x + y = 4$

By cubing on both sides

$(x + y)^3 = 4^3$

Expanding using formula

$x^3 + y^3 + 3xy (x + y) = 64$

Substituting the value of x + y

$x^3 + y^3 + 3xy (4) = 64$

So we get

$x^3 + y^3 + 12xy – 64 = 0$

Hence, the value of $x^3 + y^3 + 12xy – 64$ is 0.

30. Without actually calculating the cubes, find the values of:
(i) $(27)^3 + (-17)^3 + (-10)^3$
(ii) $(-28)^3 + (15)^3 + (13)^3$
Solution:

(i) $(27)^3 + (-17)^3 + (-10)^3$

Consider a = 27, b = - 17 and c = - 10

We know that

$a + b + c = 27 – 17 – 10 = 0$

So $a + b + c = 0$

$a^3 + b^3 + c^3 = 3abc$

Substituting the values

$27^3 + (-17)^3 + (-10)^3 = 3 (27) (-17) (- 10)$

$= 13770$

(ii) $(-28)^3 + (15)^3 + (13)^3$

Consider a = - 28, b = 15 and c = 13

We know that

$a + b + c = - 28 + 15 + 13 = 0$

So $a + b + c = 0$

$a^3 + b^3 + c^3 = 3abc$

Substituting the values

$(-28)^3 + (15)^3 + (13)^3 = 3 (- 28) (15) (13)$

$= - 16380$

31. Using suitable identity, find the value of:

$$\frac{\mathbf{86 \times 86 \times 86 + 14 \times 14 \times 14}}{\mathbf{86 \times 86 - 86 \times 14 + 14 \times 14}}$$

Solution:

Consider x = 86 and y = 14

$$\frac{86 \times 86 \times 86 + 14 \times 14 \times 14}{86 \times 86 - 86 \times 14 + 14 \times 14}$$

$It\ can\ be\ written\ as$

$$= \frac{x^3 + y^3}{x^2 - xy + y^2}$$

$So\ we\ get$

$$= \frac{(x + y)(x^2 - xy + y^2)}{x^2 - xy + y^2}$$

= x + y

Substituting the values

= 86 + 14

= 100

EXERCISE 3.2

1. If $x - y = 8$ and $xy = 5$, find $x^2 + y^2$.

Solution:

We know that

$(x - y)^2 = x^2 + y^2 - 2xy$

It can be written as

$x^2 + y^2 = (x - y)^2 + 2xy$

It is given that

$x - y = 8$ and $xy = 5$

Substituting the values

$x^2 + y^2 = 8^2 + 2 \times 5$

So we get

$= 64 + 10$

$= 74$

2. If $x + y = 10$ and $xy = 21$, find $2 (x^2 + y^2)$.

Solution:

We know that

$(x + y)^2 = x^2 + y^2 + 2xy$

It can be written as

$x^2 + y^2 = (x + y)^2 - 2xy$

It is given that

$(x + y) = 10$ and $xy = 21$

Substituting the values

$x^2 + y^2 = 10^2 - 2 \times 21$

By further calculation

$= 100 - 42$

$= 58$

Here

$2 (x^2 + y^2) = 2 \times 58 = 116$

3. If $2a + 3b = 7$ and $ab = 2$, find $4a^2 + 9b^2$.

Solution:

We know that
$(2a + 3b)^2 = 4a^2 + 9b^2 + 12ab$
It can be written as
$4a^2 + 9b^2 = (2a + 3b)^2 – 12ab$

It is given that
$2a + 3b = 7$
$ab = 2$
Substituting the values
$4a^2 + 9b^2 = 7^2 – 12 \times 2$
By further calculation
$= 49 – 24$
$= 25$

4. If $3x – 4y = 16$ and $xy = 4$, find the value of $9x^2 + 16y^2$.
Solution:

We know that
$(3x – 4y)^2 = 9x^2 + 16y^2 – 24xy$
It can be written as
$9x^2 + 16y^2 = (3x – 4y)^2 + 24xy$

It is given that
$3x – 4y = 16$ and $xy = 4$
Substituting the values
$9x^2 + 16y^2 = 16^2 + 24 \times 4$
By further calculation
$= 256 + 96$
$= 352$

5. If $x + y = 8$ and $x – y = 2$, find the value of $2x^2 + 2y^2$.
Solution:

We know that
$2 (x^2 + y^2) = (x + y)^2 + (x – y)^2$

It is given that
$x + y = 8$ and $x – y = 2$
Substituting the values

$2x^2 + 2y^2 = 8^2 + 2^2$
By further calculation
$= 64 + 4$
$= 68$

6. If $a^2 + b^2 = 13$ and $ab = 6$, find
(i) a + b
(ii) a – b
Solution:

(i) We know that
$(a + b)^2 = a^2 + b^2 + 2ab$
Substituting the values
$= 13 + 2 \times 6$
So we get
$= 13 + 12$
$= 25$

Here
$a + b = \pm \sqrt{25} = \pm 5$

(ii) We know that
$(a - b)^2 = a^2 + b^2 - 2ab$
Substituting the values
$= 13 - 2 \times 6$
So we get
$= 13 - 12$
$= 1$

Here
$a - b = \pm \sqrt{1} = \pm 1$

7. If a + b = 4 and ab = -12, find
(i) a – b
(ii) $a^2 - b^2$.
Solution:

(i) We know that
$(a - b)^2 = a^2 + b^2 - 2ab$

It can be written as
$(a - b)^2 = a^2 + b^2 + 2ab - 4ab$
$(a - b)^2 = (a + b)^2 - 4ab$

It is given that
$a + b = 4$ and $ab = -12$
Substituting the values
$(a - b)^2 = 4^2 - 4(-12)$
By further calculation
$(a - b)^2 = 16 + 48 = 64$
So we get
$(a - b) = \pm \sqrt{64} = \pm 8$

(ii) We know that
$a^2 - b^2 = (a + b)(a - b)$
Substituting the values
$a^2 - b^2 = 4 \times \pm 8$
$a^2 - b^2 = \pm 32$

8. If $p - q = 9$ and $pq = 36$, evaluate
(i) $p + q$
(ii) $p^2 - q^2$.
Solution:

(i) We know that
$(p + q)^2 = p^2 + q^2 + 2pq$
It can be written as
$(p + q)^2 = p^2 + q^2 - 2pq + 4pq$
$(p + q)^2 = (p - q)^2 + 4pq$

It is given that
$p - q = 9$ and $pq = 36$
Substituting the values
$(p + q)^2 = 9^2 + 4 \times 36$
By further calculation
$(p + q)^2 = 81 + 144 = 225$
So we get
$p + q = \pm \sqrt{225} = \pm 15$

(ii) We know that
$p^2 - q^2 = (p - q)(p + q)$
Substituting the values
$p^2 - q^2 = 9 \times \pm 15$
$p^2 - q^2 = \pm 135$

9. If $x + y = 6$ and $x - y = 4$, find
(i) $x^2 + y^2$
(ii) xy
Solution:

We know that
$(x + y)^2 - (x - y)^2 = 4xy$
Substituting the values
$6^2 - 4^2 = 4xy$
By further calculation
$36 - 16 = 4xy$
$20 = 4xy$
$4xy = 20$
So we get
$xy = 20/4 = 5$

(i) $x^2 + y^2 = (x + y)^2 - 2xy$
Substituting the values
$= 6^2 - 2 \times 5$
By further calculation
$= 36 - 10$
$= 26$

(ii) $xy = 5$

10. If $x - 3 = 1/x$, find the value of $x^2 + 1/x^2$.
Solution:

It is given that
$x - 3 = 1/x$
We can write it as
$x - 1/x = 3$

Here
$(x - 1/x)^2 = x^2 + 1/x^2 - 2$
So we get
$x^2 + 1/x^2 = (x - 1/x)^2 + 2$
Substituting the values
$x^2 + 1/x^2 = 3^2 + 2$
By further calculation
$= 9 + 2$
$= 11$

11. If x + y = 8 and xy = 3 ¾, find the values of
(i) $x - y$
(ii) $3(x^2 + y^2)$
(iii) $5(x^2 + y^2) + 4(x - y)$.
Solution:

(i) We know that
$(x - y)^2 = x^2 + y^2 - 2xy$
It can be written as
$(x - y)^2 = x^2 + y^2 + 2xy - 4xy$
$(x - y)^2 = (x + y)^2 - 4xy$

It is given that
$x + y = 8$ and $xy = 3\tfrac{3}{4} = 15/4$
Substituting the values
$(x - y)^2 = 8^2 - 4 \times 15/4$
So we get
$(x - y)^2 = 65 - 15 = 49$
$x - y = \pm \sqrt{49} = \pm 7$

(ii) We know that
$(x + y)^2 = x^2 + y^2 + 2xy$
We can write it as
$x^2 + y^2 = (x + y)^2 - 2xy$

It is given that
$x + y = 8$ and $xy = 3\tfrac{3}{4} = 15/4$
Substituting the values
$x^2 + y^2 = 8^2 - 2 \times 15/4$

So we get
$x^2 + y^2 = 64 – 15/2$
Taking LCM
$x^2 + y^2 = (128 – 15)/ 2 = 113/2$

We get
$3 (x^2 + y^2) = 3 \times 113/2 = 339/2 = 169 \frac{1}{2}$

(iii) We know that
$5 (x^2 + y^2) + 4 (x – y) = 5 \times 113/2 + 4 \times \pm 7$
By further calculation
$= 565/2 \pm 28$
We can write it as
$= 565/2 + 28$ or $565/2 – 28$
$= 621/2$ or $509/2$
It can be written as
$= 310 \frac{1}{2}$ or $254 \frac{1}{2}$

12. If $x^2 + y^2 = 34$ and $xy = 10 \frac{1}{2}$, find the value of $2 (x + y)^2 + (x – y)^2$.
Solution:

It is given that
$x^2 + y^2 = 34$ and $xy = 10 \frac{1}{2} = 21/2$
We know that
$(x + y)^2 = x^2 + y^2 + 2xy$
Substituting the values
$(x + y)^2 = 34 + 2 (21/2)$
So we get
$(x + y)^2 = 55$ ….. (1)

We know that
$(x – y)^2 = x^2 + y^2 – 2xy$
Substituting the values
$(x – y)^2 = 34 - 2 (21/2)$
So we get
$(x – y)^2 = 34 – 21 = 13$ ….. (2)

Using both the equations
$2 (x + y)^2 + (x – y)^2 = 2 \times 55 + 13 = 123$

13. If $a - b = 3$ and $ab = 4$, find $a^3 - b^3$.
Solution:

We know that
$a^3 - b^3 = (a - b)^3 + 3ab(a + b)$
Substituting the values
$a^3 - b^3 = 3^3 + 3 \times 4 \times 3$
By further calculation
$a^3 - b^3 = 27 + 36 = 63$

14. If $2a - 3b = 3$ and $ab = 2$, find the value of $8a^3 - 27b^3$.
Solution:

We know that
$8a^3 - 27b^3 = (2a)^3 - (3b)^3$
According to the formula
$= (2a - 3b)^3 + 3 \times 2a \times 3b\,(2a - 3b)$
By further simplification
$= (2a - 3b)^3 + 18ab\,(2a - 3b)$
Substituting the values
$= 3^3 + 18 \times 2 \times 3$
By further calculation
$= 27 + 108$
$= 135$

15. If $x + 1/x = 4$, find the values of
(i) $x^2 + 1/x^2$
(ii) $x^4 + 1/x^4$
(iii) $x^3 + 1/x^3$
(iv) $x - 1/x$.
Solution:

(i) We know that
$(x + 1/x)^2 = x^2 + 1/x^2 + 2$
It can be written as
$x^2 + 1/x^2 = (x + 1/x)^2 - 2$
Substituting the values
$= 4^2 - 2$
$= 16 - 2$

= 14

(ii) We know that
$(x^2 + 1/x^2)^2 = x^4 + 1/x^4 + 2$
It can be written as
$x^4 + 1/x^4 = (x^2 + 1/x^2)^2 - 2$
Substituting the values
$= 14^2 - 2$
$= 196 - 2$
$= 194$

(iii) We know that
$x^3 + 1/x^3 = (x + 1/x)^3 - 3x\ (1/x)\ (x + 1/x)$
It can be written as
$(x + 1/x)^3 - 3(x + 1/x) = 4^3 - 3 \times 4$
By further calculation
$= 64 - 12$
$= 52$

(iv) We know that
$(x - 1/x)^2 = x^2 + 1/x^2 - 2$
Substituting the values
$= 14 - 2$
$= 12$
So we get
$x - 1/x = \pm 2\sqrt{3}$

16. If $x - 1/x = 5$, find the value of $x^4 + 1/x^4$.
Solution:

We know that
$(x - 1/x)^2 = x^2 + 1/x^2 - 2$
It can be written as
$x^2 + 1/x^2 = (x - 1/x)^2 + 2$
Substituting the values
$x^2 + 1/x^2 = 5^2 + 2 = 27$

Here
$x^4 + 1/x^4 = (x^2 + 1/x^2)^2 - 2$

Substituting the values
$x^4 + 1/x^4 = 27^2 - 2$
So we get
$= 729 - 2$
$= 727$

17. If $x - 1/x = \sqrt{5}$, find the values of
(i) $x^2 + 1/x^2$
(ii) $x + 1/x$
(iii) $x^3 + 1/x^3$
Solution:

(i) $x^2 + 1/x^2 = (x - 1/x)^2 + 2$
Substituting the values
$= (\sqrt{5})^2 + 2$
$= 5 + 2$
$= 7$

(ii) $(x + 1/x)^2 = x^2 + 1/x^2 + 2$
Substituting the values
$= 7 + 2$
$= 9$
Here
$(x + 1/x)^2 = 9$
So we get
$(x + 1/x) = \pm \sqrt{9} = \pm 3$

(iii) $x^3 + 1/x^3 = (x + 1/x)^3 - 3x (1/x) (x + 1/x)$
Substituting the values
$= (\pm 3)^3 - 3 (\pm 3)$
By further calculation
$= (\pm 27) - (\pm 9)$
$= \pm 18$

18. If $x + 1/x = 6$, find
(i) $x - 1/x$
(ii) $x^2 - 1/x^2$.
Solution:

(i) We know that
$(x - 1/x)^2 = x^2 + 1/x^2 - 2$
It can be written as
$(x - 1/x)^2 = x^2 + 1/x^2 + 2 - 4$
$(x - 1/x)^2 = (x + 1/x)^2 - 4$
Substituting the values
$(x - 1/x)^2 = 6^2 - 4 = 32$
So we get
$x - 1/x = \pm \sqrt{32} = \pm 4\sqrt{2}$

(ii) We know that
$x^2 - 1/x^2 = (x - 1/x)(x + 1/x)$
Substituting the values
$x^2 - 1/x^2 = (\pm 4\sqrt{2})(6) = \pm 24\sqrt{2}$

19. If $x + 1/x = 2$, prove that $x^2 + 1/x^2 = x^3 + 1/x^3 = x^4 + 1/x^4$.
Solution:

We know that
$x^2 + 1/x^2 = (x + 1/x) - 2$
Substituting the values
$x^2 + 1/x^2 = 2^2 - 2$
So we get
$x^2 + 1/x^2 = 4 - 2 = 2$ …. (1)

$x^3 + 1/x^3 = (x + 1/x)^3 - 3(x + 1/x)$
Substituting the values
$x^3 + 1/x^3 = 2^3 - 3 \times 2$
So we get
$x^3 + 1/x^3 = 8 - 6 = 2$ …… (2)

$x^4 + 1/x^4 = (x^2 + 1/x^2)^2 - 2$
Substituting the values
$x^4 + 1/x^4 = 2^2 - 2$
So we get
$x^4 + 1/x^4 = 4 - 2 = 2$ …. (3)

From equation (1), (2) and (3)
$x^2 + 1/x^2 = x^3 + 1/x^3 = x^4 + 1/x^4$

Hence, it is proved.

20. If $x - 2/x = 3$, find the value of $x^3 - 8/x^3$.
Solution:

We know that
$(x - 2/x)^3 = x^3 - 8/x^3 - 3 (x) (2/x) (x - 2/x)$
By further simplification
$(x - 2/x)^3 = x^3 - 8/x^3 - 6 (x - 2/x)$
It can be written as
$x^3 - 8/x^3 = (x - 2/x)^3 + 6 (x - 2/x)$
Substituting the values
$x^3 - 8/x^3 = 3^3 + 6 \times 3$
By further calculation
$x^3 - 8/x^3 = 27 + 18 = 45$

21. If $a + 2b = 5$, prove that $a^3 + 8b^3 + 30ab = 125$.
Solution:

We know that
$(a + 2b)^3 = a^3 + 8b^3 + 3 (a) (2b) (a + 2b)$
Substituting the values
$5^3 = a^3 + 8b^3 + 6ab (5)$
By further calculation
$125 = a^3 + 8b^3 + 30ab$

Therefore, $a^3 + 8b^3 + 30ab = 125$.

22. If $a + 1/a = p$, prove that $a^3 + 1/a^3 = p (p^2 - 3)$.
Solution:

We know that
$a^3 + 1/a^3 = (a + 1/a)^3 - 3a (1/a) (a + 1/a)$
Substituting the values
$a^3 + 1/a^3 = p^3 - 3p$
Taking p as common
$a^3 + 1/a^3 = p (p^2 - 3)$

Therefore, it is proved.

23. If $x^2 + 1/x^2 = 27$, find the value of $x - 1/x$.
Solution:

We know that
$(x - 1/x)^2 = x^2 + 1/x^2 - 2$
Substituting the values
$(x - 1/x)^2 = 27 - 2 = 25$
So we get
$x - 1/x = \pm \sqrt{25} = \pm 5$

24. If $x^2 + 1/x^2 = 27$, find the value of $3x^3 + 5x - 3/x^3 - 5/x$.
Solution:

We know that
$(x - 1/x)^2 = x^2 + 1/x^2 - 2$
Substituting the values
$(x - 1/x)^2 = 27 - 2 = 25$
So we get
$x - 1/x = \pm \sqrt{25} = \pm 5$

Here
$3x^3 + 5x - 3/x^3 - 5/x = 3 (x^3 - 1/x^3) + 5 (x - 1/x)$
It can be written as
$= 3 [(x - 1/x)^3 + 3 (x - 1/x)] + 5 (x - 1/x)$
Substituting the values
$= 3 [(\pm 5)^3 + 3 (\pm 5)] + 5 (\pm 5)$
By further calculation
$= 3 [(\pm 125) + (\pm 15)] + (\pm 25)$
So we get
$= (\pm 375) + (\pm 45) + (\pm 25)$
$= \pm 445$

25. If $x^2 + 1/25x^2 = 8\ 3/5$, find $x + 1/5x$.
Solution:

We know that
$(x + 1/5x)^2 = x^2 + 1/25x^2 + 2x (1/5x)$
It can be written as
$(x + 1/5x)^2 = x^2 + 1/25x^2 + 2/5$

Substituting the values

$(x + 1/5x)^2 = 8\ 3/5 + 2/5$

$(x + 1/5x)^2 = 43/5 + 2/5$

So we get

$(x + 1/5x)^2 = 45/5 = 9$

Here

$x + 1/5x = \pm \sqrt{9} = \pm 3$

26. If $x^2 + 1/4x^2 = 8$, find $x^3 + 1/8x^3$.

Solution:

We know that

$(x + 1/2x)^2 = x^2 + (1/2x)^2 + 2x\ (1/2x)$

It can be written as

$(x + 1/2x)^2 = x^2 + 1/4x^2 + 1$

Substituting the values

$(x + 1/2x)^2 = 8 + 1 = 9$

So we get

$x + 1/2x = \pm \sqrt{9} = \pm 3$

Here

$x^3 + 1/8x^3 = x^3 + (1/2x)^3$

We know that

$x^3 + 1/8x^3 = (x + 1/2x)^3 – 3x\ (1/2x)\ (x + 1/2x)$

Substituting the values

$x^3 + 1/8x^3 = (\pm 3)^3 – 3/2\ (\pm 3)$

By further calculation

$x^3 + 1/8x^3 = \pm (27 – 9/2)$

Taking LCM

$x^3 + 1/8x^3 = \pm (54 – 9)/ 2$

$x^3 + 1/8x^3 = \pm 45/2 = \pm 22\frac{1}{2}$

Therefore, $x^3 + 1/8x^3 = \pm 22\frac{1}{2}$.

27. If $a^2 – 3a + 1 = 0$, find

(i) $a^2 + 1/a^2$

(ii) $a^3 + 1/a^3$.

Solution:

It is given that
$a^2 - 3a + 1 = 0$
By dividing each term by a
$a + 1/a = 3$

(i) We know that
$(a + 1/a)^2 = a^2 + 1/a^2 + 2$
It can be written as
$a^2 + 1/a^2 = (a + 1/a)^2 - 2$
Substituting the values
$= 3^2 - 2$
$= 9 - 2$
$= 7$

(ii) We know that
$(a + 1/a)^3 = a^3 + 1/a^3 + 3 (a + 1/a)$
It can be written as
$a^3 + 1/a^3 = (a + 1/a)^3 - 3 (a + 1/a)$
Substituting the values
$= 3^3 - 3 (3)$
$= 27 - 9$
$= 18$

28. If $a = 1/ (a - 5)$, find
(i) $a - 1/a$
(ii) $a + 1/a$
(iii) $a^2 - 1/a^2$.
Solution:

It is given that
$a = 1/ (a - 5)$
We can write it as
$a^2 - 5a - 1 = 0$
Now divide each term by a
$a - 5 - 1/a = 0$
So we get
$a - 1/a = 5$

(i) $a - 1/a = 5$

(ii) We know that
$(a + 1/a)^2 = (a – 1/a)^2 + 4$
Substituting the values
$(a + 1/a)^2 = 5^2 + 4$
So we get
$(a + 1/a)^2 = 25 + 4 = 29$
$a + 1/a = \pm \sqrt{29}$

(ii) We know that
$a^2 – 1/a^2 = (a + 1/a) (a – 1/a)$
Substituting the values
$a^2 – 1/a^2 = \pm \sqrt{29} \times 5$
$a^2 – 1/a^2 = \pm 5\sqrt{29}$

29. If $(x + 1/x)^2 = 3$, find $x^3 + 1/x^3$.
Solution:

It is given that
$(x + 1/x)^2 = 3$
$(x + 1/x) = \pm \sqrt{3}$

We know that
$x^3 + 1/x^3 = (x + 1/x)^3 – 3 (x + 1/x)$
Substituting the values
$x^3 + 1/x^3 = (\pm \sqrt{3})^3 – 3 (\pm \sqrt{3})$
By further calculation
$x^3 + 1/x^3 = (\pm 3\sqrt{3}) – (\pm 3\sqrt{3}) = 0$

30. If $x = 5 - 2\sqrt{6}$, find the value of $\sqrt{x} + 1/\sqrt{x}$.
Solution:

It is given that
$x = 5 - 2\sqrt{6}$
We can write it as

$$\frac{1}{x}=\frac{1}{5-2\sqrt{6}}=\frac{5+2\sqrt{6}}{(5-2\sqrt{6})(5+2\sqrt{6})}$$

By further calculation

$$=\frac{5+2\sqrt{6}}{5^2-4\times 6}$$

So we get

$$=\frac{5+2\sqrt{6}}{25-24}$$

$= 5 + 2\sqrt{6}$

Here
$x + 1/x = 5 - 2\sqrt{6} + 5 + 2\sqrt{6} = 10$
So we get
$(\sqrt{x} + 1/\sqrt{x})^2 = x + 1/x + 2$
Substituting the values
$= 10 + 2$
$= 12$

31. If $a + b + c = 12$ and $ab + bc + ca = 22$, find $a^2 + b^2 + c^2$.
Solution:

We know that
$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
We can write it as
$a^2 + b^2 + c^2 = (a + b + c)^2 – 2 (ab + bc + ca)$
Substituting the values
$a^2 + b^2 + c^2 = 12^2 – 2 (22)$
By further calculation
$a^2 + b^2 + c^2 = 144 – 44 = 100$

32. If $a + b + c = 12$ and $a^2 + b^2 + c^2 = 100$, find $ab + bc + ca$.
Solution:

We know that
$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
It can be written as

$2ab + 2bc + 2ca = (a + b + c)^2 – (a^2 + b^2 + c^2)$
Taking out 2 as common
$2 (ab + bc + ca) = 12^2 – 100 = 144 – 100 = 44$
By further calculation
$ab + bc + ca = 44/2 = 22$

33. If $a^2 + b^2 + c^2 = 125$ and $ab + bc + ca = 50$, find $a + b + c$.
Solution:

We know that
$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
Substituting the values
$(a + b + c)^2 = 125 + 2 (50)$
By further calculation
$(a + b + c)^2 = 125 + 100 = 225$
So we get
$a + b + c = \pm \sqrt{225} = \pm 15$

34. If $a + b – c = 5$ and $a^2 + b^2 + c^2 = 29$, find the value of $ab – bc – ca$.
Solution:

It is given that
$a + b – c = 5$
By squaring on both sides
$(a + b – c)^2 = 5^2$
Expanding using formula
$a^2 + b^2 + c^2 + 2ab – 2bc – 2ca = 25$
Substituting the values and taking 2 as common
$29 + 2 (ab – bc – ca) = 25$
By further calculation
$2 (ab – bc – ca) = 25 – 29 = - 4$
So we get
$ab – bc – ca = - 4/2 = - 2$

Therefore, $ab – bc – ca = - 2$.

35. If $a – b = 7$ and $a^2 + b^2 = 85$, then find the value of $a^3 – b^3$.
Solution:

We know that
$(a - b)^2 = a^2 + b^2 - 2ab$
Substituting the values
$7^2 = 85 - 2ab$
By further calculation
$49 = 85 - 2ab$
So we get
$2ab = 85 - 49 = 36$
Dividing by 2
$ab = 36/2 = 18$

Here
$a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$
Substituting the values
$a^3 - b^3 = 7(85 + 18)$
By further calculation
$a^3 - b^3 = 7 \times 103$
So we get
$a^3 - b^3 = 721$

36. If the number x is 3 less than the number y and the sum of the squares of x and y is 29, find the product of x and y.
Solution:

It is given that
$x = y - 3$ and $x^2 + y^2 = 29$
It can be written as
$x - y = -3$
By squaring on both sides
$(x - y)^2 = (-3)^2$
Expanding using formula
$x^2 + y^2 - 2xy = 9$
Substituting the values
$29 - 2xy = 9$
By further calculation
$-2xy = 9 - 29 = -20$
Dividing by 2
$xy = -20/-2 = 10$
So we get

$xy = 10$

37. If the sum and the product of two numbers are 8 and 15 respectively, find the sum of their cubes.
Solution:

Consider x and y as the two numbers
$x + y = 8$ and $xy = 15$
By cubing on both sides
$(x + y)^3 = 8^3$
Expanding using formula
$x^3 + y^3 + 3xy (x + y) = 512$
Substituting the values
$x^3 + y^3 + 3 \times 15 \times 8 = 512$
By further calculation
$x^3 + y^3 + 360 = 512$
So we get
$x^3 + y^3 = 512 – 360 = 152$

CHAPTER TEST

1. Find the expansions of the following :
(i) $(2x + 3y + 5)(2x + 3y – 5)$
(ii) $(6 – 4a - 7b)^2$
(iii) $(7 – 3xy)^3$
(iv) $(x + y + 2)^3$
Solution:
(i) $(2x + 3y + 5)(2x + 3y – 5)$
Let us simplify the expression, we get
$(2x + 3y + 5)(2x + 3y – 5) = [(2x + 3y) + 5][(2x - 3y) – 5]$
By using the formula, $(a)^2 – (b)^2 = [(a + b)(a - b)]$
$= (2x + 3y)^2 – (5)^2$
$= (2x)^2 + (3y)^2 + 2 \times 2x \times 3y – 5 \times 5$
$= 4x^2 + 9y^2 + 12xy - 25$

(ii) $(6 – 4a – 7b)^2$
Let us simplify the expression, we get
$(6 – 4a – 7b)^2 = [6 + (-4a) + (-7b)]^2$
$= (6)^2 + (-4a)^2 + (-7b)^2 + 2(6)(-4a) + 2(-4a)(-7b) + 2(-7b)(6)$
$= 36 + 16a^2 + 49b^2 – 48a + 56ab – 84b$

(iii) $(7 – 3xy)^3$
Let us simplify the expression
By using the formula, we get
$(7 – 3xy)^3 = (7)^3 – (3xy)^3 – 3(7)(3xy)(7 – 3xy)$
$= 343 – 27x^3y^3 – 63xy(7 – 3xy)$
$= 343 – 27x^3y^3 – 441xy + 189x^2y^2$

(iv) $(x + y + 2)^3$
Let us simplify the expression
By using the formula, we get
$(x + y + 2)^3 = [(x + y) + 2]^3$
$= (x + y)^3 + (2)^3 + 3(x + y)(2)(x + y + 2)$
$= x^3 + y^3 + 3x^2y + 3xy^2 + 8 + 6(x + y)[(x + y) + 2]$
$= x^3 + y^3 + 3x^2y + 3xy^2 + 8 + 6(x + y)^2 + 12(x + y)$
$= x^3 + y^3 + 3x^2y + 3xy^2 + 8 + 6(x^2 + y^2 + 2xy) + 12x + 12y = x^3 + y^3 + 3x^2y + 3xy^2 + 8 + 6x^2 + 6y^2 + 12xy + 12x + 12y$
$= x^3 + y^3 + 3x^2y + 3xy^2 + 8 + 6x^2 + 6y^2 + 12x + 12y + 12xy$

2. Simplify: $(x - 2)(x + 2)(x^2 + 4)(x^4 + 16)$

Solution:

Let us simplify the expression, we get

$$
\begin{aligned}
(x - 2)(x + 2)(x^4 + 4)(x^4 + 16) &= (x^2 - 4)(x^4 + 4)(x^4 + 16) \\
&= [(x^2)^2 - (4)^2](x^4 + 16) \\
&= (x^4 - 16)(x^4 + 16) \\
&= (x^4)^2 - (16)^2 \\
&= x^8 - 256
\end{aligned}
$$

3. Evaluate 1002×998 by using a special product.

Solution:

Let us simplify the expression, we get

$$
\begin{aligned}
1002 \times 998 &= (1000 + 2)(1000 - 2) \\
&= (1000)^2 - (2)^2 \\
&= 1000000 - 4 \\
&= 999996
\end{aligned}
$$

4. If $a + 2b + 3c = 0$, Prove that $a^3 + 8b^3 + 27c^3 = 18\,abc$

Solution:

Given:

$a + 2b + 3c = 0$, $a + 2b = -3c$

Let us cube on both the sides, we get

$(a + 2b)^3 = (-3c)^3$

$a^3 + (2b)^3 + 3(a)(2b)(a + 2b) = -27c^3$

$a^3 + 8b^3 + 6ab(-3c) = -27c^3$

$a^3 + 8b^3 - 18abc = -27c^3$

$a^3 + 8b^3 + 27c^3 = 18abc$

Hence proved.

5. If $2x = 3y - 5$, then find the value of $8x^3 - 27y^3 + 90xy + 125$.

Solution:

Given:

$2x = 3y - 5$

$2x - 3y = -5$

Now, let us cube on both sides, we get

$(2x - 3y)^3 = (-5)^3$

$(2x)^3 - (3y)^3 - 3 \times 2x \times 3y(2x - 3y) = -125$

$8x^3 - 27y^3 - 18xy(2x - 3y) = -125$

Now, substitute the value of $2x - 3y = -5$

$8x^3 – 27y^3 – 18xy (-5) = -125$
$8x^3 – 27y^3 + 90xy = -125$
$8x^3 – 27y^3 + 90xy + 125 = 0$

6. If $a^2 – 1/a^2 = 5$, evaluate $a^4 + 1/a^4$
Solution:
It is given that,
$a^2 – 1/a^2 = 5$
So,
By using the formula, $(a + b)^2$
$[a^2 – 1/a^2]^2 = a^4 + 1/a^4 – 2$
$[a^2 – 1/a^2]^2 + 2 = a^4 + 1/a^4$
Substitute the value of $a^2 – 1/a^2 = 5$, we get
$5^2 + 2 = a^4 + 1/a^4$
$a^4 + 1/a^4 = 25 + 2$
$= 27$

7. If a + 1/a = p and a – 1/a = q, Find the relation between p and q.
Solution:
It is given that,
a + 1/a = p and a – 1/a = q
so,
$(a + 1/a)^2 – (a – 1/a)^2 = 4(a) (1/a)$
$= 4$
By substituting the values, we get
$p^2 – q^2 = 4$
Hence the relation between p and q is that $p^2 – q^2 = 4$.

8. If $(a^2 + 1)/a = 4$, find the value of $2a^3 + 2/a^3$
Solution:
It is given that,
$(a^2 + 1)/a = 4$
$a^2/a + 1/a = 4$
$a + 1/a = 4$
So by multiplying the expression by 2a, we get
$2a^3 + 2/a^3 = 2[a^3 + 1/a^3]$
$= 2 [(a + 1/a)^3 – 3 (a) (1/a) (a + 1/a)]$
$= 2 [(4)^3 – 3(4)]$
$= 2 [64 - 12]$

= 2 (52)
= 104

9. If x = 1/(4 - x), find the value of
(i) x + 1/x
(ii) $x^3 + 1/x^3$
(iii) $x^6 + 1/x^6$
Solution:
It is given that,
x = 1/(4 - x)
So,
(i) x(4 - x) = 1
$4x – x^2 = 1$
Now let us divide both sides by x, we get
4 – x = 1/x
4 = 1/x + x
1/x + x = 4
1/x + x = 4

(ii) $x^3 + 1/x^3 = (x + 1/x)^2 – 3(x + 1/x)$
By substituting the values, we get
$= (4)^3 – 3(4)$
= 64 – 12
= 52

(iii) $x^6 + 1/x^6 = (x^3 + 1/x^3)^2 – 2$
$= (52)^2 – 2$
= 2704 – 2
= 2702

10. If x – 1/x = 3 + 2√2, find the value of ¼ $(x^3 – 1/x^3)$
Solution:
It is given that,
x – 1/x = 3 + 2√2
So,
$x^3 – 1/x^3 = (x – 1/x)^3 + 3(x – 1/x)$
$= (3 + 2\sqrt{2})^3 + 3(3 + 2\sqrt{2})$
By using the formula, $(a+b)^3 = a^3 + b^3 + 3ab (a + b)$
$= (3)^3 + (2\sqrt{2})^3 + 3 (3) (2\sqrt{2}) (3 + 2\sqrt{2}) + 3(3 + 2\sqrt{2})$

$= 27 + 16\sqrt{2} + 54\sqrt{2} + 72 + 9 + 6\sqrt{2}$
$= 108 + 76\sqrt{2}$

Hence,
$\frac{1}{4}(x^3 - 1/x^3) = \frac{1}{4}(108 + 76\sqrt{2})$
$= 27 + 19\sqrt{2}$

11. If $x + 1/x = 3\ 1/3$, find the value of $x^3 - 1/x^3$

Solution:

It is given that,
$x + 1/x = 3\ 1/3$
we know that,
$(x - 1/x)^2 = x^2 + 1/x^2 - 2$
$= x^2 + 1/x^2 + 2 - 4$
$= (x + 1/x)^2 - 4$
But $x + 1/x = 3\ 1/3 = 10/3$
So,
$(x - 1/x)^2 = (10/3)^2 - 4$
$= 100/9 - 4$
$= (100 - 36)/9$
$= 64/9$
$x - 1/x = \sqrt{(64/9)}$
$= 8/3$
Now,
$x^3 - 1/x^3 = (x - 1/x)^3 + 3(x)(1/x)(x - 1/x)$
$= (8/3)^3 + 3(8/3)$
$= ((512/27) + 8)$
$= 728/27$
$= 26\ 26/27$

12. If $x = 2 - \sqrt{3}$, then find the value of $x^3 - 1/x^3$

Solution:

It is given that,
$x = 2 - \sqrt{3}$
so,
$1/x = 1/(2 - \sqrt{3})$
By rationalizing the denominator, we get
$= [1(2 + \sqrt{3})] / [(2 - \sqrt{3})(2 + \sqrt{3})]$
$= [(2 + \sqrt{3})] / [(2^2) - (\sqrt{3})^2]$
$= [(2 + \sqrt{3})] / [4 - 3]$

$= 2 + \sqrt{3}$

Now,

$x – 1/x = 2 - \sqrt{3} – 2 - \sqrt{3}$

$= - 2\sqrt{3}$

Let us cube on both sides, we get

$(x – 1/x)^3 = (-2\sqrt{3})^3$

$x^3 – 1/x^3 – 3 (x) (1/x) (x – 1/x) = 24\sqrt{3}$

$x^3 – 1/x^3 – 3(-2\sqrt{3}) = -24\sqrt{3}$

$x^3 – 1/x^3 + 6\sqrt{3} = -24\sqrt{3}$

$x^3 – 1/x^3 = -24\sqrt{3} - 6\sqrt{3}$

$= -30\sqrt{3}$

Hence,

$x^3 – 1/x^3 = -30\sqrt{3}$

13. If the sum of two numbers is 11 and sum of their cubes is 737, find the sum of their squares.

Solution:

Let us consider x and y be two numbers

Then,

$x + y = 11$

$x^3 + y^3 = 735$ and $x^2 + y^2 =?$

Now,

$x + y = 11$

Let us cube on both the sides,

$(x + y)^3 = (11)^3$

$x^3 + y^3 + 3xy (x + y) = 1331$

$737 + 3x \times 11 = 1331$

$33xy = 1331 – 737$

$= 594$

$xy = 594/33$

$xy = 8$

We know that, $x + y = 11$

By squaring on both sides, we get

$(x + y)^2 = (11)^2$

$x^2 + y^2 + 2xy = 121$ ^{2} $x^2 + y^2 + 2 \times 18 = 121$

$x^2 + y^2 + 36 = 121$

$x^2 + y^2 = 121 – 36$

$= 85$

Hence sum of the squares = 85

14. If $a - b = 7$ and $a^3 - b^3 = 133$, find:
(i) ab
(ii) $a^2 + b^2$
Solution:
It is given that,
$a - b = 7$
let us cube on both sides, we get
(i) $(a - b)^3 = (7)^3$
$a^3 + b^3 - 3ab (a - b) = 343$
$133 - 3ab \times 7 = 343$
$133 - 21ab = 343$
$-21ab = 343 - 133$ 21ab
$= 210$
ab = -210/21
ab = -10

(ii) $a^2 + b^2$
Again $a - b = 7$
Let us square on both sides, we get
$(a - b)^2 = (7)^2$
$a^2 + b^2 - 2ab = 49$
$a^2 + b^2 - 2 \times (-10) = 49$
$a^2 + b^2 + 20 = 49$
$a^2 + b^2 = 49 - 20$
$= 29$
Hence, $a^2 + b^2 = 29$

15. Find the coefficient of x^2 expansion of $(x^2 + x + 1)^2 + (x^2 - x + 1)^2$
Solution:
Given:
The expression, $(x^2 + x + 1)^2 + (x^2 - x + 1)^2$
$(x^2 + x + 1)^2 + (x^2 - x + 1)^2 = [((x^2 + 1) + x)^2 + [(x^2 + 1) - x)^2]$
$= (x^2 + 1)^2 + x^2 + 2 (x^2 + 1) (x) + (x^2 + 1)^2 + x^2 - 2 (x^2 + 1) (x)$
$= (x^2)^2 + (1)^2 + 2 \times x^2 \times 1 + x^2 + (x^2)^2 + 1 + 2 \times x^2 + 1 + x^2$
$= x^4 + 1 + 2x^2 + x^2 + x^4 + 1 + 2x^2 + x^2$
$= 2x^4 + 6x^2 + 2$
$\therefore$ Co-efficient of x^2 is 6.