EXERCISE 4.1

Factorise the following (1 to 9):

1. (i) $8xy^3 + 12x^2y^2$

Solution:-

$8xy^3 + 12x^2y^2$

Take out common in both terms,

Then, $4xy^2 (2y + 3x)$

Therefore, HCF of $8xy^3$ and $12x^2y^2$ is $4xy^2$.

(ii) $15 ax^3 – 9ax^2$

Solution:-

$15 ax^3 – 9ax^2$

Take out common in both terms,

Then, $3ax^2 (5x - 3)$

Therefore, HCF of $15 ax^3$ and $9ax^2$ is $3ax^2$.

2.

(i) $21py^2 – 56py$

Solution:-

$21py^2 – 56py$

Take out common in both terms,

Then, $7py (3y - 8)$

Therefore, HCF of $21py^2$ and $56py$ is $7py$.

(ii) $4x^3 – 6x^2$

Solution:-

$4x^3 – 6x^2$

Take out common in both terms,

Then, $2x^2 (2x - 3)$

Therefore, HCF of $4x^3$ and $6x^2$ is $2x^2$.

3.

(i) $2\pi r^2 - 4\pi r$

Solution:-

$2\pi r^2 - 4\pi r$

Take out common in both terms,

Then, $2\pi r (r - 2)$

Therefore, HCF of $2\pi r^2$ and $4\pi r$ is $2\pi r$.

(ii) 18m + 16n
Solution:-
18m + 16n
Take out common in both terms,
Then, 2 (9m – 8n)
Therefore, HCF of 18m and 16n is 2.

4.
(i) $25abc^2 – 15a^2b^2c$
Solution:-
$25abc^2 – 15a^2b^2c$
Take out common in both terms,
Then, 5abc (5c – 3ab)
Therefore, HCF of $25abc^2$ and $15a^2b^2c$ is 5abc.

(ii) $28p^2q^2r – 42pq^2r^2$
Solution:-
$28p^2q^2r – 42pq^2r^2$
Take out common in both terms,
Then, $14pq^2r$ (2p – 3r)
Therefore, HCF of $28p^2q^2r$ and $42pq^2r^2$ is $14pq^2r$.

5.
(i) $8x^3 – 6x^2 + 10x$
Solution:-
$8x^3 – 6x^2 + 10x$
Take out common in both terms,
Then, $2x(4x^2 – 3x + 5)$
Therefore, HCF of $8x^3$, $6x^2$ and 10x is 2x.

(ii) 14mn + 22m – 62p
Solution:-
14mn + 22m – 62p
Take out common in both terms,
Then, 2 (7mn + 11m – 31p)

Therefore, HCF of 14mn, 22m and 62p is 2.

6.

(i) $18p^2q^2 – 24pq^2 + 30p^2q$

Solution:-

$18p^2q^2 – 24pq^2 + 30p^2q$

Take out common in both terms,

Then, $6pq (3pq – 4q + 5p)$

Therefore, HCF of $18p^2q^2$, $24pq^2$ and $30p^2q$ is $6pq$.

(ii) $27a^3b^3 – 18a^2b^3 + 75a^3b^2$

Solution:-

$27a^3b^3 – 18a^2b^3 + 75a^3b^2$

Take out common in both terms,

Then, $3a^2b^2 (9a – 6b + 25a)$

Therefore, HCF of $27a^3b^3$, $18a^2b^3$ and $75a^3b^2$ is $3a^2b^2$.

7.

(i) $15a (2p – 3q) – 10b (2p – 3q)$

Solution:-

$15a (2p – 3q) – 10b (2p – 3q)$

Take out common in both terms,

Then, $5(2p – 3q) [3a – 2b]$

Therefore, HCF of $15a (2p – 3q)$ and $10b (2p – 3q)$ is $5(2p – 3q)$.

(ii) $3a(x^2 + y^2) + 6b (x^2 + y^2)$

Solution:-

$3a(x^2 + y^2) + 6b (x^2 + y^2)$

Take out common in all terms,

Then, $3(x^2 + y^2) (a + 2b)$

Therefore, HCF of $3a(x^2 + y^2)$ and $6b (x^2 + y^2)$ is $3(x^2 + y^2)$.

8.

(i) $6(x + 2y)^3 + 8(x + 2y)^2$

Solution:-

$6(x + 2y)^3 + 8(x + 2y)^2$

Take out common in all terms,

Then, $2(x + 2y)^2$ [3(x + 2y) + 4]
Therefore, HCF of $6(x + 2y)^3$ and $8(x + 2y)^2$ is $2(x + 2y)^2$.

(ii) $14(a – 3b)^3 – 21p(a – 3b)$
Solution:-
$14(a – 3b)^3 – 21p(a – 3b)$
Take out common in all terms,
Then, $7(a – 3b) [2(a – 3b)^2 – 3p]$
Therefore, HCF of $14(a – 3b)^3$ and $21p(a – 3b)$ is $7(a – 3b)$.

9.
(i) $10a(2p + q)^3 – 15b (2p + q)^2 + 35 (2p + q)$
Solution:-
$10a(2p + q)^3 – 15b (2p + q)^2 + 35 (2p + q)$
Take out common in all terms,
Then, $5(2p + q) [2a (2p + q)^2 – 3b (2p + q) + 7]$
Therefore, HCF of $10a(2p + q)^3$, $15b (2p + q)^2$ and $35 (2p + q)$ is $5(2p + q)$.

(ii) $x(x^2 + y^2 – z^2) + y(-x^2 – y^2 + z^2) – z (x^2 + y – z^2)$
Solution:-
$x(x^2 + y^2 – z^2) + y(-x^2 – y^2 + z^2) – z (x^2 + y – z^2)$
Take out common in all terms,
Then, $(x^2 + y^2 – z^2) [x – y - z]$
Therefore, HCF of $x(x^2 + y^2 – z^2)$, $y(-x^2 – y^2 + z^2)$ and $z (x^2 + y – z^2)$ is $(x^2 + y^2 – z^2)$

EXERCISE 4.2

Factorise the following (1 to 13):

1.

(i) $x^2 + xy – x – y$

Solution:-

$x^2 + xy – x – y$

Take out common in all terms,

$x(x + y) – 1(x + y)$

$(x + y) (x - 1)$

(ii) $y^2 – yz – 5y + 5z$

Solution:-

$y^2 – yz – 5y + 5z$

Take out common in all terms,

$y(y - z) – 5(y - z)$

$(y - z) (y - 5)$

2.

(i) $5xy + 7y – 5y^2 – 7x$

Solution:-

$5xy – 7x – 5y^2 + 7y$

Take out common in all terms,

$x(5y - 7) – y(5y - 7)$

$(5y - 7) (x - y)$

(ii) $5p^2 – 8pq – 10p + 16q$

Solution:-

$5p^2 – 8pq – 10p + 16q$

Take out common in all terms,

$p(5p – 8q) – 2(5p – 8q)$

$(5p – 8q) (p - 2)$

3.

(i) $a^2b – ab^2 + 3a – 3b$

Solution:-

$a^2b – ab^2 + 3a – 3b$

Take out common in all terms,

ab(a - b) + 3(a - b)
(a - b) (ab + 3)

(ii) $x^3 – 3x^2 + x – 3$
Solution:-
$x^3 – 3x^2 + x – 3$
Take out common in all terms,
x^2 (x - 3) + 1(x - 3)
(x - 3) (x^2 + 1)

4.
(i) $6xy^2 – 3xy – 10y + 5$
Solution:-
$6xy^2 – 3xy – 10y + 5$
Take out common in all terms,
3xy(2y - 1) – 5(2y - 1)
(2y - 1) (3xy - 5)

(ii) 3ax – 6ay – 8by + 4bx
Solution:-
3ax – 6ay – 8by + 4bx
Take out common in all terms,
3a(x – 2y) + 4b (x – 2y)
(x – 2y) (3a + 4b)

5.
(i) 1 – a – b + ab
Solution:-
1 – a – b + ab
Take out common in all terms,
1(1 - a) – b(1 - a)
(1 - a) (1 - b)

(ii) a(a – 2b - c) + 2bc
Solution:-
a(a – 2b - c) + 2bc
Above question can be written as,

$a^2 – 2ab – ac + 2bc$
Take out common in all terms,
$a(a – 2b) – c(a + 2b)$
$(a – 2b) (a - c)$

6.
(i) $x^2 + xy (1 + y) + y^3$
Solution:-
$x^2 + xy (1 + y) + y^3$
Above question can be written as,
$x^2 + xy + xy^2 + y^3$
Take out common in all terms,
$x(x + y) + y^2(x + y)$
$(x + y) (x + y^2)$

(ii) $y^2 – xy (1 - x) – x^3$
Solution:-
$y^2 – xy (1 - x) – x^3$
Above question can be written as,
$y^2 – xy + x^2y – x^3$
Take out common in all terms,
$y(y - x) + x^2 (y - x)$
$(y - x) (y + x^2)$

7.
(i) $ab^2 + (a - 1)b – 1$
Solution:-
$ab^2 + (a - 1)b – 1$
Above question can be written as,
$ab^2 + ab – b – 1$
Take out common in all terms,
$ab(b + 1) – 1(b + 1)$
$(b + 1) (ab - 1)$

(ii) $2a – 4b – xa + 2bx$
Solution:-
$2a – 4b – xa + 2bx$

Take out common in all terms,
2(a – 2b) – x(a – 2b)
(a – 2b) (2 - x)

8.
(i) 5ph – 10qk + 2rph – 4qrk
Solution:-
5ph – 10qk + 2rph – 4qrk
Re-arranging the given question we get,
5ph + 2rph – 10qk – 4qrk
Take out common in all terms,
ph(5 + 2r) – 2qk(5 + 2r)
(5 + 2r) (ph – 2qk)

(ii) $x^2 – x(a + 2b) + 2ab$
Solution:-
$x^2 – x(a + 2b) + 2ab$
Above question can be written as,
$x^2 – xa – 2xb + 2ab$
Take out common in all terms,
x(x - a) – 2b(x - a)
(x - a) (x – 2b)

9.
(i) $ab(x^2 + y^2) – xy(a^2 + b^2)$
Solution:-
$ab(x^2 + y^2) – xy(a^2 + b^2)$
Above question can be written as,
$abx^2 + aby^2 – xya^2 – xyb^2$
Re-arranging the above we get,
$abx^2 – xyb^2 + aby^2 – xya^2$
Take out common in all terms,
bx(ax - by) + ay(by - ax)
bx(ax - by) – ay (ax - by)
(ax - by) (bx - ay)

(ii) $(ax + by)^2 + (bx - ay)^2$

Solution:-

By expanding the give question, we get,

$(ax)^2 + (by)^2 + 2axby + (bx)^2 + (ay)^2 – 2bxay$

$a^2x^2 + b^2y^2 + b^2x^2 + a^2y^2$

Re-arranging the above we get,

$a^2x^2 + a^2y^2 + b^2y^2 + b^2x^2$

Take out common in all terms,

$a^2 (x^2 + y^2) + b^2 (x^2 + y^2)$

$(x^2 + y^2) (a^2 + b^2)$

10.

(i) $a^3 + ab(1 – 2a) – 2b^2$

Solution:-

$a^3 + ab(1 – 2a) – 2b^2$

Above question can be written as,

$a^3 + ab – 2a^2b – 2b^2$

Re-arranging the above we get,

$a^3 – 2a^2b + ab – 2b^2$

Take out common in all terms,

$a^2(a – 2b) + b(a – 2b)$

$(a – 2b) (a^2 + b)$

(ii) $3x^2y – 3xy + 12x – 12$

Solution:-

$3x^2y – 3xy + 12x – 12$

Take out common in all terms,

$3xy(x - 1) + 12(x - 1)$

$(x - 1) (3xy + 12)$

11. $a^2b + ab^2 –abc – b^2c + axy + bxy$

Solution:-

$a^2b + ab^2 –abc – b^2c + axy + bxy$

Re-arranging the above we get,

$a^2b – abc + axy + ab^2 – b^2c + bxy$

Take out common in all terms,

$a(ab – bc + xy) + b(ab – bc + xy)$

$(a + b) (ab – bc + xy)$

12. $ax^2 – bx^2 + ay^2 – by^2 + az^2 – bz^2$

Solution:-

$ax^2 – bx^2 + ay^2 – by^2 + az^2 – bz^2$

Re-arranging the above we get,

$ax^2 + ay^2 + az^2 – bx^2 – by^2 – bz^2$

Take out common in all terms,

$a(x^2 + y^2 + z^2) – b(x^2 + y^2 + z^2)$

$(x^2 + y^2 + z^2) (a - b)$

13. $x – 1 – (x - 1)^2 + ax – a$

Solution:-

$x – 1 – (x - 1)^2 + ax – a$

By expanding the above we get,

$X – 1 – (x^2 + 1 – 2x) + ax – a$

$x – 1 – x^2 -1 + 2x + ax – a$

$2x – x^2 + ax – 2 + x – a$

Take out common in all terms,

$x(2 – x + a) – 1(2 – x + a)$

$(2 – x + a) (x - 1)$

EXERCISE 4.3

Factorise the following (1 to 17):

1. $4x^2 – 25y^2$

Solution:-

We know that, $a^2 – b^2 = (a + b) (a - b)$

So, $(2x)^2 – (5y)^2$

Then, $(2x + y) (2x – 5y)$

(ii) $9x^2 – 1$

Solution:-

We know that, $a^2 – b^2 = (a + b) (a - b)$

So, $(3x)^2 – 1^2$

Then, $(3x + 1) (3x - 1)$

2.

(i) $150 – 6a^2$

Solution:-

$150 – 6a^2$

Take out common in all terms,

$6(25 – a^2)$

$6(5^2 – a^2)$

We know that, $a^2 – b^2 = (a + b) (a - b)$

So, $6(5 + a) (5 - a)$

(ii) $32x^2 – 18y^2$

Solution:-

$32x^2 – 18y^2$

Take out common in all terms,

$2(16x^2 – 9y^2)$

$2((4x)^2 – (3y)^2)$

We know that, $a^2 – b^2 = (a + b) (a - b)$

$2(4x + 3y) (4x – 3y)$

3.

(ii) $(x - y)^2 – 9$

Solution:-

$(x - y)^2 – 9$

$(x - y)^2 – 3^2$
We know that, $a^2 – b^2 = (a + b) (a - b)$
$(x – y + 3) (x – y - 3)$

(ii) $9(x + y)^2 – x^2$
Solution:-
$9[(x + y)^2 – x^2]$
We know that, $a^2 – b^2 = (a + b) (a - b)$
$9[(x + y + x) (x + y - x)]$
So, $9(2x + y) y$
$9y(2x + y)$

4.
(i) $20x^2 – 45y^2$
Solution:-
$20x^2 – 45y^2$
Take out common in all terms,
$5(4x^2 – 9y^2)$
$5((2x)^2 – (3y)^2)$
We know that, $a^2 – b^2 = (a + b) (a - b)$
$5(2x + 3y) (2x – 3y)$

(ii) $9x^2 – 4(y + 2x)^2$
Solution:-
$9x^2 – 4(y + 2x)^2$
Above question can be written as,
$(3x)^2 – [2(y + 2x)]^2$
$(3x)^2 – (2y + 4x)^2$
We know that, $a^2 – b^2 = (a + b) (a - b)$
$(3x + 2y + 4x) (3x – 2y – 4x)$
$(7x + 2y) (-x – 2y)$

5.
(i) $2(x – 2y)^2 – 50y^2$
Solution:-
$2(x – 2y)^2 – 50y^2$
Take out common in all terms,

$2[(x – 2y)^2 – 25y^2]$
$2[(x – 2y)^2 – (5y)^2]$
We know that, $a^2 – b^2 = (a + b) (a - b)$
$2[(x – 2y + 5y) (x – 2y – 5y)]$
$2[(x + 3y) (x – 7y)]$
$2(x + 3y) (x – 7y)$

(ii) $32 – 2(x - 4)^2$
Solution:-
$32 – 2(x - 4)^2$
Take out common in all terms,
$2[16 – (x - 4)^2]$
$2[4^2- (x - 4)^2]$
We know that, $a^2 – b^2 = (a + b) (a - b)$
$2[(4 + x - 4) (4 – x + 4)]$
$2[(x) (8 - x)]$
$2x (8 - x)$

6.
(i) $108a^2 – 3(b - c)^2$
Solution:-
$108a^2 – 3(b - c)^2$
Take out common in all terms,
$3[36a^2 – (b - c)^2]$
$3[(6a)^2 – (b - c)^2]$
We know that, $a^2 – b^2 = (a + b) (a - b)$
$3[(6a + b - c) (6a – b + c)]$

(ii) $\pi a^5 – \pi^3 ab^2$
Solution:-
$\pi a^5 – \pi^3 ab^2$
Take out common in all terms,
$\pi a(a^4 – \pi^2 b^2)$
$\pi a((a^2)^2 – (\pi b)^2)$
We know that, $a^2 – b^2 = (a + b) (a - b)$
$\pi a(a^2 + \pi b) (a^2 - \pi b)$

7.

(i) $50x^2 – 2(x - 2)^2$

Solution:-

$50x^2 – 2(x - 2)^2$

Take out common in all terms,

$2[25x^2 – (x - 2)^2]$

$2[(5x)^2 – (x - 2)^2]$

We know that, $a^2 – b^2 = (a + b) (a - b)$

$2[(5x + x - 2) (5x – x + 2)]$

$2[(6x - 2) (4x + 2)]$

$2(6x - 2) (4x + 2)$

(ii) (x - 2)(x + 2) + 3

Solution:-

We know that, $a^2 – b^2 = (a + b) (a - b)$

$(x^2 – 2^2) + 3$

$X^2 – 4 + 3$

$X^2 – 1$

Then,

$(x + 1) (x - 1)$

8.

(i) $x – 2y – x^2 + 4y^2$

Solution:-

$x – 2y – x^2 + 4y^2$

$x – 2y – (x^2 + (2y)^2)$

We know that, $a^2 – b^2 = (a + b) (a - b)$

$x – 2y – [(x + 2y) (x – 2y)]$

Take out common in all terms,

$(x – 2y) (1 – (x + 2y))$

$(x – 2y) (1 – x – 2y)$

(ii) $4a^2 – b^2 + 2a + b$

Solution:-

$4a^2 – b^2 + 2a + b$

$(2a)^2 – b^2 + 2a + b$

We know that, $a^2 – b^2 = (a + b) (a - b)$

((2a + b) (2a - b)) + 1(2a + b)
Take out common in all terms,
(2a + b) (2a – b + 1)

9.
(i) a(a - 2) – b(b - 2)
Solution:-
a(a - 2) – b(b - 2)
Above question can be written as,
$a^2 – 2a – b^2 – 2b$
Rearranging the above terms, we get,
$a^2 – b^2 – 2a – 2b$
We know that, $a^2 – b^2 = (a + b) (a - b)$
[(a + b)(a - b)] – 2(a - b)
Take out common in all terms,
(a - b) (a + b - 2)

(ii) a(a - 1) – b(b - 1)
Solution:-
a(a - 1) – b(b - 1)
Above question can be written as,
$a^2 – a – b^2 + b$
Rearranging the above terms, we get,
$a^2 – b^2 – a + b$
We know that, $a^2 – b^2 = (a + b) (a - b)$
[(a + b) (a - b)] – 1 (a - b)
Take out common in all terms,
(a - b) (a + b - 1)

10.
(i) $9 – x^2 + 2xy – y^2$
Solution:-
$9 – x^2 + 2xy – y^2$
$9 – x^2 + 2xy – y^2$
Above terms can be written as,
$9 – x^2 + xy + xy – y^2$
Now,

$9 – x^2 + xy + 3x – 3x + 3y – 3y + xy – y^2$
Rearranging the above terms, we get,
$9 – 3x + 3y + 3x – x^2 + xy + xy – 3y – y^2$
Take out common in all terms,
$3(3 – x + y) + x(3 – x + y) + y (-3 – y + x)$
$3(3 – x + y) + x(3 – x + y) – y(3 – x + y)$
$(3 – x + y) (3 + x - y)$

(ii) $9x^4 – (x^2 + 2x + 1)$
Solution:-
$9x^4 – (x^2 + 2x + 1)$
Above terms can be written as,
$(3x^2)^2 – (x + 1)^2$ … [because $(a + b)^2 = a^2 + 2ab + b^2$]
We know that, $a^2 – b^2 = (a + b) (a - b)$
So, $(3x^2 + x + 1) (3x^2 – x - 1)$

11.
(i) $9x^4 – x^2 - 12x - 36$
Solution:-
$9x^4 – x^2 - 12x - 36$
Above terms can be written as,
$9x^4 – (x^2 + 12x + 36)$
We know that, $(a + b)^2 = a^2 + 2ab + b^2$
$(3x^2)^2 – (x^2 + (2 \times 6 \times x) + 6^2)$
So, $(3x^2)^2 – (x + 6)^2$
We know that, $a^2 – b^2 = (a + b) (a - b)$
$(3x^2 + x + 6) (3x^2 – x - 6)$

(ii) $x^3 – 5x^2 – x + 5$
Solution:-
$x^3 – 5x^2 – x + 5$
Take out common in all terms,
$x^2(x - 5) – 1(x - 5)$
$(x - 5) (x^2 - 1)$
$(x - 5) (x^2 – 1^2)$
We know that, $a^2 – b^2 = (a + b) (a - b)$
$(x - 5) (x + 1) (x - 1)$

12.

(i) $a^4 – b^4 + 2b^2 – 1$

Solution:-

$a^4 – b^4 + 2b^2 – 1$

Above terms can be written as,

$a^4 – (b^4 – 2b^2 + 1)$

We know that, $(a - b)^2 = a^2 - 2ab + b^2$

$a^4 – ((b^2)^2) – (2 \times b^2 \times 1) + 1^2)$

$(a^2)^2 – (b^2 - 1)^2$

We know that, $a^2 – b^2 = (a + b) (a - b)$

$(a^2 + b^2 - 1) (a^2 – b^2 + 1)$

(ii) $x^3 – 25x$

Solution:-

$x^3 – 25x$

Take out common in all terms,

$x(x^2 - 25)$

Above terms can be written as,

$x(x^2 – 5^2)$

We know that, $a^2 – b^2 = (a + b) (a - b)$

$x(x + 5) (x - 5)$

13.

(i) $2x^4 – 32$

Solution:-

$2x^4 – 32$

Take out common in all terms,

$2(x^4 - 16)$

Above terms can be written as,

$2((x^2)^2 – 4^2)$

We know that, $a^2 – b^2 = (a + b) (a - b)$

$2(x^2 + 4) (x^2 - 4)$

$2(x^2 + 4) (x^2 – 2^2)$

$2(x^2 + 4) (x + 2) (x - 2)$

(ii) $a^2(b + c) – (b + c)^3$

Solution:-

$a^2(b + c) – (b + c)^3$
Take out common in all terms,
$(b + c) (a^2 – (b + c)^2)$
We know that, $a^2 – b^2 = (a + b) (a - b)$
$(b + c) (a + b + c) (a – b - c)$

14.
(i) $(a + b)^3 – a – b$
Solution:-
$(a + b)^3 – a – b$
Above terms can be written as,
$(a + b)^3 – (a + b)$
Take out common in all terms,
$(a + b) [(a + b)^2 - 1]$
$(a + b) [(a + b)^2 – 1^2]$
We know that, $a^2 – b^2 = (a + b) (a - b)$
$(a + b) (a + b + 1) (a + b - 1)$

(ii) $x^2 – 2xy + y^2 – a^2 – 2ab – b^2$
Solution:-
$x^2 – 2xy + y^2 – a^2 – 2ab – b^2$
Above terms can be written as,
$(x^2 – 2xy + y^2) - (a^2 + 2ab + b^2)$
We know that, $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$
$(x^2 – (2 \times x \times y) + y^2) – (a^2 + (2 \times a \times b) + b^2)$
$(x - y)^2 – (a + b)^2$
We know that, $a^2 – b^2 = (a + b) (a - b)$
$[(x - y) + (a + b)] [(x - y) – (a + b)]$
$(x – y + a + b) (x – y – a - b)$

15.
(i) $(a^2 – b^2) (c^2 – d^2) – 4abcd$
Solution:-
$(a^2 – b^2) (c^2 – d^2) – 4abcd$
$a^2(c^2 – d^2) – b^2 (c^2 – d^2) – 4abcd$
$a^2c^2 – a^2d^2 – b^2c^2 + b^2d^2 – 4abcd$
$a^2c^2 + b^2d^2 – a^2d^2 – b^2c^2 – 2abcd – 2abcd$

Rearranging the above terms, we get,
$a^2c^2 + b^2d^2 – 2abcd – a^2d^2 – b^2c^2 – 2abcd$
We know that, $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$
$(ac - bd)^2 – (ad - bc)^2$
$(ac – bd + ad - bc) (ac – bd – ad + bc)$

(ii) $4x^2 – y^2 – 3xy + 2x – 2y$
Solution:-
$4x^2 – y^2 – 3xy + 2x – 2y$
Above terms can be written as,
$x^2 + 3x^2 – y^2 – 3xy + 2x – 2y$
Rearranging the above terms, we get,
$(x^2 – y^2) + (3x^2 – 3xy) + (2x – 2y)$
We know that, $a^2 – b^2 = (a + b) (a - b)$ and take out common terms,
$(x + y) (x - y) + 3x(x - y) + 2(x - y)$
$(x - y) [(x + y) + 3x + 2]$
$(x - y) (x + y + 3x + 2)$
$(x - y) (4x + y + 2)$

16.
(i) $x^2 + 1/x^2 – 11$
Solution:-
$x^2 + 1/x^2 – 11$
Above terms can be written as,
$x^2 + (1/x^2) – 2 – 9$
Then, $(x^2 + (1/x^2) – 2) – 3^2$
We know that, $(a - b)^2 = a^2 - 2ab + b^2$,
$(x^2 – (2 \times x^2 \times (1/x^2)) + (1/x)^2)$
$(x – 1/x)^2 – 3^2$
We know that, $a^2 – b^2 = (a + b) (a - b)$
$(x – 1/x + 3) (x – 1/x - 3)$

(ii) $x^4 + 5x^2 + 9$
Solution:-
$x^4 + 5x^2 + 9$
$x^4 + 6x^2 – x^2 + 9$
$(x^4 + 6x^2 + 9) – x^2$

$((x^2)^2 + (2 \times x^2 \times 3) + 3^2)$
We know that, $(a + b)^2 = a^2 + 2ab + b^2$,
$((x^2)^2 + (2 \times x^2 \times 3) + 3^2)$
So, $(x^2 + 3)^2 – x^2$
We know that, $a^2 – b^2 = (a + b)(a - b)$
$(x^2 + 3 + x)(x^2 + 3 - x)$

17.
(i) $a^4 + b^4 – 7a^2b^2$
Solution:-
$a^4 + b^4 – 7a^2b^2$
Above terms can be written as,
$a^4 + b^4 + 2a^2b^2 – 9a^2b^2$
We know that, $(a + b)^2 = a^2 + 2ab + b^2$,
$[(a^2)^2 + (b^2)^2 + (2 \times a^2 \times b^2)] – (3ab)^2$
$(a^2 + b^2)^2 – (3ab)^2$
We know that, $a^2 – b^2 = (a + b)(a - b)$
$(a^2 + b^2 + 3ab)(a^2 + b^2 – 3ab)$

(ii) $x^4 – 14x^2 + 1$
Solution:-
$x^4 – 14x^2 + 1$
Above terms can be written as,
$x^4 + 2x^2 + 1 – 16x^2$
We know that, $(a + b)^2 = a^2 + 2ab + b^2$,
So, $[(x^2)^2 + (2 \times x^2 \times 1) + 1^2] – 16x^2$
$(x^2 + 1)^2 – (4x)^2$
We know that, $a^2 – b^2 = (a + b)(a - b)$
$(x^2 + 1 + 4x)(x^2 + 1 – 4x)$

18. Express each of the following as the difference of two squares:
(i) $(x^2 – 5x + 7)(x^2 + 5x + 7)$
Solution:-
$(x^2 – 5x + 7)(x^2 + 5x + 7)$
Rearranging the above terms, we get,
$((x^2 + 7) – 5x)((x^2 + 7) + 5x)$
As, we know that, $a^2 – b^2 = (a + b)(a - b)$

So, $(x^2 + 7)^2 – (5x)^2$
$(x^2 + 7)^2 -25x^2$

(ii) $(x^2 – 5x + 7) (x^2 – 5x - 7)$
Solution:-
$(x^2 – 5x + 7) (x^2 – 5x - 7)$
$[(x^2 – 5x) + 7) ((x^2 – 5x) - 7)$
As, we know that, $a^2 – b^2 = (a + b) (a - b)$
$(x^2 – 5x)^2 – 7^2$
$(x^2 – 5x)^2 – 49$

(iii) $(x^2 + 5x - 7) (x^2 – 5x + 7)$
Solution:-
$(x^2 + 5x - 7) (x^2 – 5x + 7)$
$[x^2 + (5x – 7)] [x^2 – (5x - 7)]$
As, we know that, $a^2 – b^2 = (a + b) (a - b)$
$x^2 – (5x - 7)^2$
We know that, $(a - b)^2 = a^2 - 2ab + b^2$,
$X^2 – [(5x)^2 – (2 \times 5x \times 7) + 7^2]$
$X^2 – (25x^2 – 70x + 49)$
$X^2 – 25x^2 + 70x – 49$
$-24x^2 + 70x – 49$

19. Evaluate the following by using factors:
(i) $(979)^2 – (21)^2$
(ii) $(99.9)^2 – (0.1)^2$
Solution:

(i) $(979)^2 – (21)^2$
We know that
$= (979 + 21) (979 – 21)$
So we get
$= 1000 \times 958$
$= 958000$

(ii) $(99.9)^2 – (0.1)^2$
We know that

= (99.9 + 0.1) (99.9 – 0.1)
So we get
= 100 × 99.8
= 9980

EXERCISE 4.4

Factorise the following (1 to 18):

1.

(i) $x^2 + 5x + 6$

Solution:-

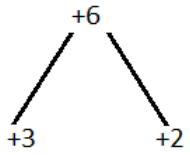
$x^2 + 5x + 6$

$x^2 + 3x + 2x + 6$

Take out common in all terms we get,

$x(x + 3) + 2 (x + 3)$

$(x + 3) (x + 2)$



(ii) $x^2 – 8x + 7$

Solution:-

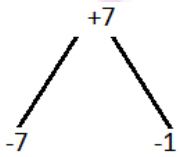
$x^2 – 8x + 7$

$x^2 – 7x – x + 7$

Take out common in all terms we get,

$x(x – 7) – 1(x - 7)$

$(x - 7) (x - 1)$



2.

(i) $x^2 + 6x – 7$

Solution:-

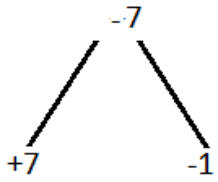
$x^2 + 6x – 7$

$x^2 + 7x – x – 7$

Take out common in all terms we get,

$x(x + 7) – 1(x + 7)$

$(x + 7) (x - 1)$



(ii) $y^2 + 7y – 18$

Solution:-

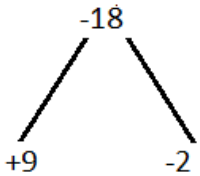
$y^2 + 7y – 18$

$y^2 + 9y – 2y – 18$

Take out common in all terms we get,

$y(y + 9) – 2(y + 9)$

$(y + 9) (y - 2)$



3.

(i) $y^2 – 7y – 18$

Solution:-

$y^2 – 7y – 18$

$y^2 + 2y – 9y – 18$

Take out common in all terms we get,

$y(y + 2) – 9(y + 2)$

$(y + 2) (y - 9)$

(ii) $a^2 – 3a – 54$

Solution:-

$a^2 – 3a – 54$

$a^2 + 6a – 9a – 54$

Take out common in all terms we get,

$a(a + 6) – 9(a + 6)$

So, $(a + 6) (a - 9)$

4.

(i) $2x^2 – 7x + 6$

Solution:-

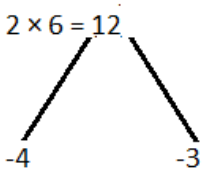
$2x^2 – 7x + 6$

$2x^2 – 4x – 3x + 6$

Take out common in all terms we get,

$2x(x - 2) – 3(x - 2)$

$(x - 2) (2x - 3)$



(ii) $6x^2 + 13x – 5$

Solution:-

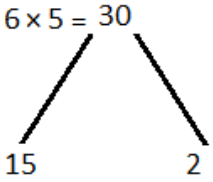
$6x^2 + 13x – 5$

$6x^2 + 15x – 2x - 5$

Take out common in all terms we get,

$3x(2x + 5) – 1(2x + 5)$

$(2x + 5) (3x - 1)$



5.

(i) $6x^2 + 11x – 10$

Solution:-

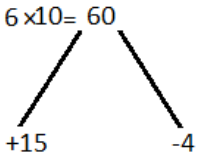
$6x^2 + 11x – 10$

$6x^2 + 15x – 4x – 10$

Take out common in all terms we get,

$3x(2x + 5) – 2(2x + 5)$

$(2x + 5) (3x - 2)$



(ii) $6x^2 – 7x - 3$

Solution:-

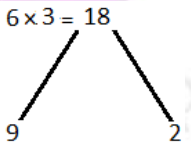
$6x^2 – 7x - 3$

$6x^2 – 9x + 2x – 3$

Take out common in all terms we get,

$3x(2x - 3) + 1(2x - 3)$

$(2x - 3) (3x + 1)$



6.

(i) $2x^2 – x – 6$

Solution:-

$2x^2 – x – 6$

$2x^2 – 4x + 3x – 6$

Take out common in all terms we get,

$2x(x - 2) + 3(x - 2)$

$(x - 2) (2x + 3)$

(ii) $1 – 18y – 63y^2$

Solution:-

$1 – 18y – 63y^2$

$1 – 21y + 3y – 63y^2$

Take out common in all terms we get,

$1(1 – 21y) + 3y(1 – 21y)$

$(1 – 21y) (1 + 3y)$

7.

(i) $2y^2 + y – 45$

Solution:-

$2y^2 + y – 45$

$2y^2 + 10y – 9y – 45$

Take out common in all terms we get,

$2y (y + 5) – 9(y + 5)$

$(y + 5) (2y - 9)$

(ii) $5 – 4x – 12x^2$

Solution:-

$5 – 4x – 12x^2$

$5 – 10x + 6x – 12x^2$

Take out common in all terms we get,

$5(1 – 2x) + 6x(1 – 2x)$

$(1 – 2x) (5 + 6x)$

8.

(i) $x(12x + 7) – 10$

Solution:-

$x(12x + 7) – 10$

Above terms can be written as,

$12x^2 + 7x – 10$

$12x^2 + 15x – 8x – 10$

Take out common in all terms we get,

$3x(4x + 5) – 2(4x + 5)$

$(4x + 5) (3x - 2)$

(ii) $(4 - x)^2 – 2x$

Solution:-

$(4 - x)^2 – 2x$

We know that, $(a - b)^2 = a^2 – 2ab + b^2$

So, $(4^2 – (2 \times 4 \times x) + x^2) – 2x$

$16 – 8x + x^2 – 2x$

$x^2 – 10x + 16$

$x^2 – 8x – 2x + 16$

Take out common in all terms we get,

$x(x - 8) – 2(x - 8)$

$(x - 8) (x -2)$

9.

(i) $60x^2 – 70x – 30$

Solution:-

$60x^2 – 70x – 30$

Take out common in all terms we get,

$10(6x^2 – 7x - 3)$

$10(6x^2 – 9x + 2x - 3)$

Again, take out common in all terms we get,

$10(3x(2x - 3) + 1(2x - 3))$

$10(2x - 3) (3x + 1)$

(ii) $x^2 – 6xy – 7y^2$

Solution:-

$x^2 – 6xy – 7y^2$

$x^2 – 7xy + xy – 7y^2$

Take out common in all terms we get,

$x(x – 7y) + y(x – 7y)$

$(x – 7y) (x + y)$

10.

(i) $2x^2 + 13xy – 24y^2$

Solution:-

$2x^2 + 13xy – 24y^2$

$2x^2 + 16xy – 3xy – 24y^2$

Take out common in all terms we get,

$2x(x + 8y) – 3y(x + 8y)$

$(x + 8y) (2x – 3y)$

(ii) $6x^2 – 5xy – 6y^2$

Solution:-

$6x^2 – 5xy – 6y^2$

$6x^2 – 9xy + 4xy – 6y^2$

Take out common in all terms we get,

$3x(2x – 3y) + 2y (2x – 3y)$

$(2x – 3y) (3x + 2y)$

11.

(i) $5x^2 + 17xy – 12y^2$
Solution:-
$5x^2 + 17xy – 12y^2$
$5x^2 + 20xy – 3xy – 12y^2$
Take out common in all terms we get,
$5x(x + 4y) – 3y(x + 4y)$
$(x + 4y) (5x – 3y)$

(ii) $x^2y^2 – 8xy – 48$
Solution:-
$x^2y^2 – 8xy – 48$
$x^2y^2 – 12xy + 4xy – 48$
Take out common in all terms we get,
$xy(xy - 12) + 4(xy - 12)$
$(xy - 12) (xy + 4)$

12.
(i) $2a^2b^2 – 7ab – 30$
Solution:-
$2a^2b^2 – 7ab – 30$
$2a^2b^2 – 12ab + 5ab – 30$
Take out common in all terms we get,
$2ab(ab - 6) + 5 (ab - 6)$
$(ab - 6) (2ab + 5)$

(ii) $a(2a - b) – b^2$
Solution:-
$a(2a - b) – b^2$
Above terms can be written as,
$2a^2 – ab – b^2$
$2a^2 – 2ab + ab – b^2$
Take out common in all terms we get,
$2a(a - b) + b(a - b)$
$(a - b) (2a + b)$

13.
(i) $(x - y)^2 – 6(x - y) + 5$

Solution:-

$(x - y)^2 – 6(x - y) + 5$

Above terms can be written as,

$(x - y)^2 – 5(x - y) – (x - y) + 5$

$(x - y) (x – y - 5) – 1(x – y - 5)$

Then,

$(x – y - 5) (x – y - 1)$

(ii) $(2x - y)^2 – 11(2x - y) + 28$

Solution:-

$(2x - y)^2 – 11(2x - y) + 28$

Above terms can be written as,

$(2x - y)^2 – 7(2x - y) – 4(2x - y) + 28$

$(2x - y) (2x – y - 7) – 4(2x – y - 7)$

$(2x – y - 7) (2x – y - 4)$

14.

(i) $4(a - 1)^2 – 4(a - 1) – 3$

Solution:-

$4(a - 1)^2 – 4(a - 1) – 3$

Above terms can be written as,

$4(a - 1)^2 – 6(a - 1) + 2(a - 1) - 3$

Take out common in all terms we get,

$2(a - 1) [2(a - 1) - 3] + 1[2(a - 1) - 3]$

$(2(a - 1) - 3) (2(a - 1) + 1)$

$(2a – 2 - 3) (2a – 2 + 1)$

$(2a - 5) (2a - 1)$

(ii) $1 – 2a – 2b – 3(a + b)^2$

Solution:-

$1 – 2a – 2b – 3(a + b)^2$

Above terms can be written as,

$1 – 2(a + b) – 3(a + b)^2$

$1 – 3(a + b) + (a + b) – 3(a + b)^2$

Take out common in all terms we get,

$1(1 – 3(a + b)) + (a + b) (1 – (a + b))$

$(1 – 3(a + b)) (1 + (a + b))$

$(1 – 3a + 3b) (1 + a + b)$

15.

(i) $3 – 5a – 5b – 12(a + b)^2$

Solution:-

$3 – 5a – 5b – 12(a + b)^2$

Above terms can be written as,

$3 – 5(a + b) – 12(a + b)^2$

$3 – 9(a + b) + 4(a + b) – 12(a + b)^2$

Take out common in all terms we get,

$3(1 – 3(a + b)) + 4(a + b) (1 – 3(a + b))$

$(1 – 3(a + b)) (3 + 4(a + b))$

$(1 – 3a – 3b) (3 + 4a + 4b)$

(ii) $a^4 – 11a^2 + 10$

Solution:-

$a^4 – 11a^2 + 10$

Above terms can be written as,

$a^4 – 10a^2 – a^2 + 10$

Take out common in all terms we get,

$a^2 (a^2 - 10) – 1(a^2 - 10)$

$(a^2 - 10) (a^2 - 1)$

16.

(i) $(x + 4)^2 – 5xy -20y – 6y^2$

Solution:-

$(x + 4)^2 – 5xy -20y – 6y^2$

Above terms can be written as,

$(x + 4)^2 – 5y(x + 4) – 6y^2$

$(x + 4)^2 – 6y(x + 4) + y(x + 4) – 6y^2$

Take out common in all terms we get,

$(x + 4) (x + 4 – 6y) + y(x + 4 – 6y)$

$(x – 6y + 4) (x + 4 + y)$

(ii) $(x^2 – 2x^2) – 23(x^2 – 2x) + 120$

Solution:-

$(x^2 – 2x^2) – 23(x^2 – 2x) + 120$

Above terms can be written as,
$(x^2 – 2x)^2 – 15(x^2 – 2x) – 8(x^2 – 2x) + 120$
Take out common in all terms we get,
$(x^2 – 2x) (x^2 – 2x - 15) – 8(x^2 – 2x - 15)$
$(x^2 – 2x - 15) (x^2 – 2x - 8)$

17. $4(2a - 3)^2 – 3(2a - 3) (a - 1) – 7 (a - 1)^2$
Solution:-
$4(2a - 3)^2 – 3(2a - 3) (a - 1) – 7 (a - 1)^2$
Let us assume, $2a – 3 = p$ and $a – 1 = q$
So, $4p^2 – 3pq – 7q^2$
Then, $4p^2 – 7pq + 4pq – 7q^2$
Take out common in all terms we get,
$P(4p – 7q) + q(4p – 7q)$
$(4p – 7q) (p + q)$
Now, substitute the value of p and q we get,
$(4(2a - 3) – 7(a - 1)) (2a – 3 + a - 1)$
$(8a – 12 – 7a + 7) (3a - 4)$
$(a - 5) (3a - 4)$

18. $(2x^2 + 5x) (2x^2 + 5x - 19) + 84$
Solution:-
$(2x^2 + 5x) (2x^2 + 5x - 19) + 84$
Let us assume, $2x^2 + 5x = p$
So, $(p) (p - 19) + 84$
$p^2 – 19p + 84$
$p^2 – 12p – 7p + 84$
$p(p - 12) – 7(p - 12)$
$(p - 12) (p - 7)$
Now, substitute the value of p we get,
$(2x^2 + 5x - 12) (2x^2 + 5x - 7)$

EXERCISE 4.5

Factorise the following (1 to 13):

1.

(i) $8x^3 + y^3$

Solution:-

$8x^3 + y^3$

Above terms can be written as,

$(2x)^3 + y^3$

We know that, $a^3 + b^3 = (a + b)(a^2 – ab + b^2)$

Where, $a = 2x$, $b = y$

Then, $(2x)^3 + y^3 = (2x + y)((2x)^2 – (2x \times y) + y^2)$

$= (2x + y)(4x^2 – 2xy + y^2)$

(ii) $64x^3 – 125y^3$

Solution:-

$64x^3 – 125y^3$

Above terms can be written as,

$(4x)^3 – (5y)^3$

We know that, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Where, $a = 4x$, $b = 5y$

Then, $(4x)^3 – (5y)^3 = (4x - 5y)((4x)^2 + (4x \times 5y) + 5y^2)$

$= (4x - 5y)(16x^2 + 2oxy + 25y^2)$

2.

(i) $64x^3 + 1$

Solution:-

$64x^3 + 1$

Above terms can be written as,

$(4x)^3 + 1^3$

We know that, $a^3 + b^3 = (a + b)(a^2 – ab + b^2)$

Where, $a = 4x$, $b = 1$

Then, $(4x)^3 + 1^3 = (4x + 1)((4x)^2 – (4x \times 1) + 1^2)$

$= (4x + 1)(16x^2 – 4x + 1)$

(ii) $7a^3 + 56b^3$

Solution:-

$7a^3 + 56b^3$
Take out common in all terms we get,
$7(a^3 + 8b^3)$
Above terms can be written as,
$7(a^3 + (2b)^3)$
We know that, $a^3 + b^3 = (a + b)(a^2 – ab + b^2)$
Where, a = a, b = 2b
Then, $7[(a)^3 + (2b)^3] = 7[(a + 2b)((a)^2 – (a \times 2b) + (2b)^2)]$
$= 7(a + 2b)(a^2 – 2ab + 4b^2)$

3.
(i) $(x^6/343) + (343/x^6)$
Solution:-
$(x^6/343) + (343/x^6)$
Above terms can be written as,
$(x^2/7)^3 + (7/x^2)^3$
We know that, $a^3 + b^3 = (a + b)(a^2 – ab + b^2)$
Where, $a = (x^2/7)$, $b = (7/x^2)$
Then, $(x^2/7)^3 + (7/x^2)^3 = [(x^2/7) + (7/x^2)][(x^2/7)^2 - ((x^2/7) \times (7/x^2)) + (7/x^2)^2]$
$= [(x^2/7) + (7/x^2)][(x^4/49) – 1 + (49/x^4)]$

(ii) $8x^3 – 1/27y^3$
Solution:-
$8x^3 – 1/27y^3$
Above terms can be written as,
$(2x)^3 – (1/3y)^3$
We know that, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
Where, a = 2x, b = (1/3y)
Then, $(2x)^3 – (1/3y)^3 = (2x – (1/3y))((2x)^2 + (2x \times (1/3y)) + (3y)^2)$
$= (2x – (1/3y))(4x^2 + (2x/3y) + 9y^2)$

4.
(i) $x^2 + x^5$
Solution:-
$x^2 + x^5$
Take out common in all terms we get,
$x^2(1 + x^3)$

$x^2(1^3 + x^3)$
We know that, $a^3 + b^3 = (a + b)(a^2 – ab + b^2)$
Where, a = 1, b = x
$= x^2 [(1 + x)(1^2 – (1 \times x) + x^2)]$
$= x^2 (1 + x)(1 – x + x^2)$

(ii) $32x^4 – 500x$
Solution:-
$32x^4 – 500x$
Take out common in all terms we get,
$4x(8x^3 - 125)$
Above terms can be written as,
$4x((2x)^3 – 5^3)$
We know that, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
Where, a = 2x, b = 5
$= 4x(2x - 5)((2x)^2 + (2x \times 5) + 5^2)$
$= 4x(2x - 5)(4x^2 + 10x + 25)$

5.
(i) $27x^3y^3 – 8$
Solution:-
$27x^3y^3 – 8$
Above terms can be written as,
$(3xy)^3 – 2^3$
We know that, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
Where, a = 3xy, b = 2
$= (3xy - 2)((3xy)^2 + (3xy \times 2) + 2^2)$
$= (3xy - 2)(9x^2y^2 + 6xy + 4)$

(ii) $27(x + y)^3 + 8(2x - y)^3$
Solution:-
$27(x + y)^3 + 8(2x - y)^3$
Above terms can be written as,
$3^3(x + y)^3 + 2^3(2x - y)^3$
$(3(x + y))^3 + (2(x - y))^3$
We know that, $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
Where, a = 3(x + y), b = 2(x - y)

$= [3(x + y) + 2(2x - y)] [(3(x + y))^3 – (3(x + y) \times 2(2x - y)) + (2(2x - y))^2]$
$= [3x + 3y + 4x – 2y] [9(x + y)^2 – 6(x + y)(2x - y) + 4(2x - y)^2]$
$= (7x - y) [9(x^2 + y^2 + 2xy) – 6(2x^2 – xy + 2xy – y^2) + 4(4x^2 + y^2 – 4xy)]$
$= (7x - y) [9x^2 + 9y^2 + 18xy – 12x^2 – 6xy – 6y^2 + 16x^2 + 4y^2 – 16xy]$
$= (7x - y) [13x^2 – 4xy + 19y^2]$

6.
(i) $a^3 + b^3 + a + b$
Solution:-
$a^3 + b^3 + a + b$
$(a^3 + b^3) + (a + b)$
We know that, $a^3 + b^3 = (a + b) (a^2 - ab + b^2)$
$[(a + b) (a^2 – ab + b^2)] + (a + b)$
$(a + b) (a^2 – ab + b^2 + 1)$

(ii) $a^3 – b^3 – a + b$
Solution:-
$a^3 – b^3 – a + b$
$(a^3 – b^3) – (a - b)$
We know that, $a^3 - b^3 = (a - b) (a^2 + ab + b^2)$
$[(a - b) (a^2 + ab + b^2)] - (a - b)$
$(a - b) (a^2 + ab + b^2 - 1)$

7.
(i) $x^3 + x + 2$
Solution:-
$x^3 + x + 2$
Above terms can be written as,
$x^3 + x + 1 + 1$
Rearranging the above terms, we get
$(x^3 + 1) (x + 1)$
$(x^3 + 1^3) (x + 1)$
We know that, $a^3 + b^3 = (a + b) (a^2 - ab + b^2)$
$[(x + 1) (x^2 – x + 1)] + (x + 1)$
$(x + 1) (x^2 – x + 1 + 1)$
$(x + 1) (x^2 – x + 2)$

(ii) $a^3 – a – 120$

Solution:-

$a^3 – a – 120$

Above terms can be written as,

$a^3 – a – 125 + 5$

Rearranging the above terms, we get

$a^3 - 125 – a + 5$

$(a^3 - 125) – (a - 5)$

$(a^3 – 5^3) – (a - 5)$

We know that, $a^3 - b^3 = (a - b) (a^2 + ab + b^2)$

$[(a - 5) (a^2 + 5a + 5^2)] – (a - 5)$

$(a - 5) (a^2 + 5a + 25) – (a - 5)$

$(a - 5) (a^2 + 5a + 25 - 1)$

$(a - 5) (a^2 + 5a + 24)$

8.

(i) $x^3 + 6x^2 + 12x + 16$

Solution:-

$x^3 + 6x^2 + 12x + 16$

$x^3 + 6x^2 + 12x + 8 + 8$

Above terms can be written as,

$(x^3 + (3 \times 2 \times x^2) + (3 \times 2^2 \times x) + 2^3) + 8$

We know that, $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

Now a = x and b = 2

So, $(x + 2)^3 + 2^3$

We know that, $a^3 + b^3 = (a + b) (a^2 - ab + b^2)$

$(x + 2 + 2) ((x + 2)^2 – (2 \times (x + 2)) + 2^2)$

$(x + 4) (x^2 + 4 + 4x – 2x – 4 + 4)$

$(x + 4) (x^2 + 2x + 4)$

(ii) $a^3 – 3a^2b + 3ab^2 – 2b^3$

Solution:-

$a^3 – 3a^2b + 3ab^2 – 2b^3$

Above terms can be written as,

$a^3 – 3a^2b + 3ab^2 – b^3 – b^3$

We know that, $(a - b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$

So, $(a - b)^3 + b^3$

We also know that, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
Where, a = a – b, b = b
$(a – b - b)((a - b)^2 + (a - b)b + b^2)$
$(a – 2b)(a^2 + b^2 – 2ab + ab – b^2 + b^2)$
$(a – 2b)(a^2 + b^2 - ab)$

9.
(i) $2a^3 + 16b^3 – 5a – 10b$
Solution:-
$2a^3 + 16b^3 – 5a – 10b$
Above terms can be written as,
$2(a^3 + 8b^3) – 5(a + 2b)$
$2(a^3 + (2b)^3) – 5(a + 2b)$
We know that, $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
$2[(a + 2b)(a^2 – 2ab + 4b^2)] – 5(a + 2b)$
$(a + 2b)(2a^2 – 4ab + 8b^2 - 5)$

(ii) $a^3 – (1/a^3) – 2a + 2/a$
Solution:-
$a^3 – (1/a^3) – 2a + 2/a$
$(a^3 – (1/a)^3) – 2a + 2/a$
We know that, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
$[(a – 1/a) – (a^2 + (a \times 1/a) + (1/a)^2] – 2(a – 1/a)$
$(a – 1/a)(a^2 + 1 + 1/a^2) – 2(a – 1/a)$
$(a – 1/a)(a^2 + 1 + 1/a^2 - 2)$
$(a – 1/a)(a^2 + (1/a^2) - 1)$

10.
(i) $a^6 – b^6$
Solution:-
$a^6 – b^6$
Above terms can be written as,
$(a^2)^3 – (b^2)^3$
We know that, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
So, $a = a^2$, $b = b^2$
$(a^2 – b^2)((a^2)^2) + a^2b^2 + (b^2)^2)$
$(a^2 – b^2)(a^4 + a^2b^2 + b^4)$

(ii) $x^6 – 1$

Solution:-

$x^6 – 1$

Above terms can be written as,

$(x^2)^3 – 1^3$

We know that, $a^3 - b^3 = (a - b) (a^2 + ab + b^2)$

So, $a = x^2$, $b = 1$

$(x^2 - 1) ((x^2)^2 + (x^2 \times 1) + 1^2)$

$(x^2 - 1) (x^4 + x^2 + 1)$

11.

(i) $64x^6 – 729y^6$

Solution:-

$64x^6 – 729y^6$

Above terms can be written as,

$(2x)^6 – (3y)^6$

$[(2x)^2]^3 – [(3y)^2]^3$

We know that, $a^3 - b^3 = (a - b) (a^2 + ab + b^2)$

So, $a = (2x)^2$, $b = (3y)^2$

$[(2x)^2 – (3y)^2] [((2x)^2)^2 + ((2x)^2 \times (3y)^2) + ((3y)^2)^2]$

$(4x^2 – 9y^2) [16x^4 + (4x^2 \times 9y^2) + (9y^2)^2]$

$(4x^2 – 9y^2) [16x^4 + 36x^2y^2 + 81y^4]$

$[(2x)^2 – (3y)^2] [16x^4 + 36x^2y^2 + 81y^4]$

$(2x + 3y) (2x – 3y) (16x^4 + 36x^2y^2 + 81y^4)$

(ii) $x^3 – (8/x)$

Solution:-

$x^3 – (8/x)$

Above terms can be written as,

$(1/x) (x^3 - 8)$

$(1/x) [(x)^3 – (2)^3]$

We know that, $a^3 - b^3 = (a - b) (a^2 + ab + b^2)$

So, $a = x$, $b = 2$

$(1/x) (x - 2) (x^2 + 2x + 4)$

12.

(i) $250 (a - b)^3 + 2$

Solution:-

$250 (a - b)^3 + 2$

Take out common in all terms we get,

$2(125(a - b)^3 + 1)$

$2[(5(a - b))^3 + 1^3]$

We know that, $a^3 + b^3 = (a + b) (a^2 - ab + b^2)$

$= 2[(5a – 5b + 1) ((5a – 5b)^2 – (5a – 5b)1 + 1^2)]$

$= 2(5a – 5b + 1) (25a^2 + 25b^2 – 50ab – 5a + 5b + 1)$

(ii) $32a^2x^3 – 8b^2x^3 – 4a^2y^3 + b^2y^3$

Solution:-

$32a^2x^3 – 8b^2x^3 – 4a^2y^3 + b^2y^3$

Take out common in all terms we get,

$8x^3(4a^2 – b^2) – y^3(4a^2 – b^2)$

$(4a^2 – b^2) (8x^3 – y^3)$

Above terms can be written as,

$((2a)^2 – b^2) ((2x)^3 – y^3)$

We know that, $a^3 - b^3 = (a - b) (a^2 + ab + b^2)$ and $(a^2 – b^2) = (a + b) (a - b)$

$(2a + b) (2a - b) [(2x - y) ((2x)^2 + 2xy + y^2)]$

$(2a + b) (2a - b) (2x - y) (4x^2 + 2xy + y^2)$

13.

(i) $x^9 + y^9$

Solution:-

$x^9 + y^9$

Above terms can be written as,

$(x^3)^3 + (y^3)^3$

We know that, $a^3 + b^3 = (a + b) (a^2 - ab + b^2)$

Where, $a = x^3$, $b = y^3$

$(x^3 + y^3) ((x^3)^2 – x^3y^3 + (y^3)^2)$

$(x^3 + y^3) (x^6 – x^3y^3 + y^6)$

Then, $(x^3 + y^3)$ in the form of $(a^3 + b^3)$

$(x + y)(x^2 – xy + y^2) (x^6 – x^3y^3 + y^6)$

(ii) $x^6 – 7x^3 – 8$

Solution:-

$X^6 – 7x^3 – 8$

Above terms can be written as,

$(x^2)^3 – 7x^3 – x^3 + x^3 – 8$

$(x^2)^3 – 8x^3 + x^3 – 2^3$

$(((x^2)^3) – (2x)^3) + (x^3 – 2^3)$

We know that, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$(x^2 – 2x)((x^2)^2 + (x^2 \times 2x) + (2x)^2) + (x - 2)(x^2 + 2x + 2^2)$

$(x^2 – 2x)(x^4 + 2x^3 + 4x^2) + (x - 2)(x^2 + 2x + 4)$

$x(x - 2)\, x^2(x^2 + 2x + 4) + (x - 2)(x^2 + 2x + 4)$

Take out common in all terms we get,

$(x - 2)(x^2 + 2x + 4)((x \times x^2) + 1)$

$(x - 2)(x^2 + 2x + 4)(x^3 + 1)$

So, above terms are in the form of $a^3 + b^3$

Therefore, $(x - 2)(x^2 + 2x + 4)(x + 1)(x^2 – x + 1)$

CHAPTER TEST

Factorise the following (1 to 12):

1.

(i) $15(2x - 3)^3 – 10(2x - 3)$

Solution:-

$15(2x - 3)^3 – 10(2x - 3)$

Take out common in both terms,

Then, $5(2x – 3) [3(2x - 3)^2 – 2]$

(ii) $a(b - c) (b + c) – d(c - b)$

Solution:-

$a(b - c) (b + c) – d(c - b)$

Above terms can be written as,

$a(b - c) (b + c) + d(b - c)$

Take out common in both terms,

$(b - c) [a(b + c) + d]$

$(b - c) (ab + ac + d)$

2.

(i) $2a^2x – bx + 2a^2 – b$

Solution:-

$2a^2x – bx + 2a^2 – b$

Rearrange the above terms we get,

$2a^2x + 2a – bx – b$

Take out common in both terms,

$2a^2(x + 1) – b(x + 1)$

$(x + 1) (2a^2 - b)$

(ii) $p^2 – (a + 2b)p + 2ab$

Solution:-

$p^2 – (a + 2b)p + 2ab$

Above terms can be written as,

$p^2 – ap - 2bp + 2ab$

Take out common in both terms,

$p(p - a) - 2b(p - a)$

$(p – a) (p – 2b)$

3.

(i) $(x^2 – y^2)z + (y^2 – z^2)x$

Solution:-

$(x^2 – y^2)z + (y^2 – z^2)x$

Above terms can be written as,

$zx^2 – zy^2 + xy^2 – xz^2$

Rearrange the above terms we get,

$zx^2 – xz^2 + xy^2 – zy^2$

Take out common in both terms,

$zx(x - z) + y^2(x - z)$

$(x - z) (zx + y^2)$

(ii) $5a^4 – 5a^3 + 30a^2 – 30a$

Solution:-

$5a^4 – 5a^3 + 30a^2 – 30a$

Take out common in both terms,

$5a(a^3 – a^2 + 6a - 6)$

$5a[a^2(a - 1) + 6(a - 1)]$

$5a(a - 1) (a^2 + 6)$

4.

(i) $b(c -d)^2 + a(d - c) + 3c – 3d$

Solution:-

$b(c -d)^2 + a(d - c) + 3c – 3d$

Above terms can be written as,

$b(c - d)^2 – a(c - d) + 3c – 3d$

$b(c - d)^2 – a(c - d) + 3(c - d)$

Take out common in both terms,

$(c - d) [b(c - d) – a + 3]$

$(c - d) (bc – bd – a + 3)$

(ii) $x^3 – x^2 – xy + x + y – 1$

Solution:-

$x^3 – x^2 – xy + x + y – 1$

Rearrange the above terms we get,

$x^3 – x^2 – xy + y + x – 1$

Take out common in both terms,

$x^2(x - 1) – y(x - 1) + 1(x - 1)$
$(x - 1) (x^2 – y + 1)$

5.
(i) x(x + z) – y (y + z)
Solution:-
$x(x + z) – y (y + z)$
$x^2 + xz – y^2 – yz$
Rearrange the above terms we get,
$x^2 – y^2 + xz – yz$
We know that, $(a^2 – b^2) = (a + b) (a - b)$
So, $(x + y) (x - y) + z(x - y)$
$(x - y) (x + y + z)$

(ii) $a^{12}x^4 – a^4x^{12}$
Solution:-
$a^{12}x^4 – a^4x^{12}$
Take out common in both terms,
$a^4x^4 (a^8 – x^8)$
$a^4x^4((a^4)^2 – (x^4)^2)$
We know that, $(a^2 – b^2) = (a + b) (a - b)$
$a^4x^4 (a^4 + x^4) (a^4 – x^4)$
$a^4x^4 (a^4 + x^4) ((a^2)^2 – (x^2)^2)$
$a^4x^4(a^4 + x^4) (a^2 + x^2) (a^2 – x^2)$
$a^4x^4 (a^4 + x^4) (a^2 + x^2) (a + x) (a - x)$

6.
(i) $9x^2 + 12x + 4 – 16y^2$
Solution:-
$9x^2 + 12x + 4 – 16y^2$
Above terms can be written as,
$(3x)^2 + (2 \times 3x \times 2) + 2^2 – 16y^2$
Then, $(3x + 2)^2 + (4y)^2$
$(3x + 2 + 4y) (3x + 2 – 4y)$

(ii) $x^4 + 3x^2 + 4$
Solution:-

$x^4 + 3x^2 + 4$
Above terms can be written as,
$(x^2)^2 + 3(x^2) + 4$
$(x^2)^2 + (2)^2 + 4x^2 – x^2$
$(x^2 + 2)^2 – (x^2)$
We know that, $(a^2 – b^2) = (a + b) (a - b)$
$(x^2 + 2 + x) (x^2 + 2 - x)$
$(x^2 + x + 2) (x^2 – x + 2)$

7.
(i) $21x^2 – 59xy + 40y^2$
Solution:-
$21x^2 – 59xy + 40y^2$
By multiplying the first and last term we get, $21 \times 40 = 840$
Then, $(-35) \times (-24) = 840$
So, $21x^2 – 35xy – 24xy + 40y^2$
$7x(3x – 5y) – 8y(3x – 5y)$
$(3x – 5y) (7x – 8y)$

(ii) $4x^3y – 44x^2y + 112xy$
Solution:-
$4x^3y – 44x^2y + 112xy$
Take out common in all terms,
$4xy(x^2 – 11x + 28)$
Then, $4xy (x^2 – 7x – 4x + 28)$
$4xy [x(x - 7) – 4(x + 7)]$
$4xy (x - 7) (x - 4)$

8.
(i) $x^2y^2 – xy - 72$
Solution:-
$x^2y^2 – xy – 72$
$x^2y^2 – 9xy + 8xy – 72$
Take out common in all terms,
$xy(xy - 9) + 8(xy - 9)$
$(xy - 9) (xy + 8)$

(ii) $9x^3y + 41x^2y^2 + 20xy^3$

Solution:-

$9x^3y + 41x^2y^2 + 20xy^3$

Take out common in all terms,

$xy(9x^2 + 41xy + y^2)$

Above terms can be written as,

$xy\ (9x^2 + 36xy + 5xy + 20y^2)$

$xy\ [9x(x + 4y) + 5y(x + 4y)]$

$xy\ (x + 4y)\ (9x + 5y)$

9.

(i) $(3a – 2b)^2 + 3(3a – 2b) - 10$

Solution:-

$(3a – 2b)^2 + 3(3a – 2b) - 10$

Let us assume, $(3a – 2b) = p$

$p^2 + 3p – 10$

$p^2 + 5p – 2p – 10$

Take out common in all terms,

$p(p + 5) – 2(p + 5)$

$(p + 5)\ (p - 2)$

Now, substitute the value of p

$(3a – 2b + 5)\ (3a – 2b - 2)$

(ii) $(x^2 – 3x)\ (x^2 – 3x + 7) + 10$

Solution:-

$(x^2 – 3x)\ (x^2 – 3x + 7) + 10$

Let us assume, $(x^2 – 3x) = q$

$q\ (q + 7) + 10$

$q^2 + 7q + 10$

$q^2 + 5q + 2q + 10$

$q(q + 5) + 2(q + 5)$

$(q + 5)\ (q + 2)$

Now, substitute the value of q

$(x^2 – 3x + 5)\ (x^2 – 3x + 2)$

10.

(i) $(x^2 - x)\ (4x^2 – 4x - 5) - 6$

Solution:-

$(x^2 - x)(4x^2 – 4x - 5) – 6$

$(x^2 - x)[(4x^2 – 4x) - 5] – 6$

$(x^2 - x)[4(x^2 - x) - 5] – 6$

Let us assume $x^2 – x = q$

So, $q[4q - 5] – 6$

$4q^2 – 5q – 6$

$4q^2 – 8q + 3q – 6$

$4q(q - 2) + 3(q - 2)$

$(q - 2)(4q + 3)$

Now, substitute the value of q

$(x^2 – x - 2)(4(x^2 - x) + 3)$

$(x^2 – x – 2)(4x^2 – 4x + 3)$

$(x^2 – 2x + x - 2)(4x^2 – 4x + 3)$

$[x(x - 2) + 1(x - 2)](4x^2 – 4x + 3)$

$(x - 2)(x + 1)(4x^2 – 4x + 3)$

(ii) $x^4 + 9x^2y^2 + 81y^4$

Solution:-

$x^4 + 9x^2y^2 + 81y^4$

Above terms can be written as,

$x^4 + 18x^2y^2 + 81y^4 – 9x^2y^2$

$((x^2)^2 + (2 \times x^2 \times 9y^2) + (9y^2)^2) – 9x^2y^2$

We know that, $(a + b)^2 = a^2 + 2ab + b^2$

$(x^2 + 9y^2)^2 – (3xy)^2$

$(x^2 + 9y^2 + 3xy)(x^2 + 9y^2 – 3xy)$

11.

(i) $(8/27)x^3 – (1/8)y^3$

Solution:-

$(8/27)x^3 – (1/8)y^3$

Above terms can be written as,

$((2/3)x)^3 – (½y)^3$

We know that, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$((2/3)x – ½y)[(2/3)x + (2/3)x(1/2)y + ((1/2)y)^2]$

$((2/3)x – (1/2)y)[(4/9)x^2 + (xy/3) + (y^2/4)]$

(ii) $x^6 + 63x^3 – 64$

Solution:-

$x^6 + 63x^3 – 64$

Above terms can be written as,

$x^6 + 64x^3 – x^3 – 64$

Take out common in all terms,

$x^3 (x^3 + 64) – 1(x^3 + 64)$

$(x^3 + 64) (x^3 - 1)$

$(x^3 + 4^3) (x^3 – 1^3)$

We know that, $a^3 - b^3 = (a - b) (a^2 + ab + b^2)$ and $a^3 + b^3 = (a + b) (a^2 - ab + b^2)$

So, $(x + 4) [x^2 – 4x + 4^2] (x - 1) [x^2 + x + 1^2]$

$(x + 4) (x^2 – 4x + 16) (x - 1) (x^2 + x + 1)$

12.

(i) $x^3 + x^2 – (1/x^2) + (1/x^3)$

Solution:-

$x^3 + x^2 – (1/x^2) + (1/x^3)$

Rearranging the above terms, we get,

$x^3 + (1/x^3) + x^2 – (1/x^2)$

We know that, $a^3 - b^3 = (a - b) (a^2 + ab + b^2)$ and $(a^2 – b^2) = (a + b) (a - b)$

$(x + 1/x) (x^2 – 1 + 1/x^2) + (x + 1/x) (x – 1/x)$

$(x + 1/x) [x^2 – 1 + 1/x^2 + x – 1/x]$

(ii) $(x + 1)^6 – (x - 1)^6$

Solution:-

$(x + 1)^6 – (x - 1)^6$

Above terms can be written as,

$((x + 1)^3)^2 – ((x - 1)^3)^2$

We know that, $(a^2 – b^2) = (a + b) (a - b)$

$[(x + 1)^3 + (x - 1)^3] [(x + 1)^3 – (x - 1)^3]$

$[(x + 1) + (x - 1)][(x + 1)^2 – (x - 1) (x + 1) + (x - 1)^2] [(x + 1) - (x - 1)][(x + 1)^2 + (x - 1) (x + 1) + (x - 1)^2]$

$(x + 1 + x - 1) [x^2 + 2x + 1 – x^2 + 1 + x^2 + 1 – 2x(x + 1) – x + 1] [x^2 + 2x + 1 + x^2 – 1 + x^2 – 2x + 1]$

By simplifying we get,

$2x(x^2 + 3) 2(3x^2 + 1)$

$4x(x^2 + 3) (3x^2 + 1)$

13. Show that $(97)^3 + (14)^3$ is divisible by 111

Solution:-

From the question,

$(97)^3 + (14)^3$

We know that, $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

So, $(97 + 14)[(97)^2 – (97 \times 14) + (14)^2]$

$111[(97)^2 – (97 \times 14) + (14)^2]$

Therefore, it is clear that the given expression is divisible by 111.

14. If a + b = 8 and ab = 15, find the value of $a^4 + a^2b^2 + b^4$.

Solution:-

$a^4 + a^2b^2 + b^4$

Above terms can be written as,

$a^4 + 2a^2b^2 + b^4 – a^2b^2$

$(a^2)^2 + 2a^2b^2 + (b^2)^2 – (ab)^2$

$(a^2 + b^2)^2 – (ab)^2$

$(a^2 + b^2 + ab)(a^2 + b - ab)$

$a + b = 8, ab = 15$

So, $(a + b)^2 = 8^2$

$a^2 + 2ab + b^2 = 64$

$a^2 + 2(15) + b^2 = 64$

$a^2 + b^2 + 30 = 64$

By transposing,

$a^2 + b^2 = 64 – 30$

$a^2 + b^2 = 34$

Then, $a^4 + a^2b^2 + b^4$

$= (a^2 + b^2 + ab)(a^2 + b^2 - ab)$

$= (34 + 15)(34 - 15)$

$= 49 \times 19$

$= 931$