Exercise 5.1

Solve the following systems of simultaneous linear equations by the substitution method (1 to 4):

1. (i) x + y = 14

x − y = 4

(ii) s − t = 3

s/3 + t/2 = 6

(iii) 2x + 3y = 9

3x + 4y = 5

(iv) 3x − 5y = 4

9x − 2y = 7

Solution:

(i) x + y = 14

x − y = 4

It can be written as

x = 4 + y

By substituting the value in the above equation

4 + y + y = 14

By further calculation

2y = 14 − 4 = 10

Dividing by 2

y = 10/2 = 5

So we get

x = 4 + 5 = 9

Hence, x = 9 and y = 5.

(ii) s − t = 3

s/3 + t/2 = 6

By taking LCM

2s + 3t = 6 × 6 = 36

We know that

s − t = 3 …. (1)

2s + 3t = 36 ….. (2)

So we get

s = 3 + t …. (3)

By substituting the value of s in equation (2)

2 (3 + t) + 3t = 36

By further calculation

6 + 2t + 3t = 36

So we get

5t = 36 − 6 = 30

By division

t = 30/5 = 6

Substituting t in equation (3)

s = 3 + 6 = 9

Hence, s = 9 and t = 6.

(iii) 2x + 3y = 9 …. (1)

3x + 4y = 5 ….. (2)
Equation (1) can be written as
2x = 9 – 3y
x = (9 – 3y)/ 2 …. (3)
By substituting the value of x in equation (2)
3 × (9 – 3y)/ 2 + 4y = 5
By further calculation
(27 – 9y)/ 2 + 4y = 5
By taking LCM
27 – 9y + 8y = 10
So we get
-y = - 17
y = 17

Substituting y in equation (3)
x = [9 – (3 × 17)]/ 2
By further calculation
x = (9 – 51)/ 2
x = - 21

Hence, x = - 21 and y = 17.

(iv) 3x – 5y = 4 ….. (1)
9x – 2y = 7 …. (2)
Multiply equation (1) by 3
9x – 15y = 12
9x – 2y = 7
By subtracting both the equations
- 13y = 5
y = -5/13

Equation (1) can be written as
3x – 5y = 4
x = (4 + 5y)/ 3 ….. (3)
By substituting the value of x in equation (2)
9 [(4 + 5y)/ 3] – 2y = 7
By further calculation
12 + 15y – 2y = 7
13y = - 5
So we get
y = -5/13

Substituting y in equation (3)

$$x = \frac{4 + 5 \times \frac{-5}{13}}{3}$$

By further calculation

$$= \frac{4 - \frac{25}{13}}{3}$$

Taking LCM

$$= \frac{\frac{52 - 25}{13}}{3}$$

So we get

$$= \frac{27}{13 \times 3}$$

$$= \frac{9}{13}$$

Hence, x = 9/13 and y = - 5/13.

2. (i) a + 3b = 5
7a – 8b = 6
(ii) 5x + 4y – 4 = 0
x – 20 = 12y
Solution:

(i) a + 3b = 5 …. (1)
7a – 8b = 6 ….. (2)
Now multiply equation (1) by 7
7a + 21b = 35 …. (3)
7a – 8b = 6 ….. (4)
By subtracting both the equations
29b = 29
So we get
b = 29/29 = 1

Now substituting b = 1 in equation (1)
a + 3 (1) = 5
By further calculation
a + 3 = 5
So we get
a = 5 – 3 = 2

Therefore, a = 2 and b = 1.

(ii) 5x + 4y – 4 = 0

x – 20 = 12y

We can write it as

5x + 4y = 4 …. (1)

x – 12y = 20 ….. (2)

Now multiply equation (2) by 5

5x + 4y = 4 …. (3)

5x – 60y = 100

By subtracting both the equations

64y = - 96

So we get

y = -96/64 = - 3/2

Now substitute the value of y in equation (1)

5x + 4 (-3/2) = 4

By further calculation

5x + 2 (-3) = 4

So we get

5x – 6 = 4

5x = 4 + 6 = 10

By division

x = 10/5 = 2

Therefore, x = 2 and y = - 3/2.

3. (i) 2x – 3y/4 = 3

5x – 2y – 7 = 0

(ii) 2x + 3y = 23

5x – 20 = 8y

Solution:

(i) 2x – 3y/4 = 3

5x – 2y – 7 = 0

We can write it as

2x/1 – 3y/4 = 3

By taking LCM

(8x – 3y)/ 4 = 3

By cross multiplication

8x – 3y = 12 ….. (1)

5x – 2y = 7 …. (2)

Now multiply equation (1) by 2 and (2) by 3

16x – 6y = 24

15x – 6y = 21

By subtracting both the equations

x = 3

Now substituting the value of x in equation (1)

8 × 3 – 3y = 12

By further calculation

24 – 3y = 12

- 3y = 12 – 24

So we get
- 3y = - 12
y = - 12/-3 = 4

Therefore, x = 3 and y = 4.

(ii) 2x + 3y = 23
5x – 20 = 8y
We can write it as
2x + 3y = 23 …. (1)
5x – 8y = 20 …. (2)
By multiplying equation (1) by 5 and equation (2) by 2
10x + 15y = 115
10x – 16y = 40
By subtracting both the equations
31y = 75
So we get
y = 75/31 = 2 13/31

By substituting the value of y in equation (1)
2x + 3 (75/31) = 23
By further calculation
2x + 225/31 = 23
We can write it as
2x = 23/1 – 225/31
Taking LCM
2x = (713 – 225)/ 31 = 488/31
So we get
x = 488/ (31 × 2) = 244/ 31 = 7 27/31

Therefore, x = 7 27/31 and y = 2 13/31.

4. (i) mx – ny = m² + n²
x + y = 2m
(ii) 2x/a + y/b = 2
x/a – y/b = 4
Solution:

(i) mx – ny = m² + n² …. (1)
x + y = 2m …. (2)
We can write it as
x = 2m – y ….. (3)
Now substitute the value of x in (1)
m (2m – y) – ny = m² + n²
By further calculation
2m² – my – ny = = m² + n²
Taking out y as common
m² – y (m + n) = n²
It can be written as
m² – n² – y (m + n) = 0

Expanding using formula
(m − n) (m + n) − y (m + n) = 0
Taking (m + n) as common
(m + n) [(m − n) − y] = 0
So we get
m − n − y = 0
y = m − n

From equation (3)
x = 2m − (m − n)
By further calculation
x = 2m − m + n = m + n

Hence, x = m + n and y = m − n.

(ii) 2x/a + y/b = 2 …. (1)
x/a − y/b = 4 …. (2)
Adding both the equations
3x/a = 6
So we get
x = 6a/3 = 2a

Substituting x in equation (1)
2 (2a)/ a + y/b = 2
By further calculation
4a/a + y/b = 2
So we get
4 + y/b = 2
y/b = 2 − 4 = - 2
Here
y = - 2b

Therefore, x = 2a and y = - 2b.

5. Solve 2x + y = 35, 3x + 4y = 65. Hence, find the value of x/y.
Solution:

It is given that
2x + y = 35 …. (1)
3x + 4y = 65 ….. (2)
Now multiply equation (1) by 4
8x + 4y = 140 …. (3)
3x + 4y = 65 …. (4)
By subtracting both the equations
5x = 75
x = 75/5 = 15

Now substituting the value of x in equation (1)
8 × 15 + 4y = 140
By further calculation

120 + 4y = 140
4y = 140 – 120
So we get
4y = 20
y = 20/4 = 5
Here
x/y = 15/5 = 3

Therefore, x/y = 3.

6. Solve the simultaneous equations 3x – y = 5, 4x – 3y = - 1. Hence, find p, if y = px – 3.
Solution:

It is given that
3x – y = 5 ….. (1)
4x – 3y = - 1 ….. (2)
Now multiply equation (1) by 3
9x – 3y = 15 ….. (3)
4x – 3y = - 1 ….. (4)
Subtracting equation (3) and (4)
5x = 16
x = 16/5

Substitute the value of x in equation (3)
3 × 16/5 – y = 5
By further calculation
48/5 – y = 5
48/5 – 5 = y
Taking LCM
(48 – 25)/ 5 = y
So we get
y = 23/5

We know that
y = px - 3
23/5 = p × 16/5 – 3
Substitute the value of x and y
23/5 + 3 = 16p/5
Taking LCM
(23 + 15)/ 5 = 16p/5
By further calculation
38/5 = 16p/5
So we get
16p = 38
p = 19/8

Therefore, x = 16/5, y = 23/5 and p = 19/8.

Exercise 5.2

Solve the following systems of simultaneous linear equations by the elimination method (1 to 9):

1. (i) 3x + 4y = 10
2x – 2y = 2
(ii) 2x = 5y + 4
3x – 2y + 16 = 0
Solution:

(i) 3x + 4y = 10 ….. (1)
2x – 2y = 2 ….. (2)
Multiplying equation (1) by 1 and (2) by 2
3x + 4y = 10
4x – 4y = 4
By adding both the equations
7x = 14
By division
x = 14/7 = 2

Substituting the value of x in equation (2)
2 × 2 – 2y = 2
By further calculation
4 – 2y = 2
So we get
2y = 4 – 2 = 2
y = 2/2 = 1

Therefore, x = 2 and y = 1.

(ii) 2x = 5y + 4
3x – 2y + 16 = 0
We can write it as
2x – 5y = 4 …. (1)
3x – 2y = - 16 ….. (2)
Now multiply equation (1) by 3 and (2) by 2
6x – 15y = 12 ….. (3)
6x – 4y = - 32 ….. (4)
By subtracting both the equations
- 11y = 44
y = -44/11 = - 4

Substitute the value of y in equation (1)
2x – 5 (-4) = 4
By further calculation
2x + 20 = 4
So we get
2x = 4 – 20 = - 16
x = - 16/2 = - 8

Therefore, x = - 8 and y = - 4.

2. (i) ¾ x – 2/3 y = 1
3/8 x – 1/6 y = 1
(ii) 2x – 3y – 3 = 0
2x/3 + 4y + ½ = 0.
Solution:

(i) ¾ x – 2/3 y = 1
3/8 x – 1/6 y = 1
We can write it as
¾ x – 2/3 y = 1
(9x – 8y)/ 12 = 1
By cross multiplication
9x – 8y = 12 ….. (1)

3/8 x – 1/6 y = 1
(9x – 4y)/ 24 = 1
By cross multiplication
9x – 4y = 24 ….. (2)

Subtracting equations (1) and (2)
- 4y = - 12
By division
y = - 12/ - 4 = 3

Substitute the value of y in (1)
9x – 8 × 3 = 12
By further calculation
9x – 24 = 12
9x = 12 + 24 = 36
By division
x = 36/ 9 = 4

Therefore, x = 4 and y = 3.

(ii) 2x – 3y – 3 = 0
2x/3 + 4y + ½ = 0
We can write it as
2x – 3y – 3 = 0
2x – 3y = 3 ….. (1)

2x/3 + 4y + ½ = 0
2x/3 + 4y = - ½
Taking LCM
(2x + 12y)/ 3 = - ½
By cross multiplication
2 (2x + 12y) = - 1 × 3
So we get
4x + 24y = - 3 …. (2)
Multiply equation (1) by 2
4x – 6y = 6

4x + 24y = - 3
By subtracting both the equations
- 30y = 9
So we get
y = -9/30 = - 3/10

Substitute the value of y in equation (1)
2x – 3 (-3/10) = 3
By further calculation
2x + 9/10 = 3
We can write it as
2x = 3 – 9/10
By taking LCM
2x = (30 – 9)/ 10
So we get
2x = 21/10
x = 21/20

Therefore, x = 21/20 and y = - 3/10.

3. (i) 15x – 14y = 117
14x – 15y = 115
(ii) 41x + 53y = 135
53x + 41y = 147.
Solution:

(i) 15x – 14y = 117 ….. (1)
14x – 15y = 115 ….. (2)
Now multiply equation (1) by 14 and (2) by 15
210x – 196y = 1638 …. (3)
210x – 225y = 1725 ….. (4)
By subtracting both the equations
29y = - 87
So we get
y = -87/29 = - 3

Substitute the value of y in equation (1)
15x – 14 (-3) = 117
By further calculation
15x + 42 = 117
So we get
15x = 117 – 42 = 75
By division
x = 75/15 = 5

Therefore, x = 5 and y = - 3.

(ii) 41x + 53y = 135 ….. (1)
53x + 41y = 147 ….. (2)
Now multiply equation (1) by 53 and (2) by 41

2173x + 2809y = 7155 ….. (3)
2173x + 1681y = 6027 ….. (4)
By subtracting both the equations
1128y = 1128
So we get
y = 1128/1128 = 1

Substitute the value of y in equation (1)
41x + 53 × 1 = 135
By further calculation
41x + 53 = 135
So we get
41x = 135 – 53 = 82
By division
x = 82/41 = 2

Therefore, x = 2 and y = 1.

4. (i) x/6 = y – 6
3x/4 = 1 + y
(ii) x – 2/3 y = 8/3
2x/5 – y = 7/5.
Solution:

(i) x/6 = y – 6
3x/4 = 1 + y
We can write it as
x = 6 (y – 6)
x = 6y – 36
x – 6y = - 36 ….. (1)

3x/4 = 1 + y
By cross multiplication
3x = 4 (1 + y)
So we get
3x = 4 + 4y
3x – 4y = 4 …. (2)

Multiply equation (1) by 3
3x – 18y = - 108
3x – 4y = 4
Subtracting both the equations
- 14y = - 112
So we get
y = - 112/- 14 = 8

Substitute the value of y in equation (1)
x – 6 × 8 = - 36
By further calculation
x – 48 = - 36

x = - 36 + 48
x = 12

Therefore, x = 12 and y = 8.

(ii) x – 2/3 y = 8/3
2x/5 – y = 7/5
We can write it as
x – 2/3 y = 8/3
Taking LCM
(3x – 2y)/ 3 = 8/3
By cross multiplication
3x – 2y = 8/3 × 3 = 8
3x – 2y = 8 ….. (1)

2x/5 – y = 7/5
Taking LCM
(2x – 5y)/ 5 = 7/5
By cross multiplication
2x – 5y = 7/5 × 5 = 7
2x – 5y = 7 …… (2)

Multiply equation (1) by 2 and (2) by 3
6x – 4y = 16 ….. (3)
6x – 15y = 21 …… (4)
Subtracting both the equations
11y = - 5
y = - 5/11

Substitute the value of y in equation (1)
3x – 2 (-5/11) = 8
By further calculation
3x + 10/11 = 8
We can write it as
3x = 8 – 10/11
Taking LCM
3x = (88 – 10)/ 11 = 78/11
By cross multiplication
x = 78/ (11 × 3) = 26/11

Therefore, x = 26/11 and y = - 5/11.

5. (i) 9 – (x – 4) = y + 7
2 (x + y) = 4 – 3y
(ii) 2x + (x – y)/ 6 = 2
x – (2x + y)/3 = 1.
Solution:

(i) 9 – (x – 4) = y + 7
2 (x + y) = 4 – 3y

We can write it as

9 – (x – 4) = y + 7

9 – x + 4 = y + 7

By further calculation

13 – x = y + 7

- x – y = 7 – 13 = - 6

x + y = 6 ….. (1)

2 (x + y) = 4 – 3y

2x + 2y = 4 – 3y

By further calculation

2x + 2y + 3y = 4

So we get

2x + 5y = 4 ….. (2)

Now multiply equation (1) by 5 and (2) by 1

5x + 5y = 30

2x + 5y = 4

By subtracting both the equations

3x = 26

So we get

x = 26/3

Substitute the value of x in (1)

26/3 + y = 6

We can write it as

y = 6 – 26/3

Taking LCM

y = (18 – 26)/3

So we get

y = - 8/3

Therefore, x = 26/3 and y = - 8/3.

(ii) 2x + (x – y)/ 6 = 2

x – (2x + y)/3 = 1

2x + (x – y)/ 6 = 2

Multiply by 6

12x + x – y = 12

By further calculation

13x – y = 12 …… (2)

x – (2x + y)/3 = 1

Multiply by 3

3x – 2x – y = 3

By further calculation

x – y = 3 …… (2)

So we get

x = 3 + y ….. (3)

Substitute the value of x in (1)
13 (3 + y) – y = 12
By further calculation
39 + 13y – y = 12
So we get
12y = 12 – 39 = - 27
By division
y = - 27/12 = - 9/4

Substitute the value of y in (3)
x = 3 + y
x = 3 + (-9)/4
By further calculation
x = 3 – 9/4
Taking LCM
x = (12 – 9)/ 4
x = ¾

Therefore, x = ¾ and y = - 9/4.

6. x – 3y = 3x – 1 = 2x – y.
Solution:

It is given that
x – 3y = 3x – 1 = 2x – y
Here
x- 3y = 3x – 1
x – 3x – 3y = - 1
By further calculation
- 2x – 3y = - 1
2x + 3y = 1 ….. (1)

3x – 1 = 2x – y
3x – 2x + y = 1
By further simplification
x + y = 1 ….. (2)

Multiply equation (2) by 2 and subtract from equation (1)
2x + 3y = 1
2x + 2y = 2
So we get
y = - 1

Substitute the value of y in equation (1)
2x + 3 (-1) = 1
So we get
2x – 3 = 1
2x = 1 + 3 = 4
By division
x = 4/2 = 2

Therefore, x = 2 and y = - 1.

7. (i) 4x + (x – y)/ 8 = 17
2y + x – (5y + 2)/3 = 2
(ii) (x + 1)/2 + (y – 1)/3 = 8
(x – 1)/3 + (y + 1)/ 2 = 9.
Solution:

(i) 4x + (x – y)/ 8 = 17
2y + x – (5y + 2)/3 = 2
We can write it as
4x + (x – y)/ 8 = 17
(32 + x – y)/ 8 = 17
By further calculation
(33x – y)/ 8 = 17
By cross multiplication
33x – y = 136 ….. (1)

2y + x – (5y + 2)/3 = 2
Taking LCM
[3 (2y + x) – 5 (5y + 2)]/ 3 = 2
By further calculation
6y + 3x – 5y – 2 = 2 × 3
So we get
y + 3x – 2 = 6
3x + y = 6 + 2
3x + y = 8 ….. (2)

By adding both the equations
36x = 144
By division
x = 144/36 = 4

Substitute the value of x in equation (1)
33 × 4 – y = 136
By further calculation
132 – y = 136
- y = 136 – 132
So we get
- y = 4
y = - 4

Therefore, x = 4 and y = - 4.

(ii) (x + 1)/2 + (y – 1)/3 = 8
(x – 1)/3 + (y + 1)/ 2 = 9
We can write it as
(x + 1)/2 + (y – 1)/3 = 8
Taking LCM
(3x + 3 + 2y – 2)/6 = 8

By further calculation
3x + 2y + 1 = 48
So we get
3x + 2y = 47 ….. (1)

(x – 1)/3 + (y + 1)/ 2 = 9
Taking LCM
(2x – 2 + 3y + 3)/6 = 9
By further calculation
2x + 3y + 1 = 54
So we get
2x + 3y = 53 ….. (2)

By adding equation (1) and (2)
5x + 5y = 100
Dividing by 5
x + y = 20 …… (3)
By subtracting equation (1) and (2)
x – y = - 6 ….. (4)

Now add equation (3) and (4)
2x = 14
x = 14/2 = 7
Subtracting equation (4) and (3)
2y = 26
y = 26/2 = 13

Therefore, x = 7 and y = 13.

8. (i) 3/x + 4y = 7
5/x + 6y = 13
(ii) 5x – 9 = 1/y
x + 1/y = 3.
Solution:

(i) 3/x + 4y = 7 ….. (1)
5/x + 6y = 13 ….. (2)
Substitute 1/x = a in equation (1) and)@)
3a + 4y = 7 …. (3)
5a + 6y = 13 ….. (4)
Multiply equation (3) by 5 and (4) by 3
15a + 20y = 35
15a + 18y = 39
Subtracting both the equations
2y = -4
So we get
y = - 4/2 = - 2

Substitute the value of y in equation (3)
3a + 4 (-2) = 7

By further calculation

3a – 8 = 7

3a = 7 + 8 = 15

So we get

3a = 15

a = 15/3 = 5

Here x = 1/a = 1/5

Therefore, x = 1/5 and y = - 2.

(ii) 5x – 9 = 1/y ….. (1)

x + 1/y = 3 ….. (2)

Substitute 1/y = b in (1) and (2)

5x – 9 = b

5x – b = 9 ….. (3)

x + b = 3 ….. (4)

By adding equation (3) and (4)

5x – b = 9 ….. (3)

x + b = 3 ….. (4)

So we get

6x = 12

By division

x = 12/6 = 2

Substitute the value of x in equation (4)

2 + b = 3

b = 3 – 2

b = 1

Here 1/y = 1

b = 1/y

y = 1

Therefore, x = 2 and y = 1.

9. (i) px + qy = p – q

qx – py = p + q

(ii) x/a – y/b = 0

ax + by = a² + b².

Solution:

(i) px + qy = p – q …. (1)

qx – py = p + q ….. (2)

Now multiply equation (1) by p and (2) by q

$p^2x + pqy = p^2 – pq$

$q^2x – pqy = pq + q^2$

By adding both the equations

$(p^2 + q^2) x = p^2 + q^2$

By further calculation

$x = (p^2 + q^2)/ (p^2 + q^2) = 1$

From equation (1)

$p \times 1 + qy = p - q$

By further calculation

$p - qy = p - q$

So we get

$qy = p - q - p = -q$

Here

$y = -q/q = -1$

Therefore, $x = 1$ and $y = -1$.

(ii) $x/a - y/b = 0$

$ax + by = a^2 + b^2$

We can write it as

$x/a - y/b = 0$

Taking LCM

$(bx - ay)/ab = 0$

By cross multiplication

$bx - ay = 0$ …… (1)

$ax + by = a^2 + b^2$ ….. (2)

Multiply equation (1) by b and equation (2) by a

$b^2x - aby = 0$

$a^2x + aby = a^2 + ab^2$

By adding both the equations

$(a^2 + b^2)x = a^2 + ab^2 = a (a^2 + b^2)$

So we get

$x = a (a^2 + b^2)/ a^2 + b^2 = a$

From equation (2)

$b \times a - ay = 0$

By further calculation

$ab - ay = 0$

$ay = ab$

So we get

$y = ab/a = b$

Therefore, $x = a$ and $y = b$.

10. Solve $2x + y = 23$, $4x - y = 19$. Hence, find the values of $x - 3y$ and $5y - 2x$.
Solution:

It is given that

$2x + y = 23$ …. (1)

$4x - y = 19$ ….. (2)

Adding both the equations

$6x = 42$

$x = 42/6 = 7$

Substitute the value of x in equation (1)
$2 \times 7 + y = 23$
By further calculation
$14 + y = 23$
So we get
$y = 23 – 14 = 9$

Therefore, x = 7 and y = 9.

$x – 3y = 7 – 3 \times 9 = 7 – 27 = - 20$
$5y – 2x = 5 \times 9 – 2 \times 7 = 45 – 14 = 31$

11. The expression ax + by has value 7 when x = 2, y = 1. When x = - 1, y = 1, it has value 1, find a and b.
Solution:

It is given that
$ax + by = 7$ when x = 2 and y = 1
Substituting the values
$a (2) + b (1) = 7$
$2a + b = 7$ ….. (1)

Here
$ax + by = 1$ when x = - 1 and y = 1
Substituting the values
$a (-1) + b (1) = 1$
$- a + b = 1$ ….. (2)

By subtracting both the equations
$- 3a = - 6$
So we get
$a = - 6/ - 3 = 2$

Substituting the value of a in equation (1)
$2 \times 2 + b = 7$
By further calculation
$4 + b = 7$
$b = 7 – 4 = 3$

Therefore, a = 2 and b = 3.

12. Can the following equations hold simultaneously?
3x – 7y = 7
11x + 5y = 87
5x + 4y = 43.
If so, find x and y.
Solution:

$3x – 7y = 7$ …… (1)
$11x + 5y = 87$ ….. (2)
$5x + 4y = 43$ ….. (3)

Now multiply equation (1) by 5 and (2) by 7
15x – 35y = 35
77x + 35y = 609
By adding both the equations
92x = 644
By division
x = 644/92 = 7

Substitute the value of x in equation (1)
$3 \times 7 – 7y = 7$
By further calculation
21 – 7y = 7
So we get
- 7y = 7 - 21 = - 14
y = - 14/ - 7 = 2

Therefore, x = 7 and y = 2.

If x = 7 and y – 2 satisfy the equation (3) then we can say that the equations hold simultaneously
Substitute the value of x and y in equation (3)
5x + 4y = 43
By further calculation
$5 \times 7 + 4 \times 2 = 43$
So we get
35 + 8 = 43
43 = 43 which is true.

Therefore, the equations hold simultaneously.

Exercise 5.3

1. Solve the following systems of simultaneous linear equations by cross-multiplication method:

(i) 3x + 2y = 4

8x + 5y = 9

(ii) 3x – 7y + 10 = 0

y – 2x = 3.

Solution:

(i) 3x + 2y = 4

8x + 5y = 9

We can write it as

3x + 2y – 4 = 0

8x + 4y – 9 = 0

By cross multiplication method

x/ (-18 + 20) = y/ (-32 + 27) = 1/ (15 – 16)

By further calculation

x/2 = y/-5 = 1/-1

So we get

x/2 = - 1

x = - 2

y = - 5 (-1) = 5

Therefore, x = - 2 and y = 5.

(ii) 3x – 7y + 10 = 0

y – 2x = 3

We can write it as

3x – 7y + 10 = 0

y – 2x – 3 = 0

By cross multiplication method

x/ (21 – 10) = y/ (-20 + 9) = 1/ (3 – 14)

By further calculation

x/11 = y/-11 = 1/-11

So we get

x/11 = 1/-11

x = - 1

Similarly

y/-11 = 1/ -11

y = 1

Therefore, x = - 1 and y = 1.

2. Solve the following pairs of linear equations by cross-multiplication method:

(i) x – y = a + b

ax + by = a² – b²

(ii) 2bx + ay = 2ab

bx – ay = 4ab.

Solution:

(i) x – y = a + b

ax + by = a^2 – b^2
We can write it as
x – y – (a + b) = 0
ax + by – (a^2 – b^2) = 0
By cross multiplication method
x/ [a^2 - b^2 + b (a + b)] = y/ [- a (a + b) + a^2 – b^2] = 1/ (b + a)
By further calculation
x/ (a^2 – b^2 + ab + b^2) = y/ (-a^2 – ab + a^2 – b^2) = 1/ (a + b)
So we get
x/ [a (a + b)] = y/ [-b (a + b)] = 1/ (a + b)
x = a (a + b)/ (a + b) = a
y = [-b (a + b)]/ (a + b) = - b

Therefore, x = a and y = - b.

(ii) 2bx + ay = 2ab
bx – ay = 4ab
We can write it as
2bx + ay – 2an = 0
bx – ay – 4ab = 0

By cross multiplication method
x/ (- 4a^2b – 2a^2b) = y/ (-2ab^2 + 8ab^2) = 1/ (-2ab – ab)
By further calculation
x/ -6a^2b = y/6ab^2= 1/-3ab
So we get
x = -6a^2b/ -3ab = 2a
y = 6ab^2/-3ab = - 2b

Therefore, x = 2a and b = - 2b.

Exercise 5.4

Solve the following pairs of linear equations (1 to 5):
1. (i) 2/x + 2/3y = 1/6
2/x – 1/y = 1
(ii) 3/2x + 2/3y = 5
5/x – 3/y = 1.
Solution:

(i) 2/x + 2/3y = 1/6 ….. (1)
2/x – 1/y = 1 ….. (2)
By subtracting both the equations
5/3y = -5/6
By cross multiplication
- 15y = 30
By division
y = 30/ -15 = - 2

Substitute the value of y in equation (1)
2/x + 2/ (3 × (-2)) = 1/6
By further calculation
2/x – 1/3 = 1/6
So we get
2/x = 1/6 + 1/3
Taking LCM
2/x = (1 + 2)/ 6 = 3/6
By cross multiplication
x = (2 × 6)/3 = 12/3 = 4

Therefore, x = 4 and y = - 2.

(ii) 3/2x + 2/3y = 5 ….. (1)
5/x – 3/y = 1 ….. (2)
Multiply equation (1) by 1 and (2) by 2/9
3/2x + 2/3y = 5
10/9x – 2/3y = 2/9
By adding both the equations
(3/2 + 10/9)1/x = 5 + 2/9
Taking LCM
(27 + 20)/ 18 × 1/x = (45 + 2)/ 9
By further calculation
47/18x = 47/9
By cross multiplication
x = (47 × 9)/ (47 × 18) = ½

Substitute the value of x in equation (2)
5/ ½ - 3/y = 1
By further calculation
10 – 3/y = 1
3/y = 10 – 1 = 9
So we get

y = 3/9 = 1/3

Therefore, x = ½ and y = 1/3.

2. (i) (7x – 2y)/ xy = 5
(8x + 7y)/ xy = 15
(ii) 99x + 101y = 499xy
101x + 99y = 501xy.
Solution:

(i) (7x – 2y)/ xy = 5
(8x + 7y)/ xy = 15
We can write it as
7x/xy – 2y/xy = 5
8x/xy + 7y/xy = 15
By further simplification
7/y – 2/x = 5 …. (1)
8/y + 7/x = 15 ….. (2)

Now multiply equation (1) by 7 and (2) by 2
49/y – 14/x = 35
16/y + 14/x = 30
By adding both the equations
65/y = 65
So we get
y = 65/65 = 1

Substitute the value of y in equation (1)
7/1 – 2/x = 5
By further calculation
2/x = 7 – 5 = 2
So we get
x = 2/2 = 1

Therefore, x = 1 and y = 1.

(ii) 99x + 101y = 499xy
101x + 99y = 501xy
Now divide each term by xy
99x/xy + 101y/xy = 499xy/xy
101y/xy + 99x/xy = 501xy/xy
By further calculation
99/y + 101/x = 499 ….. (1)
101/y + 99/x = 501 ….. (2)
By adding both the equations
200/y + 200/x = 1000
Divide by 200
1/y + 1/x = 5 …… (3)
Subtracting both the equations
-2/y + 2/x = - 2

Divide by 2

$-1/y + 1/x = -1$ …. (4)

By adding equation (3) and (4)

$2/x = 4$

So we get

$x = 2/4 = ½$

By subtracting equation (3) and (4)

$2/y = 6$

So we get

$y = 2/6 = 1/3$

Therefore, $x = ½$ and $y = 1/3$ if $x ≠ 0, y ≠ 0$.

3. (i) 3x + 14y = 5xy

21y – x = 2xy

(ii) 3x + 5y = 4xy

2y – x = xy.

Solution:

(i) $3x + 14y = 5xy$

$21y – x = 2xy$

Now dividing each equation by xy of $x ≠ 0, y ≠ 0$

$3x/xy + 14y/xy = 5xy/xy$

By further calculation

$3/y = 14/x = 5$ ….. (1)

(ii) $3x + 5y = 4xy$

$2y – x = xy$

We can write it as

$3x + 5y = 4xy$

$- x + 2y = xy$

Divide each equation by xy if $x≠ 0$ and $y ≠ 0$

$3x/xy + 5y/xy = 4xy/xy$

So we get

$3/y + 5/x = 4$ ….. (1)

$-x/xy + 2y/xy = xy/xy$

So we get

$-1/y + 2/x = 1$ ….. (2)

Now multiply equation (1) by 1 and (2) by 3

$3/y + 5/x = 4$

$-3/y + 6/x = 3$

By adding both the equations

$11/x = 7$

So we get

$x = 11/7$

Substitute the value of x in equation (2)
-1/y + 2/11/7 = 1
By further calculation
-1/y + (2 × 7)/ 11 = 1
-1/y + 14/11 = 1
We can write it as
-1/y = 1 – 14/11
Taking LCM
-1/y = (11 – 14)/ 11
So we get
-1/y = -3/11
By cross multiplication
-3y = - 11
y = - 11/-3 = 11/3

Therefore, x = 11/7 and y = 11/3.

4. (i) 20/ (x + 1) + 4/ (y – 1) = 5
10/ (x + 1) – 4/ (y – 1) = 1
(ii) 3/ (x + y) + 2/ (x – y) = 3
2/ (x + y) + 3/ (x – y) = 11/3.
Solution:

(i) 20/ (x + 1) + 4/ (y – 1) = 5 ….. (1)
10/ (x + 1) – 4/ (y – 1) = 1 ….. (2)
Add equation (1) and (2)
30/ (x + 1) = 6
By cross multiplication
30 = 6 (x + 1)
By further calculation
30/6 = x + 1
5 = x + 1
So we get
x = 5 – 1 = 4

Substitute the value of x in equation (1)
20/ (x + 1) + 4/ (y – 1) = 5
20/ (4 + 1) + 4/ (y – 1) = 5
By further calculation
20/5 + 4/ (y – 1) = 5
4 + 4/ (y – 1) = 5
We can write it as
4/ (y – 1) = 5 – 4 = 1
4/ (y – 1) = 1
By cross multiplication
4 = 1 (y – 1)
So we get
4 = y – 1
y = 4 + 1 = 5

Therefore, x = 4 and y = 5.

(ii) 3/ (x + y) + 2/ (x – y) = 3 …. (1)
2/ (x + y) + 3/ (x – y) = 11/3 ….. (2)
Multiply equation (1) by 3 and (2) by 2
9/ (x + y) + 6/ (x – y) = 9 ….. (3)
4/ (x + y) + 6/ (x – y) = 22/3 ….. (4)
Subtracting both the equations
5/ (x + y) = 9 – 22/3
Taking LCM
5/ (x + y) = 5/3
By cross multiplication
5 × 3 = 5 (x + y)
By further calculation
(5 × 3)/ 5 = x + y
x + y = (3 × 1)/ 3
x + y = 3 …… (5)

Substitute equation (5) in (1)
3/3 + 2/ (x – y) = 3
By further calculation
1 + 2/ (x – y) = 3
2/ (x – y) = 3 – 1 = 2
So we get
2/2 = x – y
Here
1 = x – y ….. (6)
We can write it as
x – y = 1
x + y = 3
By adding both the equations
2x = 4
x = 4/2 = 2
Substitute x = 2 in equation (5)
2 + y = 3
y = 3 – 2 = 1

Therefore, x = 2 and y = 1.

5. (i) 1/ 2(2x + 3y) + 12/ 7(3x – 2y) = ½
7/ (2x + 3y) + 4/ (3x – 2y) = 2
(ii) 1/ 2(x + 2y) + 5/ 3(3x – 2y) = - 3/2
5/ 4(x + 2y) – 3/ 5 (3x – 2y) = 61/60.
Solution:

(i) 1/ 2(2x + 3y) + 12/ 7(3x – 2y) = ½
7/ (2x + 3y) + 4/ (3x – 2y) = 2
Consider 2x + 3y = a and 3x – 2y = b
We can write it as
1/2a + 12/7b = ½

7/a + 4/b = 2
Now multiply equation (1) by 7 and (2) by ½
7/2a + 12/b = 7/2
7/2a + 2/b = 1
Subtracting both the equations
10/b = 5/2
So we get
b = (10 × 2)/ 5 = 4
Substitute the value of b in equation (2)
7/a + 4/4 = 2
7/a + 1 = 2
So we get
7/a = 2 – 1 = 1
a = 7

Here
2x + 3y = 7 ….. (3)
3x – 2y = 4 ….. (4)
Multiply equation (3) by 2 and (4) by 3
4x + 6y = 14
9x – 6y = 12
So we get
13x = 26
x = 26/13 = 2

Substitute the value of x in (3)
2 × 2 + 3y = 7
By further calculation
4 + 3y = 7
So we get
3y = 7 – 4 = 3
y = 3/3 = 1

Therefore, x = 2 and y = 1.

(ii) 1/ 2(x + 2y) + 5/ 3(3x – 2y) = - 3/2
5/ 4(x + 2y) – 3/ 5 (3x – 2y) = 61/60
Consider x + 2y = a and 3x – 2y = b
1/2a + 5/3b = - 3/2 …. (1)
5/4a – 3/5b = 61/60 ….. (2)
Now multiply equation (1) by 5/2 and (2) by (1)
5/4a + 25/6b = - 15/4
5/4a – 3/5b = 61/60
Subtracting both the equations
25/6b + 3/5b = - 15/4 – 61/60
Taking LCM
(125 + 18)/ 30b = (-225 – 61)/ 60
By further calculation
143/30b = - 286/60
By cross multiplication

30b × (-286) = 60 × 143
So we get
b = (60 × 143)/ (30 × -286) = - 1

Substitute the value of b in equation (1)
1/2a + 5/ (3 × - 1) = -3/2
By further calculation
1/2a – 5/4 = - 3/2
We can write it as
1/2a = - 3/2 + 5/3
Taking LCM
1/2a = (-9+ 10)/ 6 = 1/6
So we get
a = 6/2 = 3

Here
x + 2y = 3 ….. (3)
3x – 2y = - 1 ….. (4)
Adding both the equations
4x = 2
x = 2/4 = ½

Substitute the value of x in equation (3)
½ + 2y = 3
By further calculation
2y = 3 – ½
Taking LCM
2y = 5/2
y = 5/ (2 × 2) = 5/4

Therefore, x = ½ and y = 5/4.

Chapter Test

Solve the following simultaneous linear equations (1 to 4):

1.(i) 2x – (¾)y = 3,
5x – 2y = 7

Solution:

$$2x - \frac{3}{4}y = 3$$

$$\frac{8x - 3y}{4} = 3$$

8x-3y = 12   …(i)
5x-2y = 7   …(ii)

Multiply (i) by 5 and (ii) by 8, we get
40x-15y = 60   (iii)
40x -16y = 56   (iv)
Subtract (iv) from (iii), we get
y = 4
Substitute y in (i)
8x-3×4 = 12
∴8x = 12+12
∴8x = 24
∴x = 24/8
∴x = 3
Hence x = 3 and y = 4.

(ii) 2(x-4) = 9y+2
x – 6y = 2

Solution:
2(x-4) = 9y+2
2x-8 = 9y+2
2x-9y = 2+8
2x-9y = 10   ….(i)
x-6y = 2   ….(ii)
Multiply (ii) by 2, we get
2x -12y = 4   ….(iii)
Subtract (iii) from (i), we get
2x-9y = 10
-2x +12y = -4

0+3y = 6
∴3y = 6
∴y = 6/3
∴ y = 2
Substitute the value of y in (i)

$\Rightarrow 2x-9\times 2 = 10$
$2x-18 = 10$
$2x = 10+18$
$2x = 28$
$\therefore x = 28/2$
$\therefore x = 14$
Hence x = 14 and y = 2.

2. (i) 97x+53y = 177
53x+97y = 573

Solution:
Given equations are as follows.
$97x+53y = 177$ …(i)
$53x+97y = 573$ …(ii)
Multiply (i) by 53 and (ii) by 97
$53(97x+53y) = 53\times 177$
$5141x+2809y = 9381$ …..(iii)
$97(53x+97y) = 97\times 573$
$5141x+9409y = 55581$ …..(iv)
Subtract (iv) from (iii)
$5141x+2809y = 9381$ …..(iii)
$5141x+9409y = 55581$ …..(iv)

$0x -6600y = -46200$
$\Rightarrow -6600y = -46200$
$\Rightarrow y = -46200/-6600$
$\Rightarrow y = 7$
Substitute the value of y in (i)
$\Rightarrow 97x+53\times 7 = 177$
$\Rightarrow 97x+371 = 177$
$\Rightarrow 97x = 177-371$
$\Rightarrow 97x = - 194$
$\Rightarrow x = -194/97$
$\Rightarrow x = -2$
Hence x = -2 and y = 7.

(ii) x+y = 5.5
x-y = 0.9

Solution:
$x+y = 5.5$ …(i)
$x-y = 0.9$ …(ii)

Adding (i) and (ii), we get
$2x = 5.5+0.9$
$2x = 6.4$
$\Rightarrow x = 6.4/2$
$\Rightarrow x = 3.2$

Substitute value of x in (i)
3.2+y = 5.5
⇒y = 5.5-3.2
⇒y = 2.3
Hence x = 3.2 and y = 2.3.

3. (i) x+y = 7xy
2x-3y+xy = 0

Solution:
x+y = 7xy   …(i)
2x-3y+xy = 0   …..(ii)
Divide (i) by xy, we get

$$\frac{x}{xy} + \frac{y}{xy} = \frac{7xy}{xy}$$

$$\frac{1}{y} + \frac{1}{x} = 7$$

$$\frac{1}{x} + \frac{1}{y} = 7 \ldots\ldots(iii)$$

Divide (ii) by xy, we get

$$\frac{2x}{xy} - \frac{3y}{xy} + \frac{xy}{xy} = 0$$

$$\frac{2}{y} - \frac{3}{x} + 1 = 0$$

$$-\frac{3}{x} + \frac{2}{y} = -1 \ldots\ldots(iv)$$

Multiplying (iii) by 3, we get

$$\frac{3}{x} + \frac{3}{y} = 3 \times 7$$

$$\frac{3}{x} + \frac{3}{y} = 21 \ldots(v)$$

Adding (v) and (iv), we get

$\frac{5}{y} = 20$

$y = \frac{5}{20}$

$y = \frac{1}{4}$

Substitute value of y in (iv)

$-\frac{3}{x} + 2 \times 4 = -1$

$-\frac{3}{x} + 8 = -1$

$-\frac{3}{x} = -1 - 8$

$-\frac{3}{x} = -9$

$x = \frac{3}{9}$

$x = \frac{1}{3}$

Hence x = 1/3 and y = 1/4.

(ii)

$\frac{30}{x-y} + \frac{44}{x+y} = 10$

$\frac{40}{x-y} + \frac{55}{x+y} = 13$

Solution:

$\frac{30}{x-y} + \frac{44}{x+y} = 10 ... (i)$

$\frac{40}{x-y} + \frac{55}{x+y} = 13 ... (ii)$

Multiply (i) by 4 and (ii) by 3, we get

$\frac{120}{x-y} + \frac{176}{x+y} = 40 ... (iii)$

$\frac{120}{x-y} + \frac{165}{x+y} = 39 ... (iv)$

Subtracting (iv) from (iii), we get

$0 + \frac{11}{x+y} = 1$

$x + y = 11 \ldots (v)$

Substitute (v) in (i), we get

$\frac{30}{x-y} + \frac{44}{11} = 10$

$\frac{30}{x-y} + 4 = 10$

$\frac{30}{x-y} = 10 - 4$

$\frac{30}{x-y} = 6$

$x - y = \frac{30}{6}$

$x - y = 5 \ldots (vi)$

Now solve for (v) and (vi)
x+y = 11
x-y = 5
Add (v) and (vi)
2x = 16
∴ x = 16/2 = 8
Substitute x in (v)
8+y = 11
∴y = 11-8
∴y = 3
Hence x = 8 and y = 3.

4. (i) ax+by = a-b
bx-ay = a+b

Solution:
ax+by = a-b   …(i)
bx-ay = a+b   …(ii)
multiplying (i) by a and (ii) by b, we get
a(ax+by) = a(a-b)
a^2x +aby = a^2-ab   …(iii)
b(bx-ay) = b(a+b)
b^2x -aby = ab+b^2 …(iv)
Adding (iii) and (iv)
a^2x +aby = a^2-ab
b^2x -aby = ab+b^2

$(a^2+b^2)x = (a^2+b^2)$
⟹x = (a^2+b^2)/ (a^2+b^2)

$\Rightarrow$x = 1

Substitute the value of x in (i), we get

a×1+by = a-b

a+by = a-b

$\Rightarrow$by = -b

$\Rightarrow$y = -b/b

$\Rightarrow$y = -1

Hence x = 1 and y = -1.

(ii) 3x + 2y = 2xy

$$\frac{1}{x} + \frac{2}{y} = 1\frac{1}{6}$$

Solution:

3x + 2y = 2xy   …(i)

$\frac{1}{x} + \frac{2}{y} = 1\frac{1}{6}$

$\frac{1}{x} + \frac{2}{y} = \frac{7}{6}$ . . . . (ii)

Divide (i) by xy

$\frac{3x}{xy} + \frac{2y}{xy} = \frac{2xy}{xy}$

$\frac{3}{y} + \frac{2}{x} = 2$ . . . (iii)

Multiply (ii) by 2, we get

$\frac{2}{x} + \frac{4}{y} = \frac{7}{3}$ . . (iv)

Subtract (iii) from (iv)

$\frac{2}{x} + \frac{4}{y} = \frac{7}{3}$

$\frac{2}{x} + \frac{3}{y} = 2$

$0 + \frac{1}{y} = \frac{7}{3} - 2$

$\frac{1}{y} = \frac{7-6}{3}$

$\frac{1}{y} = \frac{1}{3}$

$y = 3$

Substitute y in (iii)

(3/3) + (2/x) = 2

1+(2/x) = 2

(2/x) = 1

∴ x = 2

Hence x = 2 and y = 3.

5. Solve 2x -(3/y) =9
3x + (7/y) = 2.
Hence find the value of k if x = ky + 5.

Solution:
2x -(3/y) = 9   …(i)
3x + (7/y) = 2   …(ii)
Multiply (i) by 3 and (ii) by 2, we get
6x-(9/y) = 27   ..(iii)
6x+(14/y) = 4   …(iv)
Subtracting (iv) from (iii), we get
-23/y = 23
⟹y = 23/-23
⟹y = -1
Substitute y in (i)
2x-(3/-1) = 9
2x+3 = 9
2x = 9-3
2x = 6
⟹x = 6/2
⟹x = 3
Hence x = 3 and y = -1.
Given x = ky+5
Substitute x and y in above eqn
3 = k×-1+5
3 = -k+5
⟹ k = 5-3
⟹ k = 2
Hence the value of k is 2.

6. Solve,
$$\frac{1}{x+y} - \frac{1}{2x} = \frac{1}{30}$$
$$\frac{5}{x+y} + \frac{1}{3} = \frac{4}{3}$$
.
Hence find the value of 2x²-y².

Solution:
$$\frac{1}{x+y} - \frac{1}{2x} = \frac{1}{30} \ldots (i)$$
$$\frac{5}{x+y} + \frac{1}{3} = \frac{4}{3} \ldots (ii)$$
Let (x+y) = a
(1/a)-(1/2x)

$\frac{1}{a} - \frac{1}{2x} = \frac{1}{30}$ ...(iii)

$\frac{5}{a} + \frac{1}{x} = \frac{4}{3}$

Multiply (iii) by 5

$\frac{5}{a} - \frac{5}{2x} = \frac{1}{6}$ ...(iv)

$\frac{5}{a} + \frac{1}{x} = \frac{4}{3}$

Subtracting (ii) from (iv)

$-\frac{5}{2x} - \frac{1}{x} = \frac{1}{6} - \frac{4}{3}$

$\frac{(-5-2)}{2x} = \frac{(1-8)}{6}$

$-\frac{7}{2x} = -\frac{7}{6}$

$2x = 6$

$x = 3$

Substitute x in (iii)

(1/a) -1/(2×3) = 1/30

(1/a) - (1/6) = 1/30

1/a = (1/30)+(1/6)

1/a = (1+5)/30

1/a = 6/30

⟹a = 30/6

⟹a = 5

Substitute a in x+y = a

3+y = 5

∴y = 5-3

∴y = 2

Hence x = 3, y = 2.

∴2x²-y² = 2×3²-2²

= 2×9-4

= 18-4

= 14

Hence the value of 2x²-y² is 14.

7. Can x, y be found to satisfy the following equations simultaneously ?

$\frac{2}{y} + \frac{5}{x} = 19$

$\frac{5}{y} - \frac{3}{x} = 1$

$3x + 8y = 5$

If so, find them.
Solution:

$\frac{2}{y} + \frac{5}{x} = 19 \ldots (i)$

$\frac{5}{y} - \frac{3}{x} = 1 \ldots (ii)$

$3x + 8y = 5 \ldots (iii)$

Multiply (i) by 5 and (ii) by 2, we get

$\frac{10}{y} + \frac{25}{x} = 95 \ldots (iv)$

$\frac{10}{y} - \frac{6}{x} = 2 \ldots (v)$

Subtract (v) from (iv)

31/x = 95-2

31/x = 93

∴x = 31/93

∴x = 1/3

Substitute x in (i)

(2/y)+5÷(1/3) = 19

(2/y)+5×3 = 19

(2/y) = 19-15

(2/y) = 4

∴y = 2/4

∴y = 1/2

Substitute x and y in (iii)

3×(1/3) + 8×(1/2) = 5

1+4 = 5

The value of x and y satisfies (iii).

Hence the given equations are simultaneous.