Exercise 8

Simplify the following (1 to 20):

1. (i) $(81/16)^{-3/4}$

Solution:

$(81/16)^{-3/4}$
$= [(3^4/2^4)]^{-3/4}$
$= [(3/2)^4]^{-3/4}$
$= (3/2)^{-3/4 \times 4}$
$= (3/2)^{-3}$
$= (2/3)^3$
$= 2^3/3^3$
$= (2 \times 2 \times 2)/(3 \times 3 \times 3)$
$= 8/27$

(ii) $(1\frac{61}{64})^{-\frac{2}{3}}$

Solution:

$$(1\frac{61}{64})^{-\frac{2}{3}} = (\frac{125}{64})^{-\frac{2}{3}} = (\frac{5^3}{4^3})^{-\frac{2}{3}}$$

$= (5/4)^{3 \times -2/3}$
$= (5/4)^{-2}$
$= (4/5)^2$
$= 16/25$

2. (i) $(2a^{-3}b^2)^3$

Solution:

$(2a^{-3}b^2)^3$
$= 2^3\ a^{-3x3}\ b^{2x3}$
$= 8a^{-1}b^6$

(ii) $(a^{-1} + b^{-1})/(ab)^{-1}$

Solution:

$$\frac{a^{-1}+b^{-1}}{(ab)^{-1}} = \frac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{ab}} = \frac{a+b}{ab} \times \frac{ab}{1} = a+b$$

3. (i) $(x^{-1}\ y^{-1})/(x^{-1} + y^{-1})$

Solution:

$$\frac{x^{-1}y^{-1}}{x^{-1}+y^{-1}}=\frac{(xy)^{-1}}{\frac{1}{x}+\frac{1}{y}}$$
$$=\frac{\frac{1}{xy}}{\frac{xy}{x+y}}=\frac{1}{xy}\times\frac{xy}{x+y}$$
$$=\frac{1}{x+y}$$

(ii) $(4 \times 10^7)(6 \times 10^{-5})/(8 \times 10^{10})$
Solution:

$$\frac{(4\times10^7)(6\times10^{-5})}{8\times10^{10}}$$
$$=\frac{4\times6\times10^7\times10^{-5}}{8\times10^{10}}$$
$$=\frac{24\times10^{7+(-5)}}{8\times10^{10}}$$
$$=3\times\frac{10^2}{10^{10}}=3\times10^{2-10}=3\times10^{-8}$$

4. (i) $3a/b^{-1} + 2b/a^{-1}$
Solution:

$3a/b^{-1} + 2b/a^{-1}$
$= 3a/(1/b) + 2b/(1/a)$
$= (3a \times b)/1 + (2b \times a)/1$
$= 3ab + 2ab = 5ab$

(ii) $5^0 \times 4^{-1} + 8^{1/3}$
Solution:

$5^0 \times 4^{-1} + 8^{1/3}$
$= 1 \times (1/4) + (2)^{3 \times 1/3}$
$= 1/4 + 2$
$= (1 + 8)/4$
$= 9/4 = 2\frac{1}{4}$

5. (i) $(8/125)^{-1/3}$
Solution:

$(8/125)^{-1/3}$

$= [(2 \times 2 \times 2)/(5 \times 5 \times 5)]^{-1/3}$
$= (2^3/5^3)^{-1/3}$
$= (2/5)^{3 \times -1/3}$
$= (2/5)^{-1}$
$= 5/2 = 2½$

(ii) $(0.027)^{-1/3}$
Solution:

$(0.027)^{-1/3}$
$= (27/1000)^{-1/3}$
$= [(3 \times 3 \times 3)/(10 \times 10 \times 10)]^{-1/3}$
$= (3^3/10^3)^{-1/3}$
$= (3/10)^{3 \times -1/3}$
$= (3/10)^{-1}$
$= 10/3$

6. (i) $(-1/27)^{-2/3}$
Solution:

$(-1/27)^{-2/3}$
$= (-1/3^3)^{-2/3}$
$= (-1/3)^{3 \times -2/3}$
$= (-1/3)^{-2}$
$= (-3)^2$
$= 9$

(ii) $(64)^{-2/3} \div 9^{-3/2}$
Solution:

$(64)^{-2/3} \div 9^{-3/2}$
We can write it as
$= (4^3)^{-2/3} \div (3^2)^{-3/2}$
By further calculation
$= 4^{3 \times -2/3} \div 3^{2 \times -3/2}$
So we get
$= 4^{-2} \div 3^{-3}$
$= 4^{-2}/ 3^{-3}$
It can be written as
$= 1/4^2 / 1/3^3$
$= 3^3/4^2$
We get
$= 27/16$
$= 1\ 11/16$

7. (i) $\frac{(27)^{\frac{2n}{3}} \times (8)^{\frac{-n}{6}}}{(18)^{\frac{-n}{2}}}$

(ii) $\frac{5.(25)^{n+1} - 25.(5)^{2n}}{5.(5)^{2n+3} - (25)^{n+1}}$.

Solution:

$$(i)\frac{(27)^{\frac{2n}{3}} \times (8)^{\frac{-n}{6}}}{(18)^{\frac{-n}{2}}}$$

We can write it as

$$= \frac{(3 \times 3 \times 3)^{\frac{2n}{3}} \times (2 \times 2 \times 2)^{\frac{-n}{6}}}{(2 \times 3 \times 3)^{\frac{-n}{2}}}$$

By further calculation

$$= \frac{(3)^{3\times\frac{2n}{3}} \times (2)^{3\times\frac{-n}{6}}}{(2 \times 3^2)^{\frac{-n}{2}}}$$

$$= \frac{(3)^{2n} \times (2)^{\frac{-n}{2}}}{(2)^{\frac{-n}{2}} \times (3)^{2\times\frac{-n}{2}}}$$

So we get

$$= \frac{(3)^{2n} \times 1}{1 \times 3^{-n}}$$

It can be written as

$= (3)^{2n} \times (3)^{n}$

$= 3^{2n + n}$

$= 3^{3n}$

$$(ii)\frac{5.(25)^{n+1} - 25.(5)^{2n}}{5.(5)^{2n+3} - (25)^{n+1}}$$

We can write it as

$$= \frac{5.(5 \times 5)^{n+1} - (5 \times 5).(5)^{2n}}{5.(5)^{2n+3} - (5 \times 5)^{n+1}}$$

$$= \frac{5.(5)^{2(n+1)} - (5)^{2}.(5)^{2n}}{5^{3}.(5)^{2n+3} - (5)^{2(n+1)}}$$

$By\ further\ calculation$

$$= \frac{(5)^{2(n+1)+1} - (5)^{2\times 2n}}{(5)^{2n+3+1} - (5)^{2n+2}}$$

$$= \frac{(5)^{2n+3} - (5)^{2n+2}}{(5)^{2n+4} - (5)^{2n+2}}$$

$On\ further\ simplification$

$$= \frac{(5)^{2n}.(5)^3 - (5)^{2n}.(5)^2}{(5)^{2n}.(5)^4 - (5)^{2n}.(5)^2}$$

$Taking\ out\ the\ common\ terms$

$$= \frac{(5)^{2n}[(5)^3 - (5)^2]}{(5)^{2n}[(5)^4 - (5)^2]}$$

$So\ we\ get$

$$= \frac{125 - 25}{625 - 25}$$

= 100/600

= 1/6

$\mathbf{8.(i)[8^{\frac{-4}{3}} \div 2^{-2}]^{\frac{1}{2}}}$

$\mathbf{(ii)(\frac{27}{3})^{2/3} - (\frac{1}{4})^{-2} + 5^0}$.

Solution:

$(i)[8^{\frac{-4}{3}} \div 2^{-2}]^{\frac{1}{2}}$

$It\ can\ be\ written\ as$

$$= [(2 \times 2 \times 2)^{\frac{-4}{3}} \div (2)^{-2}]^{\frac{1}{2}}$$

$By\ further\ calculation$

$$= [(2)^{3\times\frac{-4}{3}} \div \frac{1}{(2)^2}]^{\frac{1}{2}}$$

$$= [(2)^{-4} \div \frac{1}{4}]^{\frac{1}{2}}$$

$On\ further\ simplification$

$$= [\frac{1}{(2)^4} \div \frac{1}{4}]^{\frac{1}{2}}$$

$$= [\frac{1}{2 \times 2 \times 2 \times 2} \times \frac{4}{1}]^{\frac{1}{2}}$$

$So\ we\ get$

$$= [\frac{4}{16}]^{\frac{1}{2}}$$

$$= [\frac{1}{4}]^{\frac{1}{2}}$$

$We\ can\ write\ it\ as$

$$= [\frac{1}{2} \times \frac{1}{2}]^{\frac{1}{2}}$$

$$= [\frac{1}{2}]^{2 \times \frac{1}{2}}$$

$= (1/2)^1$

= ½

$$(ii) (\frac{27}{8})^{2/3} - (\frac{1}{4})^{-2} + 5^0$$

$We\ can\ write\ it\ as$

$$= (\frac{3 \times 3 \times 3}{2 \times 2 \times 2})^{2/3} - (\frac{1 \times 1}{2 \times 2})^{-2} + 1$$

$$= (\frac{3}{2} \times \frac{3}{2} \times \frac{3}{2})^{2/3} - (\frac{1}{2} \times \frac{1}{2})^{-2} + 1$$

$By\ further\ calculation$

$$= [(\frac{3}{2})^3]^{2/3} - [(\frac{1}{2})^2]^{-2} + 1$$

$$= (\frac{3}{2})^{3 \times \frac{2}{3}} - (\frac{1}{2})^{2 \times -2} + 1$$

$So\ we\ get$

$$= (\frac{3}{2})^2 - (\frac{1}{2})^{-4} + 1$$

It can be written as

$$= \frac{3}{2} \times \frac{3}{2} - \frac{1}{(\frac{1}{2})^4} + 1$$

$$= \frac{9}{4} - \frac{1}{\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}} + 1$$

We get

$$= \frac{9}{4} - \frac{1}{(\frac{1}{16})} + 1$$

$$= \frac{9}{4} - \frac{1 \times 16}{1} + 1$$

By further calculation

$$= \frac{9}{4} - 16 + 1$$

$$= \frac{9}{4} - 15$$

Taking LCM

$$= \frac{9 - 60}{4}$$

$$= \frac{-51}{4}$$

$$= -12\frac{3}{4}$$

9. (i) $(3x^2)^{-3} \times (x^9)^{2/3}$
(ii) $(8x^4)^{1/3} \div x^{1/3}$.
Solution:

(i) $(3x^2)^{-3} \times (x^9)^{2/3}$
We can write it as

$$= \frac{1}{(3x^2)^3} \times (x^9)^{\frac{2}{3}}$$
$$= \frac{1}{(3)^3(x^2)^3} \times (x)^{3\times 2}$$

$By\ further\ calculation$

$$= \frac{1}{(3 \times 3 \times 3) \times (x)^{2\times 3}} \times x^6$$

$So\ we\ get$

$$= \frac{1}{27x^6} \times x^6$$
$$= \frac{x^6}{27x^6}$$
$$= \frac{1}{27}$$

(ii) $(8x^4)^{1/3} \div x^{1/3}$

We can write it as

$$= (8)^{\frac{1}{3}}(x^4)^{\frac{1}{3}} \div (x)^{\frac{1}{3}}$$
$$= \frac{(2 \times 2 \times 2)^{\frac{1}{3}}(x^4)^{\frac{1}{3}}}{x^{\frac{1}{3}}}$$

$By\ further\ calculation$

$$= \frac{(2)^{3\times\frac{1}{3}}(x)^{\frac{4}{3}}}{x^{\frac{1}{3}}}$$
$$= (2)^1(x)^{\frac{4}{3}-\frac{1}{3}}$$

$Taking\ LCM$

$$= (2)^1 \times x^{(\frac{4-1}{3})}$$

$= 2 \times x^{3/3}$

So we get

$= 2 \times x^1$

$= 2 \times x$

$= 2x$

10. (i) $(3^2)^0 + 3^{-4} \times 3^6 + (1/3)^{-2}$
(ii) $9^{5/2} - 3.(5)^0 - (1/81)^{-1/2}$
Solution:

(i) $(3^2)^0 + 3^{-4} \times 3^6 + (1/3)^{-2}$
We can write it as

$$= (3)^{2\times 0} + (3)^{-4+6} + \frac{1}{(\frac{1}{3})^2}$$

$$= (3)^0 + (3)^2 + \frac{1}{\frac{1}{3}\times\frac{1}{3}}$$

By further calculation

$$= 1 + 9 + \frac{1}{\frac{1}{9}}$$

$$= 1 + 9 + \frac{9\times 1}{1}$$

So we get
= 1 + 9 + 9
= 19

(ii) $9^{5/2} - 3.(5)^0 - (1/81)^{-1/2}$
We can write it as

$$= (3\times 3)^{\frac{5}{2}} - 3\times 1 - \frac{1}{(\frac{1}{81})^{1/2}}$$

By further calculation

$$= (3)^{2\times\frac{5}{2}} - 3 - \frac{1}{(\frac{1}{9}\times\frac{1}{9})^{1/2}}$$

$$= (3)^5 - 3 - \frac{1}{(\frac{1}{9})^{2\times\frac{1}{2}}}$$

So we get

$$= (3)^5 - 3 - \frac{1}{(\frac{1}{9})^1}$$

$$= (3\times 3\times 3\times 3\times 3) - 3 - \frac{1}{(\frac{1}{9})}$$

Here
= 243 – 3 – (9 × 1)/1
= 240 – 9
= 231

11. (i) $16^{3/4} + 2\,(1/2)^{-1}\,(3)^0$
(ii) $(81)^{3/4} - (1/32)^{-2/5} + (8)^{1/3}\,(1/2)^{-1}\,(2)^0$.
Solution:

(i) $16^{3/4} + 2\,(1/2)^{-1}\,(3)^0$
We can write it as

$$= (2 \times 2 \times 2 \times 2)^{\frac{3}{4}} + 2 \times \frac{1}{(\frac{1}{2})^1} \times 1$$

$By\ further\ calculation$

$$= (2)^{4 \times \frac{3}{4}} + 2 \times \frac{2}{1} \times 1$$

So we get
$= (2)^3 + 4$
$= 2 \times 2 \times 2 + 4$
$= 8 + 4$
$= 12$

(ii) $(81)^{3/4} - (1/32)^{-2/5} + (8)^{1/3}\,(1/2)^{-1}\,(2)^0$
We can write it as

$$= (3 \times 3 \times 3 \times 3)^{\frac{3}{4}} - (\frac{1}{2 \times 2 \times 2 \times 2 \times 2})^{\frac{-2}{5}} + (2 \times 2 \times 2)^{\frac{1}{3}} \times \frac{1}{(\frac{1}{2})^1} \times 1$$

$By\ further\ calculation$

$$= (3)^{4 \times \frac{3}{4}} - (\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2})^{\frac{-2}{5}} + (2)^{3 \times \frac{1}{3}} \times \frac{1 \times 2}{1} \times 1$$

$$= (3)^3 - (\frac{1}{2})^{5 \times \frac{-2}{5}} + 2 \times 2 \times 1$$

$So\ we\ get$

$$= (3 \times 3 \times 3) - (\frac{1}{2})^{-2} + 4$$

$$= 27 - \frac{1}{(\frac{1}{2})^2} + 4$$

$$= 27 - \frac{1}{\frac{1}{2} \times \frac{1}{2}} + 4$$

$Here$

$$= 27 - \frac{1}{\frac{1}{4}} + 4$$

$$= 27 - \frac{4 \times 1}{1} + 4$$
= 27 – 4 + 4
= 27

$$\mathbf{12.(i)}\left(\frac{64}{125}\right)^{\frac{-2}{3}} \div \frac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}} + \left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right)^{0}$$

$$\mathbf{(ii)}\frac{5^{n+3} - 6 \times 5^{n+1}}{9 \times 5^{n} - 2^{2} \times 5^{n}}.$$

Solution:

$$(i)\left(\frac{64}{125}\right)^{\frac{-2}{3}} \div \frac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}} + \left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right)^{0}$$

We can write it as

$$= \left(\frac{4 \times 4 \times 4}{5 \times 5 \times 5}\right)^{\frac{-2}{3}} \div \frac{1}{\left(\frac{4\times4\times4\times4}{5\times5\times5\times5}\right)^{\frac{1}{4}}} + 1$$

$$= \left(\frac{4 \times 4 \times 4}{5 \times 5 \times 5}\right)^{\frac{-2}{3}} \div \frac{1}{\left(\frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}\right)^{\frac{1}{4}}} + 1$$

By further calculation

$$= \left(\frac{4}{5}\right)^{3\times\frac{-2}{3}} \div \frac{1}{\left(\frac{4}{5}\right)^{4\times\frac{1}{4}}} + 1$$

$$= \left(\frac{4}{5}\right)^{-2} \div \frac{1}{\frac{4}{5}} + 1$$

So we get

$$= \frac{1}{\left(\frac{4}{5}\right)^{2}} \div \frac{5 \times 1}{4} + 1$$

$$= \frac{1}{\frac{4}{5} \times \frac{4}{5}} \div \frac{5}{4} + 1$$

On further simplification

$$= \frac{1}{\frac{16}{25}} \times \frac{4}{5} + 1$$

$$= \frac{25 \times 1}{16} \times \frac{4}{5} + 1$$

$We\ get$

$$= \frac{5}{4} + 1$$

$Taking\ LCM$

$$= \frac{5+4}{4}$$

= 9/4

= 2 ¼

$$(ii) \frac{5.(25)^{n+1} - 25.(5)^{2n}}{5.(5)^{2n+3} - (25)^{n+1}}$$

$We\ can\ write\ it\ as$

$$= \frac{5^n.5^3 - 6 \times 5^n.5}{9 \times 5^n - 4.5^n}$$

$Taking\ 5^n\ as\ common$

$$= \frac{5^n(5^3 - 6 \times 5)}{5^n(9-4)}$$

$By\ further\ calculation$

$$= \frac{5^3 - 30}{5}$$

$$= \frac{125 - 30}{5}$$

$So\ we\ get$

$$= \frac{95}{5}$$

= 19

13. (i) $[(64)^{-2/3}\ 2^{-2} + 8^0]^{-1/2}$
(ii) $3^n \times 9^{n+1} \div (3^{n-1} \times 9^{n-1})$.
Solution:

(i) $[(64)^{-2/3}\ 2^{-2} + 8^0]^{-1/2}$

We can write it as

$$= [(4 \times 4 \times 4)^{\frac{2}{3}} \times \frac{1}{(2)^2} \div 1]^{\frac{-1}{2}}$$

By further calculation

$$= [(4)^{3\times\frac{2}{3}} \times \frac{1}{2 \times 2} \div 1]^{\frac{-1}{2}}$$

$$= [(4)^2 \times \frac{1}{4} \div 1]^{\frac{-1}{2}}$$

So we get

$$= [4 \times 4 \times \frac{1}{4} \times 1]^{\frac{-1}{2}}$$

$= [4 \times 1 \times 1]^{-1/2}$
$= (4)^{-1/2}$
Here
$= (2 \times 2)^{-1/2}$
$= (2)^{2 \times -1/2}$
$= (2)^{-1}$
$= 1/(2)^1$
= ½

(ii) $3^n \times 9^{n+1} \div (3^{n-1} \times 9^{n-1})$
We can write it as
$= 3^n \times (3 \times 3)^{n+1} \div (3^{n-1} \times (3 \times 3)^{n-1})$
By further calculation
$= 3^n \times (3)^{2 \times (n+1)} \div (3^{n-1} \times (3)^{2(n-1)}])$
$= 3^n \times (3)^{2n+2} \div (3^{n-1} \times (3)^{2n-2})$
So we get
$= (3)^{n+2n+2} \div (3)^{n-1+2n-2}$
$= (3)^{3n+2} \div (3)^{3n-3}$
Here
$= (3)^{3n+2-3n+3}$
$= (3)^5$
We get
$= 3 \times 3 \times 3 \times 3 \times 3$
= 243

14. (i) $\frac{\sqrt{2^2} \times \sqrt[4]{256}}{\sqrt[3]{64}} - (\frac{1}{2})^{-2}$

(ii) $\frac{3^{-\frac{6}{7}} \times 4^{-\frac{3}{7}} \times 9^{\frac{3}{7}} \times 2^{\frac{6}{7}}}{2^2 + 2^0 + 2^{-2}}$.

Solution:

$(i) \frac{\sqrt{2^2} \times \sqrt[4]{256}}{\sqrt[3]{64}} - (\frac{1}{2})^{-2}$

$We\ can\ write\ it\ as$

$= \frac{(2^2)^{\frac{1}{2}} \times (256)^{\frac{1}{4}}}{(64)^{\frac{1}{3}}} - (\frac{1}{2})^{-2}$

$By\ further\ calculation$

$= \frac{(2)^{2\times\frac{1}{2}} \times (4 \times 4 \times 4 \times 4)^{\frac{1}{4}}}{(4 \times 4 \times 4)^{\frac{1}{3}}} - (\frac{1}{2})^{-2}$

$= \frac{(2)^1 \times (4)^{4\times\frac{1}{4}}}{(4)^{3\times\frac{1}{3}}} - \frac{1}{(\frac{1}{2})^2}$

$So\ we\ get$

$= \frac{2 \times (4)^1}{(4)^1} - \frac{1}{\frac{1}{2} \times \frac{1}{2}}$

$= \frac{2 \times 1}{1} - \frac{1}{\frac{1}{4}}$

$Here$

$= 2 - \frac{1 \times 4}{1}$

= 2 – 4

= - 2

$(ii) \frac{3^{-\frac{6}{7}} \times 4^{-\frac{3}{7}} \times 9^{\frac{3}{7}} \times 2^{\frac{6}{7}}}{2^2 + 2^0 + 2^{-2}}$

$We\ can\ write\ it\ as$

$= \frac{(3)^{-\frac{6}{7}} \times (2 \times 2)^{-\frac{3}{7}} \times (3 \times 3)^{\frac{3}{7}} \times (2)^{\frac{6}{7}}}{(2 \times 2) + 1 + \frac{1}{(2)^2}}$

$By\ further\ calculation$

$= \frac{(3)^{-\frac{6}{7}} \times (2)^{2\times-\frac{3}{7}} \times (3)^{2\times\frac{3}{7}} \times (2)^{\frac{6}{7}}}{4 + 1 + \frac{1}{(2\times2)}}$

$$= \frac{(3)^{-\frac{6}{7}} \times (2)^{-\frac{6}{7}} \times (3)^{\frac{6}{7}} \times (2)^{\frac{6}{7}}}{5 + \frac{1}{(4)}}$$

$So\ we\ get$

$$= \frac{(3)^{\frac{6}{7}+\frac{6}{7}} \times (2)^{\frac{-6}{7}+\frac{6}{7}}}{\frac{5\times4+1}{4}}$$

$$= \frac{(3)^0 \times (2)^0}{\frac{21}{4}}$$

$Here$

$$= \frac{1 \times 1}{\frac{21}{4}}$$

$$= \frac{1 \times 1 \times 4}{21}$$

$$= \frac{4}{21}$$

15. **(i)** $\frac{(32)^{\frac{2}{5}} \times (4)^{-\frac{1}{2}} \times (8)^{\frac{1}{3}}}{2^{-2} \div (64)^{-1/3}}$

(ii) $\frac{5^{2(x+6)} \times (25)^{-7+2x}}{(125)^{2x}}$.

Solution:

$(i)\ \frac{(32)^{\frac{2}{5}} \times (4)^{-\frac{1}{2}} \times (8)^{\frac{1}{3}}}{2^{-2} \div (64)^{-1/3}}$

$We\ can\ write\ it\ as$

$$= \frac{(2 \times 2 \times 2 \times 2 \times 2)^{\frac{2}{5}} \times (2 \times 2)^{-\frac{1}{2}} \times (2 \times 2 \times 2)^{\frac{1}{3}}}{\frac{1}{(2)^2} \div \frac{1}{(64)^{1/3}}}$$

$By\ further\ calculation$

$$= \frac{(2)^{5\times\frac{2}{5}} \times (2)^{2\times-\frac{1}{2}} \times (2)^{3\times\frac{1}{3}}}{\frac{1}{2\times2} \div \frac{1}{(4\times4\times4)^{\frac{1}{3}}}}$$

$$= \frac{(2)^2 \times (2)^{-1} \times (2)^1}{\frac{1}{4} \div \frac{1}{(4)^{3\times\frac{1}{3}}}}$$

$On\ further\ simplification$

$$= \frac{4 \times \frac{1}{(2)^1} \times 2}{\frac{1}{4} \div \frac{1}{(4)^1}}$$

$So\ we\ get$

$$= \frac{4 \times \frac{1}{1} \times 1}{\frac{1}{4} \times 4}$$

$$= \frac{4}{1}$$

= 4

$$(ii) \frac{5^{2(x+6)} \times (25)^{-7+2x}}{(125)^{2x}}$$

$We\ can\ write\ it\ as$

$$= \frac{5^{2(x+6)} \times (5 \times 5)^{-7+2x}}{(5 \times 5 \times 5)^{2x}}$$

$By\ further\ calculation$

$$= \frac{(5)^{2x+12} \times (5)^{2(-7+2x)}}{[(5)^3]^{2x}}$$

$$= \frac{(5)^{2x+12} \times (5)^{-14+4x}}{(5)^{6x}}$$

$On\ further\ simplification$

$$= \frac{(5)^{2x+12+(-14+4x)}}{(5)^{6x}}$$

$$= \frac{(5)^{2x+12-14+4x}}{(5)^{6x}}$$

$So\ we\ get$

$$= \frac{(5)^{6x-2}}{(5)^{6x}}$$

$= 5^{6x-2-6x}$
$= 5^{-2}$
$= 1/(5)^2$
$= 1/25$

16. $(i)\ \frac{7^{2n+3}-(49)^{n+2}}{((343)^{n+1})^{2/3}}$

$(ii)\ (27)^{\frac{4}{3}}+(32)^{0.8}+(0.8)^{-1}$.

Solution:

$$(i)\ \frac{7^{2n+3}-(49)^{n+2}}{((343)^{n+1})^{2/3}}$$

We can write it as

$$= \frac{7^{2n+3}-(7\times 7)^{n+2}}{((7\times 7\times 7)^{n+1})^{2/3}}$$

By further calculation

$$= \frac{(7)^{2n}.(7)^3-[(7)^2]^{n+2}}{((7)^{3(n+1)})^{\frac{2}{3}}}$$

$$= \frac{(7)^{2n}.(7)^3-(7)^{2(n+2)}}{(7)^{3(n+1)\times\frac{2}{3}}}$$

On further simplfication

$$= \frac{(7)^{2n}.(7)^3-(7)^{2n}.(7)^4}{(7)^{2(n+1)}}$$

$$= \frac{(7)^{2n}[(7)^3-(7)^4]}{(7)^{2n}.(7)^2}$$

So we get

$$= \frac{(7)^3-(7)^4}{(7)^2}$$

It can be written as

$$= \frac{7\times 7\times 7-7\times 7\times 7\times 7}{7\times 7}$$

$$= \frac{7\times 7(7-7\times 7)}{7\times 7}$$

$So\ we\ get$

$$= \frac{1 \times (7 - 7 \times 7)}{1}$$

= 7 – 7 × 7
= 7 – 49
= - 42

(ii) $(27)^{4/3} + (32)^{0.8} + (0.8)^{-1}$
We can write it as

$$= (3 \times 3 \times 3)^{\frac{4}{3}} + (2 \times 2 \times 2 \times 2 \times 2)^{0.8} + \frac{1}{(0.8)^1}$$

$By\ further\ calculation$

$$= (3)^{3\times\frac{4}{3}} + (2)^{5\times 0.8} + \frac{1}{0.8}$$

$$= (3)^4 + (2)^{4.0} + \frac{10}{8}$$

$So\ we\ get$

$$= (3 \times 3 \times 3 \times 3) + (2 \times 2 \times 2 \times 2) + \frac{5}{4}$$

$$= 81 + 16 + \frac{5}{4}$$

$On\ further\ simplification$

$$= \frac{97}{1} + \frac{5}{4}$$

$Taking\ LCM$

$$= \frac{97 \times 4 + 5 \times 1}{4}$$

$$= \frac{388 + 5}{4}$$

$$= \frac{393}{4}$$

= 98.25

$\mathbf{17.(i)(\sqrt{32} - \sqrt{5})^{\frac{1}{3}}(\sqrt{32} + \sqrt{5})^{\frac{1}{3}}}$

$\mathbf{(ii)(x^{\frac{1}{3}} - x^{-\frac{1}{3}})(x^{\frac{2}{3}} + 1 + x^{-\frac{2}{3}})}$.

Solution:

$(i)(\sqrt{32} - \sqrt{5})^{\frac{1}{3}}(\sqrt{32} + \sqrt{5})^{\frac{1}{3}}$

$We\ can\ write\ it\ as$

$= [(\sqrt{32} - \sqrt{5})(\sqrt{32} + \sqrt{5})]^{\frac{1}{3}}$

$= [(\sqrt{32})^2 - (\sqrt{5})^2]^{\frac{1}{3}}$

$By\ further\ calculation$

$= (32 - 5)^{\frac{1}{3}}$

$= (27)^{\frac{1}{3}}$

$So\ we\ get$

$= (3 \times 3 \times 3)^{\frac{1}{3}}$

$= (3)^{3\times\frac{1}{3}}$

$= (3)^1$

$= 3$

$(ii)(x^{\frac{1}{3}} - x^{-\frac{1}{3}})(x^{\frac{2}{3}} + 1 + x^{-\frac{2}{3}})$

$We\ can\ write\ it\ as$

$= (x^{\frac{1}{3}} - x^{-\frac{1}{3}})[(x^{\frac{1}{3}})^2 + x^{\frac{1}{3}} \times x^{-\frac{1}{3}} + (x^{-\frac{1}{3}})^2]$

$By\ further\ calculation$

$= (x^{\frac{1}{3}})^3 - (x^{-\frac{1}{3}})^3$

$= x^{3\times\frac{1}{3}} - x^{3\times-\frac{1}{3}}$

$So\ we\ get$

$$= x - \frac{1}{(x)^1}$$
$$= x - \frac{1}{x}$$

18. (i) $\left(\frac{x^m}{x^n}\right)^1 . \left(\frac{x^n}{x^1}\right)^m . \left(\frac{x^1}{x^m}\right)^n$

(ii) $\left(\frac{x^{a+b}}{x^c}\right)^{a-b} . \left(\frac{x^{b+c}}{x^a}\right)^{b-c} . \left(\frac{x^{c+a}}{x^b}\right)^{c-a}$.

Solution:

$(i)\left(\frac{x^m}{x^n}\right)^1 . \left(\frac{x^n}{x^1}\right)^m . \left(\frac{x^1}{x^m}\right)^n$

We can write it as

$= (x^{m-n})^l . (x^{n-1})^m . (x^{1-m})^n$

By further calculation

$= (x)^{(m-n)l} . (x)^{(n-1)m} . (x)^{(l-m)n}$

$= x^{ml-nl} . x^{nm-lm} . x^{ln-mn}$

So we get

$= x^{ml-nl+nm-lm+ln-mn}$

$= x^0$

$= 1$

$(ii)\left(\frac{x^{a+b}}{x^c}\right)^{a-b} . \left(\frac{x^{b+c}}{x^a}\right)^{b-c} . \left(\frac{x^{c+a}}{x^b}\right)^{c-a}$.

We can write it as

$= (x^{a+b-c})^{a-b} . (x^{b+c-a})^{b-c} . (x^{c+a-b})^{c-a}$

By further calculation

$= x^{(a+b-c)(a-b)} . x^{(b+c-a)(b-c)} . x^{(c+a-b)(c-a)}$

So we get$= x^{a^2-b^2-ac+bc+b^2-c^2-ab+ac+c^2-a^2-bc+ab}$

$= x^0$

$= 1$

19. (i) $\sqrt[lm]{\frac{x^l}{x^m}} . \sqrt[mn]{\frac{x^m}{x^n}} . \sqrt[nl]{\frac{x^n}{x^l}}$

(ii) $\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} . \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} . \left(\frac{x^c}{x^a}\right)^{c^2+ac+a^2}$

$(\mathbf{iii})(\frac{\mathbf{x^a}}{\mathbf{x^{-b}}})^{\mathbf{a^2-ab+b^2}}.(\frac{\mathbf{x^b}}{\mathbf{x^{-c}}})^{\mathbf{b^2-bc+c^2}}.(\frac{\mathbf{x^c}}{\mathbf{x^a}})^{\mathbf{c^2-ca+a^2}}.$

Solution:

$(i)\ \sqrt[lm]{\frac{x^l}{x^m}}.\ \sqrt[mn]{\frac{x^m}{x^n}}.\ \sqrt[nl]{\frac{x^n}{x^l}}$

$We\ can\ write\ it\ as$

$=\ \sqrt[lm]{x^{l-m}}.\ \sqrt[mn]{x^{m-n}}.\ \sqrt[nl]{x^{n-l}}$

$By\ further\ calculation$

$= x^{\frac{l-m}{lm}}.x^{\frac{m-n}{mn}}.x^{\frac{n-l}{nl}}$

$= x^{\frac{l-m}{lm}+\frac{m-n}{mn}+\frac{n-l}{nl}}$

$So\ we\ get$

$= x^{\frac{nl-mn+lm-nl+mn-lm}{lmn}}$

$= x^0$

$= 1$

$(ii)(\frac{x^a}{x^b})^{a^2+ab+b^2}.(\frac{x^b}{x^c})^{b^2+bc+c^2}.(\frac{x^c}{x^a})^{c^2+ac+a^2}$

$We\ can\ write\ it\ as$

$= (x^{a-b})^{a^2+ab+b^2}.(x^{b-c})^{b^2+bc+c^2}.(x^{c-a})^{c^2+ac+a^2}$

$By\ further\ calculation$

$= x^{(a-b)(a^2+ab+b^2)}.x^{(b-c)(b^2+bc+c^2)}.x^{(c-a)(c^2+ac+a^2)}$

$= x^{a^3-b^3}.x^{b^3-c^3}.x^{c^3-a^3}$

$So\ we\ get$

$= x^{a^3-b^3+b^3-c^3+c^3-a^3}$

$= x^0$

= 1

$$(iii)(\frac{x^a}{x^{-b}})^{a^2-ab+b^2}.(\frac{x^b}{x^{-c}})^{b^2-bc+c^2}.(\frac{x^c}{x^{-a}})^{c^2-ca+a^2}$$

$We\ can\ write\ it\ as$

$$= (x^{a+b})^{a^2-ab+b^2}.(x^{b+c})^{b^2-bc+c^2}.(x^{c+a})^{c^2-ca+a^2}$$

$By\ further\ calculation$

$$= x^{(a+b)(a^2-ab+b^2)}.x^{(b+c)(b^2-bc+c^2)}.x^{(c+a)(c^2-ca+a^2)}$$

$$= x^{a^3+b^3}.x^{b^3+c^3}.x^{c^3+a^3}$$

$So\ we\ get$

$$= x^{a^3+b^3+b^3+c^3+c^3+a^3}$$

$$= x^{2a^3+2b^3+2x^3}$$

$$= x^{2(a^3+b^3+c^3)}$$

20. (i) $(a^{-1} + b^{-1}) \div (a^{-2} - b^{-2})$

$$(\mathbf{ii})\frac{\mathbf{1}}{\mathbf{1+a^{m-n}}} + \frac{\mathbf{1}}{\mathbf{1+a^{n-m}}}.$$

Solution:

(i) $(a^{-1} + b^{-1}) \div (a^{-2} - b^{-2})$

We can write it as

$$= (\frac{1}{a} + \frac{1}{b}) \div (\frac{1}{a^2} - \frac{1}{b^2})$$

$Taking\ LCM$

$$= (\frac{b+a}{ab}) \div (\frac{b^2-a^2}{a^2b^2})$$

$By\ further\ calculation$

$$= \frac{b+a}{ab} \times \frac{a^2b^2}{b^2-a^2}$$

$$= \frac{b+a}{ab} \times \frac{(ab)(ab)}{(b-a)(b+a)}$$

So we get

$$= \frac{1}{1} \times \frac{ab \times 1}{(b-a) \times 1}$$
$$= \frac{ab}{b-a}$$

$$(ii) \frac{1}{1+a^{m-n}} + \frac{1}{1+a^{n-m}}$$

We can write it as

$$= \frac{1}{a^0 + a^{m-n}} + \frac{1}{a^0 + a^{n-m}}$$
$$= \frac{1}{a^{m-m} + a^{m-n}} + \frac{1}{a^{n-n} + a^{n-m}}$$

Taking out the common terms

$$= \frac{1}{a^m(a^{-m} + a^{-n})} + \frac{1}{a^n(a^{-n} + a^{-m})}$$

By further calculation

$$= \frac{1}{a^m(\frac{1}{a^m} + \frac{1}{a^n})} + \frac{1}{a^n(\frac{1}{a^n} + \frac{1}{a^m})}$$

Taking LCM

$$= \frac{1}{a^m(\frac{a^n+a^m}{a^{m+n}})} + \frac{1}{a^n(\frac{a^m+a^n}{a^{m+n}})}$$
$$= \frac{a^{m+n}}{a^m(a^n+a^m)} + \frac{a^{m+n}}{a^n(a^m+a^n)}$$

On further simplification

$$= \frac{a^n}{(a^n+a^m)} + \frac{a^m}{(a^m+a^n)}$$

So we get

$$= \frac{a^n+a^m}{a^m+a^n}$$

$$= \frac{a^m + a^n}{a^m + a^n}$$
= 1

21. Prove the following:
(i) $(a + b)^{-1} (a^{-1} + b^{-1}) = 1/ab$
$$(\mathbf{ii})\frac{\mathbf{x+y+z}}{\mathbf{x^{-1}y^{-1}+y^{-1}z^{-1}+z^{-1}x^{-1}}} = \mathbf{xyz}.$$
Solution:

(i) $(a + b)^{-1} (a^{-1} + b^{-1}) = 1/ab$
Here
LHS = $(a + b)^{-1} (a^{-1} + b^{-1})$
We can write it as
$$= (a+b)^{-1}(\frac{1}{a} + \frac{1}{b})$$

$$Taking\ LCM$$

$$= \frac{1}{(a+b)^1}(\frac{b+a}{ab})$$

$$By\ further\ calculation$$

$$= \frac{1}{a+b}.\frac{a+b}{ab}$$
$$= \frac{1}{ab}$$
= RHS

Hence, proved.

$$(ii)\frac{x+y+z}{x^{-1}y^{-1}+y^{-1}z^{-1}+z^{-1}x^{-1}}=xyz$$

$Here$

$$LHS=\frac{x+y+z}{x^{-1}y^{-1}+y^{-1}z^{-1}+z^{-1}x^{-1}}$$

$We\ can\ write\ it\ as$

$$=\frac{x+y+z}{\frac{1}{x}\frac{1}{y}+\frac{1}{y}\frac{1}{z}+\frac{1}{z}\frac{1}{x}}$$
$$=\frac{x+y+z}{\frac{1}{xy}+\frac{1}{yz}+\frac{1}{zx}}$$

$Taking\ LCM$

$$=\frac{x+y+z}{\frac{z+x+y}{xyz}}$$

$By\ further\ calculation$

$$=\frac{(x+y+z)\times xyz}{(z+x+y)}$$
$$=\frac{(x+y+z)\times xyz}{(x+y+z)}$$

$So\ we\ get$

$$=\frac{1\times xyz}{1}$$

= xyz
= RHS

Hence, proved.

22. If $a = c^z$, $b = a^x$ and $c = b^y$, prove that xyz = 1.
Solution:

It is given that
$a = c^z$, $b = a^x$ and $c = b^y$
We can write it as
$a = (b^y)^z$ where $c = b^y$
So we get
$a = b^{yz}$

Here
$a = (a^x)^{yz}$
$a^1 = a^{xyz}$

By comparing both
$xyz = 1$

Therefore, it is proved.

23. If $a = xy^{p-1}$, $b = xy^{q-1}$ and $c = xy^{r-1}$, prove that $a^{q-r}. b^{r-p}. c^{p-q} = 1$.
Solution:
It is given that
$a = xy^{p-1}$
Here
$a^{q-r} = (xy^{b-1})^{q-r} = x^{q-r}. y^{(q-r)(p-1)}$

$b = xy^{q-1}$
Here
$b^{r-p} = (xy^{q-1})^{r-p} = x^{r-p}. y^{(q-1)(r-p)}$

$c = xy^{r-1}$
Here
$c^{p-q} = (xy^{r-1})^{p-q} = x^{p-q}. y^{(r-1)(p-q)}$

Consider
LHS $= a^{q-r}. b^{r-p}. c^{p-q}$
Substituting the values
$= x^{q-r}. y^{(q-r)(p-1)}. x^{r-p}. y^{(q-1)(r-p)}. x^{p-q}. y^{(r-1)(p-q)}$
By further calculation
$= x^{q-r+r-p-q}. y^{(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)}$
So we get
$= x^0. y^{pq-pr-q+r+qr-pr-r+p+rp-qr-p+q}$
$= x^0. y^0$
$= 1 \times 1$
$= 1$
= RHS

24. If $2^x = 3^y = 6^{-z}$, prove that $1/x + 1/y + 1/z = 0$.
Solution:

Consider
$2^x = 3^y = 6^{-z} = k$
Here
$2^x = k$
We can write it as

$2 = (k)^{1/x}$

$3^y = k$
We can write it as
$3 = (k)^{1/y}$

$6^{-z} = k$
We can write it as
$6 = (k)^{-1/z}$

So we get
$2 \times 3 = 6$
$(k)^{1/x} \times (k)^{1/y} = (k)^{-1/z}$
By further calculation
$(k)^{1/x + 1/y} = (k)^{-1/z}$
We get
$1/x + 1/y = - 1/z$
$1/x + 1/y + 1/z = 0$

Therefore, it is proved.

25. If $2^x = 3^y = 12^z$, prove that $x = 2yz/y – z$.
Solution:

It is given that
$2^x = 3^y = 12^z$
Consider
$2^x = 3^y = 12^z = k$

Here
$2^x = k$ where $2 = (k)^{1/x}$
$3^y = k$ where $3 = (k)^{1/y}$
$12^z = k$ where $12 = (k)^{-1/z}$

We know that
$12 = 2 \times 2 \times 3$

$$(k)^{\frac{1}{z}}=(k)^{\frac{1}{x}}\times(k)^{\frac{1}{x}}\times(k)^{\frac{1}{y}}$$

By further calculation

$$(k)^{\frac{1}{z}}=(k)^{\frac{1}{x}+\frac{1}{x}+\frac{1}{y}}$$

$$(k)^{\frac{1}{z}}=(k)^{\frac{2}{x}+\frac{1}{y}}$$

So we get

$$\frac{1}{z}=\frac{2}{x}+\frac{1}{y}$$
$$\frac{1}{z}-\frac{1}{y}=\frac{2}{x}$$

By taking LCM

$$\frac{y-z}{yz}=\frac{2}{x}$$

By cross multiplication

$$x(y-z)=2yz$$

$$x=\frac{2yz}{y-z}$$

Therefore, it is proved.

26. Simplify and express with positive exponents:
$(3x^2)^0$, $(xy)^{-2}$, $(-27a^9)^{2/3}$.
Solution:

We know that
$(3x^2)^0 = 1$

$$(xy)^{-2} = \frac{1}{(xy)^2} = \frac{1}{x^2y^2}$$

$$(-27a^9)^{\frac{2}{3}} = [(-3) \times (-3) \times (-3)]^{\frac{2}{3}} \times (a^9)^{\frac{2}{3}}$$

$$By\ further\ calculation$$

$$= (-3)^{3\times\frac{2}{3}} \times a^{9\times\frac{2}{3}}$$

$$So\ we\ get$$

$$= (-3)^2 \times a^{3\times 2}$$

$$= (-3) \times (-3) \times a^6$$

$$= 9a^6$$

27. If a = 3 and b = - 2, find the values of:
(i) $a^a + b^b$
(ii) $a^b + b^a$.
Solution:

It is given that
a = 3 and b = - 2

(i) $a^a + b^b = (3)^3 + (-2)^{-2}$
We can write it as

$$= 3 \times 3 \times 3 + \frac{1}{(-2)^2}$$

$By\ further\ calculation$

$$= 27 + \frac{1}{4}$$

$Taking\ LCM$

$$= \frac{27 \times 4 + 1}{4}$$

$$= \frac{108 + 1}{4}$$

$$= \frac{109}{4}$$

$$= 27\frac{1}{4}$$

(ii) $a^b + b^a = (3)^{-2} + (-2)^3$

We can write it as

$$= \frac{1}{(-3)^2} + (-2) \times (-2) \times (-2)$$

$By\ further\ calculation$

$$= \frac{1}{9} + (-8)$$

$$= \frac{1}{9} - 8$$

$Taking\ LCM$

$$= \frac{1 - 8 \times 9}{9}$$

$$= \frac{1 - 72}{9}$$

$$= \frac{-71}{9}$$

$$= -7\frac{8}{9}$$

28. If $x = 10^3 \times 0.0099$, $y = 10^{-2} \times 110$, find the value of $\sqrt{\frac{x}{y}}$.
Solution:

It is given that
$x = 10^3 \times 0.0099$, $y = 10^{-2} \times 110$
We know that

$$\sqrt{\frac{x}{y}} = \sqrt{\frac{10^3 \times 0.0099}{10^{-2} \times 110}}$$

$By\ further\ calculation$

$$= \frac{10^3 \times 10^2 \times 0.0099}{110}$$

$$= \frac{10^5 \times 0.0099}{110}$$

$So\ we\ get$

$$= \sqrt{\frac{100000 \times 0.0099}{110}}$$

$$= \sqrt{\frac{990}{110}}$$

$= \sqrt{9}$
$= \sqrt{(3 \times 3)}$
$= 3$

29. Evaluate $x^{1/2}. y^{-1}. z^{2/3}$ when $x = 9$, $y = 2$ and $z = 8$.
Solution:

It is given that
$x = 9$, $y = 2$ and $z = 8$
We know that
$x^{1/2}. y^{-1}. z^{2/3} = (9)^{1/2}. (2)^{-1}. (8)^{2/3}$

$It\ can\ be\ written\ as$

$$= (3 \times 3)^{\frac{1}{2}}.\frac{1}{(2)^1}.(2 \times 2 \times 2)^{\frac{2}{3}}$$

$By\ further\ calculation$

$$= (3)^{2\times\frac{1}{2}}.\frac{1}{2}.(2)^{3\times\frac{2}{3}}$$

$$= (3)^1 \times \frac{1}{2} \times (2)^2$$

So we get

$$= 3 \times \frac{1}{2} \times 2 \times 2$$

$$= 3 \times \frac{1}{1} \times 1 \times 2$$

$$= \frac{6}{1}$$

= 6

30. If $x^4y^2z^3 = 49392$, find the values of x, y and z, where x, y and z are different positive primes.
Solution:

It is given that
$x^4y^2z^3 = 49392$
We can write it as
$x^4y^2z^3 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 7 \times 7$
$x^4y^2z^3 = (2)^4\,(3)^2\,(7)^3$ ……. (1)

2	49392
2	24696
2	12348
2	6174
3	3087
3	1029
7	343
7	49
7	7
	1

Now compare the powers of 4, 2 and 3 on both sides of equation (1)
x = 2, y = 3 and z = 7

31. If $\sqrt[3]{a^6b^{-4}} = a^x.b^{2y}$, find x and y, where a, b are different positive primes.
Solution:

It is given that

$$\sqrt[3]{a^6b^{-4}} = a^x.b^{2y}$$

We can write it as

$$(a^6b^{-4})^{\frac{1}{3}} = a^x.b^{2y}$$

$$By\ further\ calculation$$

$$(a)^{6\times\frac{1}{3}}.(b)^{-4\times\frac{1}{3}} = a^x.b^{2y}$$

$$(a)^2.(b)^{-\frac{4}{3}} = a^x.b^{2y}$$

By comparing the base on both sides
2 = x
x = 2

- 4/3 = 2y
2y = - 4/3
By further calculation
y = - 4/3 × ½ = - 2/3

32. If $(p + q)^{-1} (p^{-1} + q^{-1}) = p^a q^b$, prove that a + b + 2 = 0, where p and q are different positive primes.
Solution:

It is given that
$(p + q)^{-1} (p^{-1} + q^{-1}) = p^a q^b$
We can write it as

$$\frac{1}{p+q}(\frac{1}{p}+\frac{1}{q}) = p^a q^b$$

$$Taking\ LCM$$

$$\frac{1}{p+q}\times(\frac{q+p}{pq}) = p^a q^b$$

$$\frac{1}{pq} = p^a q^b$$

By cross multiplication
$p^{-1}q^{-1} = p^a q^b$
By comparing the powers
a = - 1 and b = - 1

Here
LHS = a + b + 2
Substituting the values
= - 1 – 1 + 2
= 0
= RHS