EXERCISE 1.1 PAGE NO: 2

Choose the correct answer from the given four options in the following questions:

1. For some integer m, every even integer is of the form:

(A) m (B) m +1

(C) 2m (D) 2m+1

Solution:

(C)2m

Explanation:

Even integers are those integers which are divisible by 2.

Hence, we can say that every integer which is a multiple of 2 must be an even integer.

Therefore, let us conclude that,

for an integer 'm', every even integer must be of the form

$2 \times m = 2m$.

Hence, **option (C)** is the correct answer.

2. For some integer q, every odd integer is of the form

(A) q (B) q +1

(C) 2g (D) 2q +1

Solution:

(D) 2q+1

Explanation:

Odd integers are those integers which are not divisible by 2.

Hence, we can say that every integer which is a multiple of 2 must be an even integer, while 1 added to every integer which is multiplied by 2 is an odd integer.

Therefore, let us conclude that,

for an integer 'q', every odd integer must be of the form

$(2 \times q)+1 = 2q+1$.

Hence, **option (D)** is the correct answer.

3. $n^2 - 1$ is divisible by 8, if *n* is

(A) an integer (B) a natural number

(C) an odd integer (D) an even integer

Solution:

(C) an odd integer

Explanation:

Let $x = n^2 - 1$

In the above equation, n can be either even or odd.

Let us assume that n= even.

So, when n = even i.e., n = 2k, where k is an integer,

We get,

$\Rightarrow x = (2k)^2-1$

$\Rightarrow x = 4k^2 - 1$

At k = -1, $x = 4(-1)^2 - 1 = 4 - 1 = 3$, is not divisible by 8.
At k = 0, $x = 4(0)^2 - 1 = 0 - 1 = -1$, is not divisible by 8
Let us assume that n= odd:
So, when n = odd i.e., n = 2k + 1, where k is an integer,
We get,
$\Rightarrow x = 2k + 1$
$\Rightarrow x = (2k+1)^2 - 1$
$\Rightarrow x = 4k^2 + 4k + 1 - 1$
$\Rightarrow x = 4k^2 + 4k$
$\Rightarrow x = 4k(k+1)$
At k = -1, x = 4(-1)(-1+1) = 0 which is divisible by 8.
At k = 0, x = 4(0)(0+1) = 0 which is divisible by 8 .
At k = 1, x = 4(1)(1+ 1) = 8 which is divisible by 8.

From the above two observation, we can conclude that, if n is odd, if n odd, n^2-1 is divisible by 8.
Hence, **option (C)** is the correct answer.

4. If the HCF of 65 and 117 is expressible in the form 65m – 117, then the value of m is

(A) 4 **(B) 2**
(C) 1 **(D) 3**

Solution:

(B) 2

Explanation:

Let us find the HCF of 65 and 117,

117 = 1×65 + 52

65 = 1× 52 + 13

52 = 4 ×13 + 0

Hence, we get the HCF of 65 and 117 = 13.

According to the question,

65m - 117 = 13

65m = 117+13 = 130

∴ m =130/65 = 2

Hence, **option (B)** is the correct answer.

5. The largest number which divides 70 and 125, leaving remainders 5 and 8, respectively, is

(A) 13 **(B) 65**
(C) 875 **(D) 1750**

Solution:

(A) 13

Explanation:

According to the question,
We have to find the largest number which divides 70 and 125, leaving remainders 5 and 8.

This can be also written as,
To find the largest number which exactly divides (70 - 5), and (125 - 8)
The largest number that divides 65 and 117 is also the Highest Common Factor of 65 and 117
Therefore, the required number is the HCF of 65 and 117

Factors of 65 = 1, 5, 13, 65
Factors of 117 = 1, 3, 9, 13, 39, 117
Common Factors = 1, 13
Highest Common factor (HCF) = 13
i.e., the largest number which divides 70 and 125, leaving remainders 5 and 8, respectively = 13
Hence, **option (A)** is the correct answer.

EXERCISE 1.2 PAGE NO: 4

1. Write whether every positive integer can be of the form 4q + 2, where q is an integer. Justify your answer.

Solution:

No, every positive integer cannot be of the form 4q + 2, where q is an integer.

Justification:

All the numbers of the form 4q + 2, where 'q' is an integer, are even numbers which are not divisible by '4'.

For example,

When q=1,

4q+2 = 4(1) + 2= 6.

When q=2,

4q+2 = 4(2) + 2= 10

When q=0,

4q+2 = 4(0) + 2= 2 and so on.

So, any number which is of the form 4q+2 will give only even numbers which are not multiples of 4.

Hence, every positive integer **cannot** be written in the form 4q+2

2. "The product of two consecutive positive integers is divisible by 2". Is this statement true or false? Give reasons.

Solution:

Yes, the statement "the product of two consecutive positive integers is divisible by 2" is true.

Justification:

Let the two consecutive positive integers = a, a + 1

According to Euclid's division lemma,

We have,

$a = bq + r$, where $0 \leq r < b$

For b = 2, we have $a = 2q + r$, where $0 \leq r < 2$ ... (i)

Substituting r = 0 in equation (i),

We get,

a = 2q, is divisible by 2.

a + 1 = 2q + 1, is not divisible by 2.

Substituting r = 1 in equation (i),

We get,

a = 2q + 1, is not divisible by 2.

a + 1 = 2q + 1+1 = 2q + 2, is divisible by 2.

Thus, we can conclude that, for $0 \leq r < 2$, one out of every two consecutive integers is divisible by 2. So, the product of the two consecutive positive numbers will also be even.

Hence, the statement "product of two consecutive positive integers is divisible by 2" is true.

3. "The product of three consecutive positive integers is divisible by 6". Is this statement true or false? Justify your answer.

Solution:

Yes, the statement "the product of three consecutive positive integers is divisible by 6" is true.

Justification:

Consider the 3 consecutive numbers 2, 3, 4

$(2 \times 3 \times 4)/6 = 24/6 = 4$

Now, consider another 3 consecutive numbers 4, 5, 6

$(4 \times 5 \times 6)/6 = 120/6 = 20$

Now, consider another 3 consecutive numbers 7, 8, 9

$(7 \times 8 \times 9)/6 = 504/6 = 84$

Hence, the statement "product of three consecutive positive integers is divisible by 6" is true.

4. Write whether the square of any positive integer can be of the form 3m + 2, where m is a natural number. Justify your answer.

Solution:

No, the square of any positive integer cannot be written in the form 3m + 2 where m is a natural number

Justification:

According to Euclid's division lemma,

A positive integer 'a' can be written in the form of bq + r

a = bq + r, where b, q and r are any integers,

For b = 3

a = 3(q) + r, where, r can be an integers,

For r = 0, 1, 2, 3..........

3q + 0, 3q + 1, 3q + 2, 3q + 3....... are positive integers,

$(3q)^2 = 9q^2 = 3(3q^2) = 3m$ (where $3q^2 = m$)

$(3q+1)^2 = (3q+1)^2 = 9q^2+1+6q = 3(3q^2+2q) +1 = 3m + 1$ (Where, $m = 3q^2+2q$)

$(3q+2)^2 = (3q+2)^2 = 9q^2+4+12q = 3(3q^2+4q) +4 = 3m + 4$ (Where, $m = 3q^2+2q$)

$(3q+3)^2 = (3q+3)^2 = 9q^2+9+18q = 3(3q^2+6q) +9 = 3m + 9$ (Where, $m = 3q^2+2q$)

Hence, there is no positive integer whose square can be written in the form 3m + 2 where m is a natural number.

5. A positive integer is of the form 3q + 1, q being a natural number. Can you write its square in any form other than 3m + 1, i.e., 3m or 3m + 2 for some integer m? Justify your answer.

Solution:

No.

Justification:

Consider the positive integer 3q + 1, where q is a natural number.

$(3q + 1)2 = 9q^2 + 6q + 1$

$= 3(3q^2 + 2q) + 1$

$= 3m + 1$, (where m is an integer which is equal to $3q^2 + 2q$.

Thus $(3q + 1)^2$ cannot be expressed in any other form apart from 3m + 1.

EXERCISE 1.3 PAGE NO: 6

1. Show that the square of any positive integer is either of the form 4q or 4q + 1 for some integer q.

Solution:

According to Euclid's division lemma,

$a=bq+r$

According to the question,

When b = 4.

$a = 4k + r, 0 \leq r < 4$

When r = 0, we get, a = 4k

$a^2 = 16k^2 = 4(4k^2) = 4q$, where $q = 4k^2$

When r = 1, we get, a = 4k + 1

$a^2 = (4k + 1)^2 = 16k^2 + 1 + 8k = 4(4k + 2) + 1 = 4q + 1$, where $q = k(4k + 2)$

When r = 2, we get, a = 4k + 2

$a^2 = (4k + 2)^2 = 16k^2 + 4 + 16k = 4(4k^2 + 4k + 1) = 4q$, where $q = 4k^2 + 4k + 1$

When r = 3, we get, a = 4k + 3

$a^2 = (4k + 3)^2 = 16k^2 + 9 + 24k = 4(4k^2 + 6k + 2) + 1$

$= 4q + 1$, where $q = 4k^2 + 6k + 2$

Therefore, the square of any positive integer is either of the form 4q or 4q + 1 for some integer q.

Hence Proved.

2. Show that cube of any positive integer is of the form 4m, 4m + 1 or 4m + 3, for some integer m.

Solution:

Let a be any positive integer and b = 4.

According to Euclid Division Lemma,

$a = bq + r \quad [0 \leq r < b]$

$a = 3q + r \quad [0 \leq r < 4]$

According to the question, the possible values of r are,

r = 0, r = 1, r = 2, r = 3

When r = 0,

$a = 4q + 0$

$a = 4q$

Taking cubes on LHS and RHS,

We have,

$a^3 = (4q)^3$

$a^3 = 4(16q^3)$

$a^3 = 4m \quad$ [where m is an integer $= 16q^3$]

When r = 1,

$a = 4q + 1$

Taking cubes on LHS and RHS,

We have,

$a^3 = (4q + 1)^3$

$a^3 = 64q^3 + 1^3 + 3 \times 4q \times 1 (4q + 1)$

$a^3 = 64q^3 + 1 + 48q^2 + 12q$

$a^3 = 4(16q^3 + 12q^2 + 3q) + 1$
$a^3 = 4m + 1$ [where m is an integer $= 16q^3 + 12q^2 + 3q$]
When r = 2,
$a = 4q + 2$
Taking cubes on LHS and RHS,
We have,
$a^3 = (4q + 2)^3$
$a^3 = 64q^3 + 2^3 + 3 \times 4q \times 2(4q + 2)$
$a^3 = 64q^3 + 8 + 96q^2 + 48q$
$a^3 = 4(16q^3 + 2 + 24q^2 + 12q)$
$a^3 = 4m$ [where m is an integer $=16q^3 + 2 + 24q^2 + 12q$]
When r = 3,
$a = 4q + 3$
Taking cubes on LHS and RHS,
We have,
$a^3 = (4q + 3)^3$
$a^3 = 64q^3 + 27 + 3 \times 4q \times 3(4q + 3)$
$a^3 = 64q^3 + 24 + 3 + 144q^2 + 108q$
$a^3 = 4(16q^3 + 36q^2 + 27q + 6) + 3$
$a^3 = 4m + 3$ [where m is an integer $=16q^3 + 36q^2 + 27q + 6$]
Hence, the cube of any positive integer is in the form of 4m, 4m+1 or 4m+3.

3. Show that the square of any positive integer cannot be of the form 5q + 2 or 5q + 3 for any integer q.
Solution:

Let the positive integer = a
According to Euclid's division lemma,
$a = bm + r$
According to the question, b = 5
$a = 5m + r$
So, r= 0, 1, 2, 3, 4
When r = 0, a = 5m.
When r = 1, a = 5m + 1.
When r = 2, a = 5m + 2.
When r = 3, a = 5m + 3.
When r = 4, a = 5m + 4.
Now,
When a = 5m
$a^2 = (5m)^2 = 25m^2$
$a^2 = 5(5m^2) = 5q$, where $q = 5m^2$
When a = 5m + 1
$a^2 = (5m + 1)^2 = 25m^2 + 10m + 1$
$a^2 = 5(5m^2 + 2m) + 1 = 5q + 1$, where $q = 5m^2 + 2m$
When a = 5m + 2
$a^2 = (5m + 2)^2$
$a^2 = 25m^2 + 20m + 4$

$a^2 = 5(5m^2 + 4m) + 4$

$a^2 = 5q + 4$ where $q = 5m^2 + 4m$

When $a = 5m + 3$

$a^2 = (5m + 3)^2 = 25m^2 + 30m + 9$

$a^2 = 5(5m^2 + 6m + 1) + 4$

$a^2 = 5q + 4$ where $q = 5m^2 + 6m + 1$

When $a = 5m + 4$

$a^2 = (5m + 4)^2 = 25m^2 + 40m + 16$

$a^2 = 5(5m^2 + 8m + 3) + 1$

$a^2 = 5q + 1$ where $q = 5m^2 + 8m + 3$

Therefore, square of any positive integer cannot be of the form 5q + 2 or 5q + 3.

Hence Proved.

4. Show that the square of any positive integer cannot be of the form 6m + 2 or 6m + 5 for any integer m.

Solution:

Let the positive integer = a

According to Euclid's division algorithm,

$a = 6q + r$, where $0 \leq r < 6$

$a^2 = (6q + r)^2 = 36q^2 + r^2 + 12qr$ [$\because (a+b)^2 = a^2 + 2ab + b^2$]

$a^2 = 6(6q^2 + 2qr) + r^2$ ...(i), where, $0 \leq r < 6$

When r = 0, substituting r = 0 in Eq.(i), we get

$a^2 = 6(6q^2) = 6m$, where, $m = 6q^2$ is an integer.

When r = 1, substituting r = 1 in Eq.(i), we get

$a^2 = 6(6q^2 + 2q) + 1 = 6m + 1$, where, $m = (6q^2 + 2q)$ is an integer.

When r = 2, substituting r = 2 in Eq(i), we get

$a^2 = 6(6q^2 + 4q) + 4 = 6m + 4$, where, $m = (6q^2 + 4q)$ is an integer.

When r = 3, substituting r = 3 in Eq.(i), we get

$a^2 = 6(6q^2 + 6q) + 9 = 6(6q^2 + 6a) + 6 + 3$

$a^2 = 6(6q^2 + 6q + 1) + 3 = 6m + 3$, where, $m = (6q + 6q + 1)$ is integer.

When r = 4, substituting r = 4 in Eq.(i) we get

$a^2 = 6(6q^2 + 8q) + 16$

$= 6(6q^2 + 8q) + 12 + 4$

$\Rightarrow a^2 = 6(6q^2 + 8q + 2) + 4 = 6m + 4$, where, $m = (6q^2 + 8q + 2)$ is integer.

When r = 5, substituting r = 5 in Eq.(i), we get

$a^2 = 6(6q^2 + 10q) + 25 = 6(6q^2 + 10q) + 24 + 1$

$a^2 = 6(6q^2 + 10q + 4) + 1 = 6m + 1$, where, $m = (6q^2 + 10q + 4)$ is integer. Hence, the square of any positive integer cannot be of the form 6m + 2 or 6m + 5 for any integer m.

Hence Proved

5. Show that the square of any odd integer is of the form 4q + 1, for some integer q.

Solution:

Let a be any odd integer and b = 4.

According to Euclid's algorithm,

$a = 4m + r$ for some integer $m \geq 0$

And r = 0,1,2,3 because $0 \le r < 4$.

So, we have that,

a = 4m or 4m + 1 or 4m + 2 or 4m + 3 So, a = 4m + 1 or 4m + 3

We know that, a cannot be 4m or 4m + 2, as they are divisible by 2.

$(4m + 1)^2 = 16m^2 + 8m + 1$

$= 4(4m^2 + 2m) + 1$

$= 4q + 1$, where q is some integer and $q = 4m^2 + 2m$.

$(4m + 3)^2 = 16m^2 + 24m + 9$

$= 4(4m^2 + 6m + 2) + 1$

$= 4q + 1$, where q is some integer and $q = 4m^2 + 6m + 2$

Therefore, Square of any odd integer is of the form 4q + 1, for some integer q.

Hence Proved.

6. If n is an odd integer, then show that $n^2 – 1$ is divisible by 8.

Solution:

We know that any odd positive integer n can be written in form $4q + 1$ or $4q + 3$.

So, according to the question,

When $n = 4q + 1$,

Then $n^2 - 1 = (4q + 1)^2 - 1 = 16q^2 + 8q + 1 - 1 = 8q(2q + 1)$, is divisible by 8.

When $n = 4q + 3$,

Then $n^2 - 1 = (4q + 3)^2 - 1 = 16q^2 + 24q + 9 - 1 = 8(2q^2 + 3q + 1)$, is divisible by 8.

So, from the above equations, it is clear that, if n is an odd positive integer $n^2 - 1$ is divisible by 8.

Hence Proved.

7. Prove that if x and y are both odd positive integers, then $x^2 + y^2$ is even but not divisible by 4.

Solution:

Let the two odd positive numbers x and y be 2k + 1 and 2p + 1, respectively

i.e., $x^2 + y^2 = (2k + 1)^2 +(2p + 1)^2$

$= 4k^2 + 4k + 1 + 4p^2 + 4p + 1$

$= 4k^2 + 4p^2 + 4k + 4p + 2$

$= 4 (k^2 + p^2 + k + p) + 2$

Thus, the sum of square is even the number is not divisible by 4

Therefore, if x and y are odd positive integer, then $x^2 + y^2$ is even but not divisible by four.

Hence Proved

EXERCISE 1.4 PAGE NO: 7

1. Show that the cube of a positive integer of the form 6q + r, q is an integer and r = 0, 1, 2, 3, 4, 5 is also of the form 6m + r.

Solution:

6q + r is a positive integer, where q is an integer and r = 0, 1, 2, 3, 4, 5
Then, the positive integers are of the form 6q, 6q+1, 6q+2, 6q+3, 6q+4 and 6q+5.
Taking cube on L.H.S and R.H.S,
For 6q,
$(6q)^3 = 216\ q^3 = 6(36q)^3 + 0$
$= 6m + 0$, (where m is an integer $= (36q)^3$)
For 6q+1,
$(6q+1)^3 = 216q^3 + 108q^2 + 18q + 1$
$= 6(36q^3 + 18q^2 + 3q) + 1$
$= 6m + 1$, (where m is an integer $= 36q^3 + 18q^2 + 3q$)
For 6q+2,
$(6q+2)^3 = 216q^3 + 216q^2 + 72q + 8$
$= 6(36q^3 + 36q^2 + 12q + 1) +2$
$= 6m + 2$, (where m is an integer $= 36q^3 + 36q^2 + 12q + 1$)
For 6q+3,
$(6q+3)^3 = 216q^3 + 324q^2 + 162q + 27$
$= 6(36q^3 + 54q^2 + 27q + 4) + 3$
$= 6m + 3$, (where m is an integer $= 36q^3 + 54q^2 + 27q + 4$)
For 6q+4,
$(6q+4)^3 = 216q^3 + 432q^2 + 288q + 64$
$= 6(36q^3 + 72q^2 + 48q + 10) + 4$
$= 6m + 4$, (where m is an integer $= 36q^3 + 72q^2 + 48q + 10$)
For 6q+5,
$(6q+5)^3 = 216q^3 + 540q^2 + 450q + 125$
$= 6(36q^3 + 90q^2 + 75q + 20) + 5$
$= 6m + 5$, (where m is an integer $= 36q^3 + 90q^2 + 75q + 20$)
Hence, the cube of a positive integer of the form 6q + r, q is an integer and r = 0, 1, 2, 3, 4, 5 is also of the form 6m + r.

2. Prove that one and only one out of n, n + 2 and n + 4 is divisible by 3, where n is any positive integer.

Solution:

According to Euclid's division Lemma,
Let the positive integer = n
And b=3
n =3q+r, where q is the quotient and r is the remainder
$0 \leq r < 3$ implies remainders may be 0, 1 and 2
Therefore, n may be in the form of 3q, 3q+1, 3q+2
When n=3q
n+2=3q+2

n+4=3q+4
Here n is only divisible by 3
When n = 3q+1
n+2=3q=3
n+4=3q+5
Here only n+2 is divisible by 3
When n=3q+2
n+2=3q+4
n+4=3q+2+4=3q+6
Here only n+4 is divisible by 3
So, we can conclude that one and only one out of n, n + 2 and n + 4 is divisible by 3.
Hence Proved