EXERCISE 2.1

Choose the correct answer from the given four options in the following questions:

1. If one of the zeroes of the quadratic polynomial (*k*–1) x^2 + *k x* + 1 is –3, then the value of *k* is

(A) 4/3 **(B) -4/3**

(C) 2/3 **(D) -2/3**

Solution:

(A) 4/3

Explanation:

According to the question,

-3 is one of the zeros of quadratic polynomial $(k-1)x^2+kx+1$

Substituting -3 in the given polynomial,

$(k-1)(-3)^2+k(-3)+1=0$

$(k-1)9+k(-3)+1 = 0$

$9k-9-3k+1=0$

$6k-8=0$

$k=8/6$

Therefore, $k=4/3$

Hence, **option (A)** is the correct answer.

2. A quadratic polynomial, whose zeroes are –3 and 4, is

(A) $x^2 - x + 12$ **(B) $x^2 + x + 12$**

(C) $(x^2/2)-(x/2)-6$ **(D) $2x^2 + 2x - 24$**

Solution:

(C) $(x^2/2)-(x/2)-6$

Explanation:

Sum of zeroes, $\alpha+\beta = -3+4 = 1$

Product of Zeroes, $\alpha\beta = -3\times 4 = -12$

Therefore, the quadratic polynomial becomes,

x^2- (sum of zeroes)x+(product of zeroes)

$= x^2- (\alpha+ \beta)x+(\alpha\beta)$

$= x^2 - (1)x + (-12)$

$= x^2 - x -12$

Divide by 2, we get

$= x^2/2 - x/2 - 12/2$

$= x^2/2 - x/2 - 6$

Hence, **option (C)** is the correct answer.

3. If the zeroes of the quadratic polynomial $x^2 + (a + 1) x + b$ are 2 and –3, then

(A) $a = -7, b = -1$ **(B) $a = 5, b = -1$**

(C) $a = 2, b = -6$ **(D) $a = 0, b = -6$**

Solution:

(D) $a = 2, b = -6$

Explanation:

According to the question,

$x^2 + (a+1)x + b$

Given that, the zeroes of the polynomial = 2 and -3,

When x = 2

$2^2 + (a+1)(2) + b = 0$

$4 + 2a+2 + b = 0$

$6 + 2a+b = 0$

$2a+b = -6$ ----- (1)

When x = -3,

$(-3)^2 + (a+1)(-3) + b = 0$

$9 - 3a-3 + b = 0$

$6 - 3a+b = 0$

$-3a+b = -6$ ----- (2)

Subtracting equation (2) from (1)

$2a+b - (-3a+b) = -6-(-6)$

$2a+b+3a-b = -6+6$

$5a = 0$

$a = 0$

Substituting the value of 'a' in equation (1), we get,

$2a + b = -6$

$2(0) +b = -6$

$b = -6$

Hence, **option (D)** is the correct answer.

4. The number of polynomials having zeroes as –2 and 5 is

(A) 1 **(B) 2**

(C) 3 **(D) more than 3**

Solution:

(D) more than 3

Explanation:

According to the question,

The zeroes of the polynomials = -2 and 5

We know that the polynomial is of the form,

$p(x) = ax^2 + bx + c.$

Sum of the zeroes = - (coefficient of x) ÷ coefficient of x^2 i.e.

Sum of the zeroes = - b/a

$- 2 + 5 = - b/a$

$3 = - b/a$

b = - 3 and a = 1

Product of the zeroes = constant term ÷ coefficient of x^2 i.e.

Product of zeroes = c/a

$(- 2)5 = c/a$

$- 10 = c$

Substituting the values of a, b and c in the polynomial $p(x) = ax^2 + bx + c$.

We get, $x^2 - 3x - 10$

Therefore, we can conclude that x can take any value.

Hence, **option (D)** is the correct answer.

5. Given that one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is zero, the product of the other two zeroes is

(A) (–c/a) **(B) c/a**

(C) 0 **(D) (–b/a)**

Solution:

(B) (c/a)

Explanation:

According to the question,

We have the polynomial,

$ax^3 + bx^2 + cx + d$

We know that,

Sum of product of roots of a cubic equation is given by c/a

It is given that one root = 0

Now, let the other roots be α, β

So, we get,

$\alpha\beta + \beta(0) + (0)\alpha = c/a$

$\alpha\beta = c/a$

Hence the product of other two roots is c/a

Hence, **option (B)** is the correct answer

EXERCISE 2.2 PAGE NO: 11

1. Answer the following and justify:

(i) Can $x^2 – 1$ be the quotient on division of $x^6 + 2x^3 + x – 1$ by a polynomial in x of degree 5?

Solution:

No, $x^2 - 1$ cannot be the quotient on division of $x^6 + 2x^3 + x - 1$ by a polynomial in x of degree 5.

Justification:

When a degree 6 polynomial is divided by degree 5 polynomial,
The quotient will be of degree 1.
Assume that $(x^2 - 1)$ divides the degree 6 polynomial with and the quotient obtained is degree 5 polynomial (1)
According to our assumption,
(degree 6 polynomial) = $(x^2 - 1)$(degree 5 polynomial) + r(x) [Since, (a = bq + r)]
= (degree 7 polynomial) + r(x) [Since, (x^2 term × x^5 term = x^7 term)]
= (degree 7 polynomial)
From the above equation, it is clear that, our assumption is contradicted.
$x^2 - 1$ cannot be the quotient on division of $x^6 + 2x^3 + x - 1$ by a polynomial in x of degree 5
Hence Proved.

(ii) What will the quotient and remainder be on division of $ax^2 + bx + c$ by $px^3 + qx^2 + rx + s, p \neq 0$?

Solution:

Degree of the polynomial $px^3 + qx^2 + rx + s$ is 3
Degree of the polynomial $ax^2 + bx + c$ is 2
Here, degree of $px^3 + qx^2 + rx + s$ is greater than degree of the $ax^2 + bx + c$
Therefore, the quotient would be zero,
And the remainder would be the dividend = $ax^2 + bx + c$.

(iii) If on division of a polynomial $p(x)$ by a polynomial $g(x)$, the quotient is zero, what is the relation between the degrees of $p(x)$ and $g(x)$?

Solution:

We know that,
p(x)= g(x) × q(x)+r(x)
According to the question,
q(x) =0
When q(x)=0, then r(x) is also = 0
So, now when we divide p(x) by g(x),
Then p(x) should be equal to zero
Hence, the relation between the degrees of p (x) and g (x) is the degree p(x)<degree g(x)

(iv) If on division of a non-zero polynomial $p(x)$ by a polynomial $g(x)$, the remainder is zero, what is the relation between the degrees of $p(x)$ and $g(x)$?

Solution:

In order to divide p(x) by g(x)

We know that,
Degree of p(x) > degree of g(x)
or
Degree of p(x)= degree of g(x)
Therefore, we can say that,
The relation between the degrees of *p* (*x*) and *g* (*x*) is degree of p(x) $\geq$ degree of g(x)

(v) Can the quadratic polynomial $x^2 + kx + k$ have equal zeroes for some odd integer $k > 1$?
Solution:

A Quadratic Equation will have equal roots if it satisfies the condition:

$b^2 - 4ac = 0$

Given equation is $x^2 + kx + k = 0$

a = 1, b = k, x = k

Substituting in the equation we get,

$k^2 - 4 (1) (k) = 0$
$k^2 - 4k = 0$
$k (k - 4) = 0$
$k = 0 , k = 4$

But in the question, it is given that k is greater than 1.
Hence the value of k is 4 if the equation has common roots.
Hence if the value of k = 4, then the equation ($x^2 + kx + k$) will have equal roots.

EXERCISE 2.3 PAGE NO: 12

Find the zeroes of the following polynomials by factorisation method.

1. $4x^2 – 3x – 1$

Solution:

$4x^2 – 3x – 1$

Splitting the middle term, we get,

$4x^2-4x+1x-1$

Taking the common factors out, we get,

4x(x-1) +1(x-1)

On grouping, we get,

(4x+1)(x-1)

So, the zeroes are,

4x+1= 0⇒ 4x=-1 ⇒x= (-1/4)

(x-1) = 0 ⇒ x=1

Therefore, zeroes are (-1/4) and 1

Verification:

Sum of the zeroes = - (coefficient of x) ÷ coefficient of x^2

α + β = - b/a

1 - 1/4 = - (- 3)/4 = ¾

Product of the zeroes = constant term ÷ coefficient of x^2

α β = c/a

1(- 1/4) = - ¼

- 1/4 = - 1/4

2. $3x^2 + 4x – 4$

Solution:

$3x^2 + 4x – 4$

Splitting the middle term, we get,

$3x^2 + 6x - 2x – 4$

Taking the common factors out, we get,

3x(x+2) -2(x+2)

On grouping, we get,

(x+2)(3x-2)

So, the zeroes are,

x+2=0 ⇒ x= -2

3x-2=0⇒ 3x=2⇒x=2/3

Therefore, zeroes are (2/3) and -2

Verification:

Sum of the zeroes = - (coefficient of x) ÷ coefficient of x^2

α + β = - b/a

- 2 + (2/3) = - (4)/3

= - 4/3 = - 4/3

Product of the zeroes = constant term ÷ coefficient of x^2

α β = c/a

Product of the zeroes = (- 2) (2/3) = - 4/3

3. $5t^2 + 12t + 7$

Solution:

$5t^2 + 12t + 7$

Splitting the middle term, we get,

$5t^2 + 5t + 7t + 7$

Taking the common factors out, we get,

5t (t+1) +7(t+1)

On grouping, we get,

(t+1)(5t+7)

So, the zeroes are,

t+1=0 ⇒ y= -1

5t+7=0 ⇒ 5t=-7⇒t=-7/5

Therefore, zeroes are (-7/5) and -1

Verification:

Sum of the zeroes = - (coefficient of x) ÷ coefficient of x^2

α + β = - b/a

(- 1) + (- 7/5) = - (12)/5

= - 12/5 = - 12/5

Product of the zeroes = constant term ÷ coefficient of x^2

α β = c/a

(- 1)(- 7/5) = 7/5

7/5 = 7/5

4. $t^3 – 2t^2 – 15t$

Solution:

$t^3 – 2t^2 – 15t$

Taking t common, we get,

t (t^2 -2t -15)

Splitting the middle term of the equation t^2 -2t -15, we get,

t(t^2 -5t + 3t -15)

Taking the common factors out, we get,

t (t (t-5) +3(t-5)

On grouping, we get,

t (t+3)(t-5)

So, the zeroes are,

t=0

t+3=0 ⇒ t= -3

t -5=0 ⇒ t=5

Therefore, zeroes are 0, 5 and -3

Verification:

Sum of the zeroes = - (coefficient of x^2) ÷ coefficient of x^3

α + β + γ = - b/a

(0) + (- 3) + (5) = - (- 2)/1
= 2 = 2
Sum of the products of two zeroes at a time = coefficient of x ÷ coefficient of x^3
αβ + βγ + αγ = c/a
(0)(- 3) + (- 3) (5) + (0) (5) = - 15/1
= - 15 = - 15
Product of all the zeroes = - (constant term) ÷ coefficient of x^3
αβγ = - d/a
(0)(- 3)(5) = 0
0 = 0

5. $2x^2$ +(7/2)*x* +3/4
Solution:

$2x^2$ +(7/2)*x* +3/4
The equation can also be written as,
$8x^2$+14x+3
Splitting the middle term, we get,
$8x^2$+12x+2x+3
Taking the common factors out, we get,
4x (2x+3) +1(2x+3)
On grouping, we get,
(4x+1)(2x+3)
So, the zeroes are,
4x+1=0 ⇒ x = -1/4
2x+3=0 ⇒ x = -3/2
Therefore, zeroes are -1/4 and -3/2
Verification:
Sum of the zeroes = - (coefficient of x) ÷ coefficient of x^2
α + β = - b/a
(- 3/2) + (- 1/4) = - (7)/4
= - 7/4 = - 7/4
Product of the zeroes = constant term ÷ coefficient of x^2
α β = c/a
(- 3/2)(- 1/4) = (3/4)/2
3/8 = 3/8

EXERCISE 2.4 PAGE NO: 14

1. For each of the following, find a quadratic polynomial whose sum and product respectively of the zeroes are as given. Also find the zeroes of these polynomials by factorisation.

(i) (–8/3), 4/3
(ii) 21/8, 5/16
(iii) -2√3, -9
(iv) (-3/(2√5)), -½

Solution:

(i) Sum of the zeroes = - 8/3
Product of the zeroes = 4/3
P(x) = x^2 - (sum of the zeroes) + (product of the zeroes)
Then, P(x)= x^2 - (-8x)/3 + 4/3
P(x)= $3x^2 + 8x + 4$
Using splitting the middle term method,
$3x^2 + 8x + 4 = 0$
$3x^2 + (6x + 2x) + 4 = 0$
$3x^2 + 6x + 2x + 4 = 0$
$3x(x + 2) + 2(x + 2) = 0$
$(x + 2)(3x + 2) = 0$
⇒ x = - 2, - 2/3

(ii) Sum of the zeroes = 21/8
Product of the zeroes = 5/16
P(x) = x^2 - (sum of the zeroes) + (product of the zeroes)
Then, P(x)= x^2 - 21x/8 + 5/16
P(x)= $16x^2 - 42x + 5$
Using splitting the middle term method,
$16x^2 - 42x + 5 = 0$
$16x^2 - (2x + 40x) + 5 = 0$
$16x^2 - 2x - 40x + 5 = 0$
$2x (8x - 1) - 5(8x - 1) = 0$
$(8x - 1)(2x - 5) = 0$
⇒ x = 1/8, 5/2

(iii) Sum of the zeroes = - 2√3
Product of the zeroes = - 9
P(x) = x^2 - (sum of the zeroes) + (product of the zeroes)
Then, P(x) = $x^2 - (-2\sqrt{3}x) - 9$
Using splitting the middle term method,
$x^2 + 2\sqrt{3}x - 9 = 0$
$x^2 + (3\sqrt{3}x - \sqrt{3}x) - 9 = 0$
$x^2 + 3\sqrt{3}x - \sqrt{3}x - 9 = 0$
$x(x + 3\sqrt{3}) - \sqrt{3}(x + 3\sqrt{3}) = 0$
$(x + 3\sqrt{3})(x - \sqrt{3}) = 0$

$\Rightarrow x = \sqrt{3}, -3\sqrt{3}$

(iv) Sum of the zeroes = $-3/2\sqrt{5}x$

Product of the zeroes = $-\frac{1}{2}$

$P(x) = x^2$ - (sum of the zeroes) + (product of the zeroes)

Then, $P(x)= x^2 \ -3/2\sqrt{5}x - \frac{1}{2}$

$P(x)= 2\sqrt{5}x^2 + 3x - \sqrt{5}$

Using splitting the middle term method,

$2\sqrt{5}x^2 + 3x - \sqrt{5} = 0$

$2\sqrt{5}x^2 + (5x - 2x) - \sqrt{5} = 0$

$2\sqrt{5}x^2 + 5x - 2x - \sqrt{5} = 0$

$\sqrt{5}x(2x + \sqrt{5}) - 1(2x + \sqrt{5}) = 0$

$(2x + \sqrt{5})(\sqrt{5}x - 1) = 0$

$\Rightarrow x = 1/\sqrt{5}, -\sqrt{5}/2$

2. Given that the zeroes of the cubic polynomial $x^3 - 6x^2 + 3x + 10$ are of the form a, $a + b$, $a + 2b$ for some real numbers a and b, find the values of a and b as well as the zeroes of the given polynomial.

Solution:

Given that a, a+b, a+2b are roots of given polynomial $x^3-6x^2+3x+10$

Sum of the roots $\Rightarrow$ a+2b+a+a+b = -coefficient of x^2/ coefficient of x^3

$\Rightarrow$ 3a+3b = -(-6)/1 = 6

$\Rightarrow$ 3(a+b) = 6

$\Rightarrow$ a+b = 2 --------- (1) b = 2-a

Product of roots $\Rightarrow$ (a+2b)(a+b)a = -constant/coefficient of x^3

$\Rightarrow$ (a+b+b)(a+b)a = -10/1

Substituting the value of a+b=2 in it

$\Rightarrow$ (2+b)(2)a = -10

$\Rightarrow$ (2+b)2a = -10

$\Rightarrow$ (2+2-a)2a = -10

$\Rightarrow$ (4-a)2a = -10

$\Rightarrow 4a-a^2 = -5$

$\Rightarrow a^2-4a-5 = 0$

$\Rightarrow a^2-5a+a-5 = 0$

$\Rightarrow$ (a-5)(a+1) = 0

a-5 = 0 or a+1 = 0

a = 5 a = -1

a = 5, -1 in (1) a+b = 2

When a = 5, 5+b=2 $\Rightarrow$ b=-3

a = -1, -1+b=2 $\Rightarrow$ b= 3

$\therefore$ If a=5 then b= -3

or
If a= -1 then b=3

3. Given that √2 is a zero of the cubic polynomial $6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$, find its other two zeroes.
Solution:

Given, √2 is one of the zero of the cubic polynomial.
Then, (x-√2) is one of the factor of the given polynomial $p(x) = 6x^3+\sqrt{2}x^2-10x-4\sqrt{2}$.
So, by dividing p(x) by x-√2

$$\begin{array}{r|l} & 6x^2 + 7\sqrt{2}x + 4 \\ (x-\sqrt{2}) & 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2} \\ & 6x^3 - 6\sqrt{2}x^2 \\ & - \quad + \\ \hline & 7\sqrt{2}x^2 - 10x - 4\sqrt{2} \\ & 7\sqrt{2}x^2 - 14x \\ & - \quad + \\ \hline & 4x - 4\sqrt{2} \\ & 4x - 4\sqrt{2} \\ \hline & 0 \end{array}$$

$6x^3+\sqrt{2}x^2-10x-4\sqrt{2} = (x-\sqrt{2})(6x^2+7\sqrt{2}x+4)$
By splitting the middle term,
We get,
$(x-\sqrt{2})(6x^2 + 4\sqrt{2}x + 3\sqrt{2}x + 4)$
$= (x-\sqrt{2})[2x(3x+2\sqrt{2}) + \sqrt{2}(3x+2\sqrt{2})]$
$= (x-\sqrt{2})(2x+\sqrt{2})(3x+2\sqrt{2})$

To get the zeroes of p(x),
Substitute p(x)= 0
$(x-\sqrt{2})(2x+\sqrt{2})(3x+2\sqrt{2}) = 0$
$x= \sqrt{2}$, $x= -\sqrt{2}/2$, $x= -2\sqrt{2}/3$
which is equal to,
$x= \sqrt{2}$, $x= -\sqrt{2}/2$, $x= -2\sqrt{2}/3$
Hence, the other two zeroes of p(x) are $-\sqrt{2}/2$ and $-2\sqrt{2}/3$