EXERCISE 5.1 PAGE NO: 45

Choose the correct answer from the given four options in the following questions:

1. In an AP, if $d = –4$, $n = 7$, $a_n = 4$, then a is

(A) 6 (B) 7 (C) 20 (D) 28

Solution:

(D) 28

Explanation:

We know that nth term of an AP is

$a_n = a + (n - 1)d$

where,

a = first term

a_n is nth term

d is the common difference

According to the question,

$4 = a + (7 - 1)(- 4)$

$4 = a – 24$

$a = 24 + 4 = 28$

2. In an AP, if $a = 3.5$, $d = 0$, $n = 101$, then a_n will be

(A) 0 (B) 3.5 (C) 103.5 (D) 104.5

Solution:

(B) 3.5

Explanation:

We know that nth term of an AP is

$a_n = a + (n - 1)d$

Where,

a = first term

a_n is nth term

d is the common difference

$a_n = 3.5 + (101 - 1)0$

$= 3.5$

(Since, d = 0, it's a constant A.P)

3. The list of numbers – 10, – 6, – 2, 2,... is

(A) an AP with d = – 16

(B) an AP with d = 4

(C) an AP with d = – 4

(D) not an AP

Solution:

(B) an AP with d = 4

Explanation:

According to the question,

$a_1 = - 10$

$a_2 = - 6$

$a_3 = -2$
$a_4 = 2$
$a_2 - a_1 = 4$
$a_3 - a_2 = 4$
$a_4 - a_3 = 4$
$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = 4$
Therefore, it's an A.P with d = 4

4. The 11th term of the AP: –5, (–5/2), 0, 5/2, ...is
(A) –20 (B) 20 (C) –30 (D) 30
Solution:
(B) 20
Explanation:
First term, a = - 5
Common difference,
d = 5 – (-5/2) = 5/2
n = 11
We know that the nth term of an AP is
$a_n = a + (n - 1)d$
Where,
a = first term
a_n is nth term
d is the common difference
$a_{11} = -5 + (11 - 1)(5/2)$
$a_{11} = -5 + 25 = 20$

5. The first four terms of an AP, whose first term is –2 and the common difference is –2, are
(A) – 2, 0, 2, 4
(B) – 2, 4, – 8, 16
(C) – 2, – 4, – 6, – 8
(D) – 2, – 4, – 8, –16
Solution:
(C) – 2, – 4, – 6, – 8
Explanation:
First term, a = - 2
Second Term, d = - 2
$a_1 = a = -2$
We know that the nth term of an AP is
$a_n = a + (n - 1)d$
Where,
a = first term
a_n is nth term
d is the common difference
Hence, we have,
$a_2 = a + d = -2 + (-2) = -4$
Similarly,

$a_3 = -6$
$a_4 = -8$
So the A.P is
- 2, - 4, - 6, - 8

6. The 21st term of the AP whose first two terms are –3 and 4 is
(A) 17 (B) 137 (C) 143 (D) –143
Solution:
(B) 137
Explanation:
First two terms of an AP are a = - 3 and $a_2 = 4$.
We know, nth term of an AP is
$a_n = a + (n - 1)d$
Where,
a = first term
a_n is nth term
d is the common difference
$a_2 = a + d$
4 = - 3 + d
d = 7
Common difference, d = 7
$a_{21} = a + 20d$
= - 3 + (20)(7)
= 137

7. If the 2nd term of an AP is 13 and the 5th term is 25, what is its 7^{th} term?
(A) 30 (B) 33 (C) 37 (D) 38
Solution:
(B) 33
Explanation:
We know that the nth term of an AP is
$a_n = a + (n - 1)d$
Where,
a = first term
a_n is nth term
d is the common difference
$a_2 = a + d = 13$ …..(1)
$a_5 = a + 4d = 25$ …… (2)
From equation (1) we have,
a = 13 – d
Using this in equation (2), we have
13 - d + 4d = 25
13 + 3d = 25
3d = 12
d = 4
a = 13 - 4 = 9

$a_7 = a + 6d$
$= 9 + 6(4)$
$= 9 + 24 = 33$

8. Which term of the AP: 21, 42, 63, 84... is 210?

(A) 9th **(B) 10th** **(C) 11th** **(D) 12th**

Solution:

(B) 10th

Explanation:

Let nth term of the given AP be 210.
According to question,
first term, a = 21
common difference, d = 42 – 21 = 21 and $a_n = 210$
We know that the nth term of an AP is
$a_n = a + (n - 1)d$
Where,
a = first term
a_n is nth term
d is the common difference
$210 = 21 + (n - 1)21$
$189 = (n - 1)21$
$n - 1 = 9$
$n = 10$
So, 10th term of an AP is 210.

9. If the common difference of an AP is 5, then what is $a_{18} – a_{13}$?

(A) 5 **(B) 20** **(C) 25** **(D) 30**

Solution:

(C) 25

Explanation:

Given, the common difference of AP i.e., d = 5
Now,
As we know, nth term of an AP is
$a_n = a + (n - 1)d$
where a = first term
a_n is nth term
d is the common difference
$a_{18} - a_{13} = a + 17d – (a + 12d)$
$= 5d$
$= 5(5)$
$= 25$

EXERCISE 5.2

1. Which of the following form an AP? Justify your answer.

(i) –1, –1, –1, –1,...

Solution:

We have $a_1 = -1$, $a_2 = -1$, $a_3 = -1$ and $a_4 = -1$

$a_2 - a_1 = 0$

$a_3 - a_2 = 0$

$a_4 - a_3 = 0$

Clearly, the difference of successive terms is same, therefore given list of numbers from an AP.

(ii) 0, 2, 0, 2,...

Solution:

We have $a_1 = 0$, $a_2 = 2$, $a_3 = 0$ and $a_4 = 2$

$a_2 - a_1 = 2$

$a_3 - a_2 = -2$

$a_4 - a_3 = 2$

Clearly, the difference of successive terms is not same, therefore given list of numbers does not form an AP.

(iii) 1, 1, 2, 2, 3, 3...

Solution:

We have $a_1 = 1$, $a_2 = 1$, $a_3 = 2$ and $a_4 = 2$

$a_2 - a_1 = 0$

$a_3 - a_2 = 1$

Clearly, the difference of successive terms is not same, therefore given list of numbers does not form an AP.

(iv) 11, 22, 33...

Solution:

We have $a_1 = 11$, $a_2 = 22$ and $a_3 = 33$

$a_2 - a_1 = 11$

$a_3 - a_2 = 11$

Clearly, the difference of successive terms is same, therefore given list of numbers form an AP.

(v) 1/2,1/3,1/4, ...

Solution:

We have $a_1 = ½$, $a_2 = 1/3$ and $a_3 = ¼$

$a_2 - a_1 = -1/6$

$a_3 - a_2 = -1/12$

Clearly, the difference of successive terms is not same, therefore given list of numbers does not form an AP.

(vi) 2, 2^2, 2^3, 2^4, ...

Solution:

We have $a_1 = 2$, $a_2 = 2^2$, $a_3 = 2^3$ and $a_4 = 2^4$

$a_2 - a_1 = 2^2 - 2 = 4 - 2 = 2$

$a_3 - a_2 = 2^3 - 2^2 = 8 - 4 = 4$

Clearly, the difference of successive terms is not same, therefore given list of numbers does not form an AP.

(vii) √3, √12, √27, √48, ...

Solution:

We have,

$a_1 = \sqrt{3}$, $a_2 = \sqrt{12}$, $a_3 = \sqrt{27}$ and $a_4 = \sqrt{48}$

$a_2 – a_1 = \sqrt{12} – \sqrt{3} = 2\sqrt{3} – \sqrt{3} = \sqrt{3}$

$a_3 – a_2 = \sqrt{27} – \sqrt{12} = 3\sqrt{3} – 2\sqrt{3} = \sqrt{3}$

$a_4 – a_3 = \sqrt{48} – \sqrt{27} = 4\sqrt{3} – 3\sqrt{3} = \sqrt{3}$

Clearly, the difference of successive terms is same, therefore given list of numbers from an AP.

2. Justify whether it is true to say that –1, -3/2, –2, 5/2,... forms an AP as $a_2 – a_1 = a_3 – a_2$.

Solution:

False

$a_1 = -1$, $a_2 = -3/2$, $a_3 = -2$ and $a_4 = 5/2$

$a_2 – a_1 = -3/2 – (-1) = - ½$

$a_3 – a_2 = - 2 – (- 3/2) = - ½$

$a_4 – a_3 = 5/2 – (-2) = 9/2$

Clearly, the difference of successive terms in not same, all though, $a_2 - a_1 = a_3 - a_2$ but $a_4 - a_3 \neq a_3 - a_2$ therefore it does not form an AP.

3. For the AP: –3, –7, –11, ..., can we find directly $a_{30} – a_{20}$ without actually finding a_{30} and a_{20}? Give reasons for your answer.

Solution:

True

Given

First term, a = - 3

Common difference, $d = a_2 - a_1 = - 7 - (- 3) = - 4$

$a_{30} - a_{20} = a + 29d - (a + 19d)$

= 10d

= - 40

It is so because difference between any two terms of an AP is proportional to common difference of that AP

4. Two APs have the same common difference. The first term of one AP is 2 and that of the other is 7. The difference between their 10th terms is the same as the difference between their 21st

terms, which is the same as the difference between any two corresponding terms. Why?
Solution:

Suppose there are two AP's with first terms a and A
And their common differences are d and D respectively
Suppose n be any term
$a_n = a + (n - 1)d$
$A_n = A + (n - 1)D$
As common difference is equal for both AP's
We have D = d
Using this we have
$A_n - a_n = a + (n - - 1)d - [A + (n - 1)D]$
$= a + (n - 1)d - A - (n - 1)d$
$= a – A$
As a - A is a constant value
Therefore, difference between any corresponding terms will be equal to a - A.

EXERCISE 5.3 PAGE NO: 51

1. Match the APs given in column A with suitable common differences given in column B.

Column A		Column B	
(A_1)	**2, – 2, – 6, –10,...**	**(B_1)**	**2/3**
(A_2)	**$a = -18$, $n = 10$, $a_n = 0$**	**(B_2)**	**– 5**
(A_3)	**$a = 0$, $a_{10} = 6$**	**(B_3)**	**4**
(A_4)	**$a_2 = 13$, $a_4 = 3$**	**(B_4)**	**– 4**
		(B_5)	**2**
		(B_6)	**1/2**
		(B_7)	**5**

Solution:

(A_1) AP is 2, - 2, - 6, - 10, ….
So common difference is simply
$a_2 - a_1 = -2 - 2 = -4 = (B_3)$

(A_2) Given
First term, $a = -18$
No of terms, $n = 10$
Last term, $a_n = 0$
By using the nth term formula
$a_n = a + (n - 1)d$
$0 = -18 + (10 - 1)d$
$18 = 9d$
$d = 2 = (B_5)$

(A_3) Given
First term, $a = 0$
Tenth term, $a_{10} = 6$
By using the nth term formula
$a_n = a + (n - 1)d$
$a_{10} = a + 9d$
$6 = 0 + 9d$
$d = 2/3 = (B_6)$

(A_4) Let the first term be a and common difference be d
Given that
$a_2 = 13$
$a_4 = 3$
$a_2 - a_4 = 10$
$a + d - (a + 3d) = 10$
$d - 3d = 10$
$-2d = 10$
$d = -5 = (B_1)$

2. Verify that each of the following is an AP, and then write its next three terms.

(i) 0, 1/4, 1/2, 3/4,...

Solution:

Here,

$a_1 = 0$

$a_2 = ¼$

$a_3 = ½$

$a_4 = ¾$

$a_2 – a_1 = ¼ - 0 = ¼$

$a_3 – a_2 = ½ - ¼ = ¼$

$a_4 – a_3 = ¾ - ½ = ¼$

Since, difference of successive terms are equal,

Hence, 0, 1/4, 1/2, 3/4... is an AP with common difference ¼.

Therefore, the next three term will be,

¾ + ¼ , ¾ + 2(¼), ¾ + 3(¼)

1, 5/4 , 3/2

(ii) 5, 14/3, 13/3, 4...

Solution:

Here,

$a_1 = 5$

$a_2 = 14/3$

$a_3 = 13/3$

$a_4 = 4$

$a_2 – a_1 = 14/3 – 5 = -1/3$

$a_3 – a_2 = 13/3 – 14/3 = -1/3$

$a_4 – a_3 = 4 – 13/3 = -1/3$

Since, difference of successive terms are equal,

Hence, 5, 14/3, 13/3, 4... is an AP with common difference -1/3.

Therefore, the next three term will be,

4 + (-1/3), 4 + 2(-1/3), 4 + 3(-1/3)

11/3 , 10/3, 3

(iii) √3 , 2√3, 3√3,...

Solution:

Here,

$a_1 = \sqrt{3}$

$a_2 = 2\sqrt{3}$

$a_3 = 3\sqrt{3}$

$a_4 = 4\sqrt{3}$

$a_2 – a_1 = 2\sqrt{3} – \sqrt{3} = \sqrt{3}$

$a_3 – a_2 = 3\sqrt{3} – 2\sqrt{3}= \sqrt{3}$

$a_4 – a_3 = 4\sqrt{3} – 3\sqrt{3}= \sqrt{3}$

Since, difference of successive terms are equal,

Hence, √3 , 2√3, 3√3,... is an AP with common difference √3.

Therefore, the next three term will be,

$4\sqrt{3} + \sqrt{3}, 4\sqrt{3} + 2\sqrt{3}, 4\sqrt{3} + 3\sqrt{3}$
$5\sqrt{3}, 6\sqrt{3}, 7\sqrt{3}$

(iv) *a* + *b*, (*a* + 1) + *b*, (*a* + 1) + (*b* + 1), ...
Solution:

Here
$a_1 = a + b$
$a_2 = (a + 1) + b$
$a_3 = (a + 1) + (b + 1)$
$a_2 - a_1 = (a + 1) + b - (a + b) = 1$
$a_3 - a_2 = (a + 1) + (b + 1) - (a + 1) - b = 1$
Since, difference of successive terms are equal,
Hence, *a* + *b*, (*a* + 1) + *b*, (*a* + 1) + (*b* + 1), ... is an AP with common difference 1.
Therefore, the next three term will be,
$(a + 1) + (b + 1) + 1, (a + 1) + (b + 1) + 1(2), (a + 1) + (b + 1) + 1(3)$
$(a + 2) + (b + 1), (a + 2) + (b + 2), (a + 3) + (b + 2)$

(v) *a*, 2*a* + 1, 3*a* + 2, 4*a* + 3,...
Solution:

Here $a_1 = a$
$a_2 = 2a + 1$
$a_3 = 3a + 2$
$a_4 = 4a + 3$
$a_2 - a_1 = (2a + 1) – (a) = a + 1$
$a_3 - a_2 = (3a + 2) – (2a + 1) = a + 1$
$a_4 - a_3 = (4a + 3) – (3a+2) = a + 1$
Since, difference of successive terms are equal,
Hence, *a*, 2*a* + 1, 3*a* + 2, 4*a* + 3,... is an AP with common difference a+1.
Therefore, the next three term will be,
$4a + 3 +(a + 1), 4a + 3 + 2(a + 1), 4a + 3 + 3(a + 1)$
$5a + 4, 6a + 5, 7a + 6$

3. Write the first three terms of the APs when *a* and *d* are as given below:

(i) $a = 1/2, d = -1/6$
(ii) $a = –5, d = –3$
(iii) $a = 2 , d = 1/\sqrt{2}$

Solution:

(i) $a = 1/2, d = -1/6$

We know that,
First three terms of AP are :
$a, a + d, a + 2d$
$½, ½ + (-1/6), ½ + 2 (-1/6)$
$½, 1/3, 1/6$

(ii) $a = –5, d = –3$

We know that,

First three terms of AP are :
a, a + d, a + 2d
-5, - 5 + 1 (- 3), - 5 + 2 (- 3)
- 5, - 8, - 11

(iii) $a = \sqrt{2}$, $d = 1/\sqrt{2}$
We know that,
First three terms of AP are :
a, a + d, a + 2d
$\sqrt{2}$, $\sqrt{2}+1/\sqrt{2}$, $\sqrt{2}+2/\sqrt{2}$
$\sqrt{2}$, $3/\sqrt{2}$, $4/\sqrt{2}$

4. Find a, b and c such that the following numbers are in AP: a, 7, b, 23, c.
Solution:

For a, 7, b, 23, c… to be in AP
it has to satisfy the condition,
$a_5 - a_4 = a_4 - a_3 = a_3 - a_2 = a_2 - a_1 = d$
Where d is thecommon difference
7 - a = b - 7 = 23 - b = c – 23 …(1)
Let us equate,
b - 7 = 23 – b
2b = 30
b = 15 (eqn 1)
And,
7 - a = b – 7
From eqn 1
7 - a = 15 – 7
a = - 1
And,
c - 23 = 23 – b
c - 23 = 23 – 15
c - 23 = 8
c = 31
So a = - 1
b = 15
c = 31
Then, we can say that, the sequence - 1, 7, 15, 23, 31 is an AP

5. Determine the AP whose fifth term is 19 and the difference of the eighth term from the thirteenth term is 20.
Solution:

We know that,
The first term of an AP = a
And, the common difference = d.
According to the question,
5^{th} term, $a_5 = 19$

Using the n^{th} term formula,
$a_n = a + (n - 1)d$
We get,
$a + 4d = 19$
$a = 19 - 4d$ …(1)
Also,
13^{th} term - 8^{th} term = 20
$a + 12d - (a + 7d) = 20$
$5d = 20$
$d = 4$
Substituting d = 4 in equation 1,
We get,
$a = 19 – 4(4)$
$a = 3$
Then, the AP becomes,
3, 3 + 4/, 3 + 2(4),...
3, 7, 11,...

EXERCISE 5.4 PAGE NO: 56

1. The sum of the first five terms of an AP and the sum of the first seven terms of the same AP is 167. If the sum of the first ten terms of this AP is 235, find the sum of its first twenty terms.

Solution:

We know that, in an A.P.,
First term = a
Common difference = d
Number of terms of an AP = n
According to the question,
We have,
$S_5 + S_7 = 167$
Using the formula for sum of n terms,
$S_n = (n/2)\ [2a + (n-1)d]$
So, we get,
$(5/2)\ [2a + (5-1)d] + (7/2)[2a + (7-1)d] = 167$
$5(2a + 4d) + 7(2a + 6d) = 334$
$10a + 20d + 14a + 42d = 334$
$24a + 62d = 334$
$12a + 31d = 167$
$12a = 167 - 31d$ …(1)
We have,
$S_{10} = 235$
$(10/2)\ [2a + (10-1)d] = 235$
$5[\ 2a + 9d] = 235$
$2a + 9d = 47$
Multiplying L.H.S and R.H.S by 6,
We get,
$12a + 54d = 282$
From equation (1)
$167 - 31d + 54d = 282$
$23d = 282 – 167$
$23d = 115$
$d = 5$
Substituting the value of d = 5 in equation (1)
$12a = 167 - 31(5)$
$12a = 167 – 155$
$12a = 12$
$a = 1$
We know that,
$S_{20} = (n/2)\ [2a + (20 – 1)d]$
$= 20/(2[2(1) + 19\ (5)])$
$= 10[\ 2 + 95]$
$= 970$
Therefore, the sum of first 20 terms is 970.

2. Find the

(i) Sum of those integers between 1 and 500 which are multiples of 2 as well as of 5.

(ii) Sum of those integers from 1 to 500 which are multiples of 2 as well as of 5 .

(iii) Sum of those integers from 1 to 500 which are multiples of 2 or 5.

[Hint (iii): These numbers will be: multiples of 2 + multiples of 5 – multiples of 2 as well as of 5]

Solution:

(i) Sum of those integers between 1 and 500 which are multiples of 2 as well as of 5.

We know that,
Multiples of 2 as well as of 5 = LCM of (2, 5) = 10
Multiples of 2 as well as of 5 between 1 and 500 = 10, 20, 30…, 490.
Hence,
We can conclude that 10, 20, 30…, 490 is an AP with common difference, d = 10
First term, a = 10
Let the number of terms in this AP = n
Using n^{th} term formula,
$a_n = a + (n - 1)d$
$490 = 10 + (n - 1)10$
$480 = (n - 1)10$
$n - 1 = 48$
$n = 49$
Sum of an AP,
$S_n = (n/2)\ [a + a_n]$, here a_n is the last term, which is given]
$= (49/2) \times [10 + 490]$
$= (49/2) \times [500]$
$= 49 \times 250$
$= 12250$
Therefore, sum of those integers between 1 and 500 which are multiples of 2 as well as of 5 = 12250

(ii) Sum of those integers from 1 to 500 which are multiples of 2 as well as of 5.

We know that,
Multiples of 2 as well as of 5 = LCM of (2, 5) = 10
Multiples of 2 as well as of 5 from 1 and 500 = 10, 20, 30…, 500.
Hence,
We can conclude that 10, 20, 30…, 500 is an AP with common difference, d = 10
First term, a = 10
Let the number of terms in this AP = n
Using n^{th} term formula,
$a_n = a + (n - 1)d$
$500 = 10 + (n - 1)10$
$490 = (n - 1)10$
$n - 1 = 49$
$n = 50$
Sum of an AP,
$S_n = (n/2)\ [\ a + a_n]$, here a_n is the last term, which is given]

= (50/2) ×[10+500]
= 25× [10 + 500]
= 25(510)
= 12750

Therefore, sum of those integers from 1 to 500 which are multiples of 2 as well as of 5= 12750

(iii) Sum of those integers from 1 to 500 which are multiples of 2 or 5.

We know that,
Multiples of 2 or 5 = Multiple of 2 + Multiple of 5 – Multiple of LCM (2, 5)
Multiples of 2 or 5 = Multiple of 2 + Multiple of 5 – Multiple of LCM (10)
Multiples of 2 or 5 from 1 to 500 = List of multiple of 2 from 1 to 500 + List of multiple of 5 from 1 to 500 - List of multiple of 10 from 1 to 500
= (2, 4, 6… 500) + (5, 10, 15… 500) - (10, 20, 30… 500)
Required sum = sum(2, 4, 6,…, 500) + sum(5, 10, 15,…, 500) - sum(10, 20, 30,., 500)
Consider the first series,
2, 4, 6, …., 500
First term, a = 2
Common difference, d = 2
Let n be no of terms
$a_n = a + (n - 1)d$
$500 = 2 + (n - 1)^2$
$498 = (n - 1)^2$
n - 1 = 249
n = 250
Sum of an AP, $S_n = (n/2) [a + a_n]$
Let the sum of this AP be S_1,
$S_1 = S_{250} = (250/2) \times[2+500]$
$S_1 = 125(502)$
$S_1 = 62750$ … (1)

Consider the second series,
5, 10, 15, …., 500
First term, a = 5
Common difference, d = 5
Let n be no of terms
By nth term formula
$a_n = a + (n - 1)d$
500 = 5 + (n - 1)
495 = (n - 1)5
n - 1 = 99
n = 100
Sum of an AP, $S_n = (n/2) [a + a_n]$
Let the sum of this AP be S_2,
$S_2 = S_{100} = (100/2) \times[5+500]$
$S_2 = 50(505)$

$S_2 = 25250$ … (2)

Consider the third series,

10, 20, 30, …., 500

First term, a = 10

Common difference, d = 10

Let n be no of terms

$a_n = a + (n - 1)d$

$500 = 10 + (n - 1)10$

$490 = (n - 1)10$

$n - 1 = 49$

$n = 50$

Sum of an AP, $S_n = (n/2)$ [$a + a_n$]

Let the sum of this AP be S_3,

$S_3 = S_{50} = (50/2) \times [2+510]$

$S_3 = 25(510)$

$S_3 = 12750$ … (3)

Therefore, the required Sum, $S = S_1 + S_2 - S_3$

$S = 62750 + 25250 - 12750$

$= 75250$

3. The eighth term of an AP is half its second term and the eleventh term exceeds one third of its fourth term by 1. Find the 15th term.

Solution:

We know that,

First term of an AP = a

Common difference of AP = d

n^{th} term of an AP, $a_n = a + (n - 1)d$

According to the question,

$a_s = ½ a_2$

$2a_8 = a_2$

$2(a + 7d) = a + d$

$2a + 14d = a + d$

$a = - 13d$ …(1)

Also,

$a_{11} = 1/3\ a_4 + 1$

$3(a + 10d) = a + 3d + 3$

$3a + 30d = a + 3d + 3$

$2a + 27d = 3$

Substituting $a = -13d$ in the equation,

$2 (- 13d) + 27d = 3$

$d = 3$

Then,

$a = - 13(3)= - 39$

Now,

$a_{15} = a + 14d$
$= -39 + 14(3)$
$= -39 + 42$
$= 3$
So 15^{th} term is 3.

4. An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three is 429. Find the AP.

Solution:

We know that,
First term of an AP = a
Common difference of AP = d
n^{th} term of an AP, $a_n = a + (n - 1)d$
Since, n = 37 (odd),
Middle term will be $(n+1)/2 = 19^{th}$ term
Thus, the three middle most terms will be,
18^{th}, 19^{th} and 20^{th} terms
According to the question,
$a_{18} + a_{19} + a_{20} = 225$
Using $a_n = a + (n - 1)d$
$a + 17d + a + 18d + a + 19d = 225$
$3a + 54d = 225$
$3a = 225 - 54d$
$a = 75 - 18d$ … (1)
Now, we know that last three terms will be 35^{th}, 36^{th} and 37^{th} terms.
According to the question,
$a_{35} + a_{36} + a_{37} = 429$
$a + 34d + a + 35d + a + 36d = 429$
$3a + 105d = 429$
$a + 35d = 143$
Substituting $a = 75 - 18d$ from equation 1,
$75 - 18d + 35d = 143$ [using eqn1]
$17d = 68$
$d = 4$
Then,
$a = 75 - 18(4)$
$a = 3$
Therefore, the AP is a, a + d, a + 2d….
i.e. 3, 7, 11….

5. Find the sum of the integers between 100 and 200 that are

(i) divisible by 9
(ii) not divisible by 9
[Hint (ii): These numbers will be: Total numbers – Total numbers divisible by 9]

Solution:

(i) The number between 100 and 200 which is divisible by 9 = 108, 117, 126, …198

Let the number of terms between 100 and 200 which is divisible by 9 = n

$a_n = a + (n - 1)d$

$198 = 108 + (n - 1)9$

$90 = (n - 1)9$

$n - 1 = 10$

$n = 11$

Sum of an AP = $S_n = (n/2)\ [a + a_n]$

$S_n = (11/2) \times [108 + 198]$

$= (11/2) \times 306$

$= 11(153)$

$= 1683$

(ii) Sum of the integers between 100 and 200 which is not divisible by 9 = (sum of total numbers between 100 and 200) – (sum of total numbers between 100 and 200 which is divisible by 9)

Sum, $S = S_1 - S_2$

Here,

S_1 = sum of AP 101, 102, 103, - - - , 199

S_2 = sum of AP 108, 117, 126, - - - , 198

For AP 101, 102, 103, - - - , 199

First term, a = 101

Common difference, d = 199

Number of terms = n

Then,

$a_n = a + (n - 1)d$

$199 = 101 + (n - 1)1$

$98 = (n - 1)$

$n = 99$

Sum of an AP = $S_n = (n/2)\ [a + a_n]$

Sum of this AP,

$S_1 = (99/2) \times [199 + 101]$

$= (99/2) \times 300$

$= 99(150)$

$= 14850$

For AP 108, 117, 126, - - - - , 198

First term, a = 108

Common difference, d = 9

Last term, $a_n = 198$

Number of terms = n

Then,

$a_n = a + (n - 1)d$

$198 = 108 + (n - 1)9$

$10 = (n - 1)$

$n = 11$

Sum of an AP = $S_n = (n/2)\ [a + a_n]$
Sum of this AP,

$S_2 = (11/2) \times [108 + 198]$
$= (11/2) \times (306)$
$= 11(153)$
$= 1683$

Substituting the value of S_1 and S_2 in the equation, $S = S_1 - S_2$

$S = S_1 + S_2$
$= 14850 – 1683$
$= 13167$