

EXERCISE 7.1

Choose the correct answer from the given four options in the following questions:

1. The distance of the point P (2, 3) from the x-axis is

- (A) 2 (B) 3 (C) 1 (D) 5

Solution:

(B) 3

We know that,

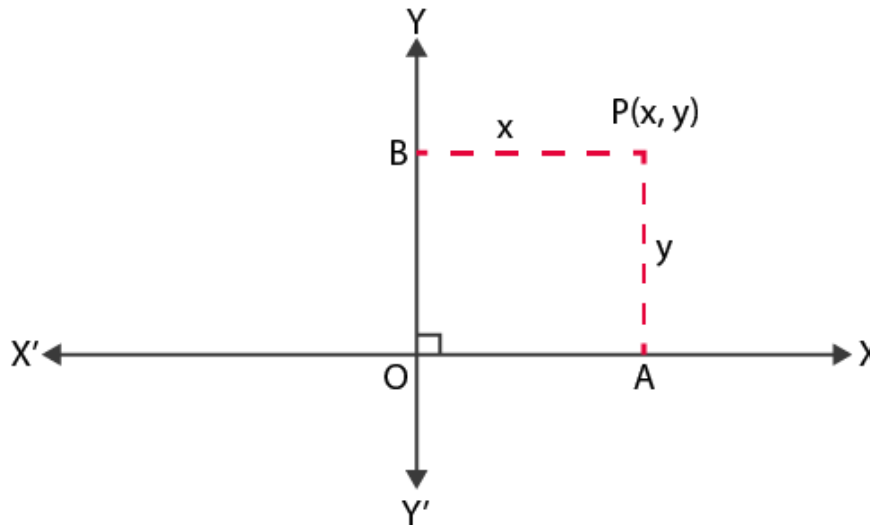
(x, y) is a point on the Cartesian plane in first quadrant.

Then,

x = Perpendicular distance from Y - axis and

y = Perpendicular distance from X – axis

Therefore, the perpendicular distance from X-axis = y coordinate = 3



2. The distance between the points A (0, 6) and B (0, -2) is

- (A) 6 (B) 8 (C) 4 (D) 2

Solution:

(B) 8

Distance formula: $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

According to the question,

We have,

$$x_1 = 0, x_2 = 0$$

$$y_1 = 6, y_2 = -2$$

$$d^2 = (0 - 0)^2 + (-2 - 6)^2$$

$$d = \sqrt{(0)^2 + (-8)^2}$$

$$d = \sqrt{64}$$

$$d = 8 \text{ units}$$

Therefore, the distance between A (0, 6) and B (0, 2) is 8

3. The distance of the point P (-6, 8) from the origin is

- (A) 8 (B) $2\sqrt{7}$ (C) 10 (D) 6

Solution: (c) 10

Distance formula: $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

According to the question,

We have;

$$x_1 = -6, x_2 = 0$$

$$y_1 = 8, y_2 = 0$$

$$d^2 = [0 - (-6)]^2 + [0 - 8]^2$$

$$d = \sqrt{((0 - (-6)))^2 + (0 - 8)^2}$$

$$d = \sqrt{(6)^2 + (-8)^2}$$

$$d = \sqrt{36 + 64}$$

$$d = \sqrt{100}$$

$$d = 10$$

Therefore, the distance between P (-6, 8) and origin O (0, 0) is 10

4. The distance between the points (0, 5) and (-5, 0) is

- (A) 5 (B) $5\sqrt{2}$ (C) $2\sqrt{5}$ (D) 10

Solution: (B) $5\sqrt{2}$

Distance formula: $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

According to the question,

We have;

$$x_1 = 0, x_2 = -5$$

$$y_1 = 5, y_2 = 0$$

$$d^2 = ((-5) - 0)^2 + (0 - 5)^2$$

$$d = \sqrt{(-5 - 0)^2 + (0 - 5)^2}$$

$$d = \sqrt{((-5)^2 + (-5)^2)}$$

$$d = \sqrt{25 + 25}$$

$$d = \sqrt{50} = 5\sqrt{2}$$

So the distance between (0, 5) and (-5, 0) = $5\sqrt{2}$

5. AOBC is a rectangle whose three vertices are vertices A (0, 3), O (0, 0) and B (5, 0). The length of its diagonal is

- (A) 5 (B) 3 (C) $\sqrt{34}$ (D) 4

Solution: (C) $\sqrt{34}$

The three vertices are: A = (0, 3), O = (0, 0), B = (5, 0)

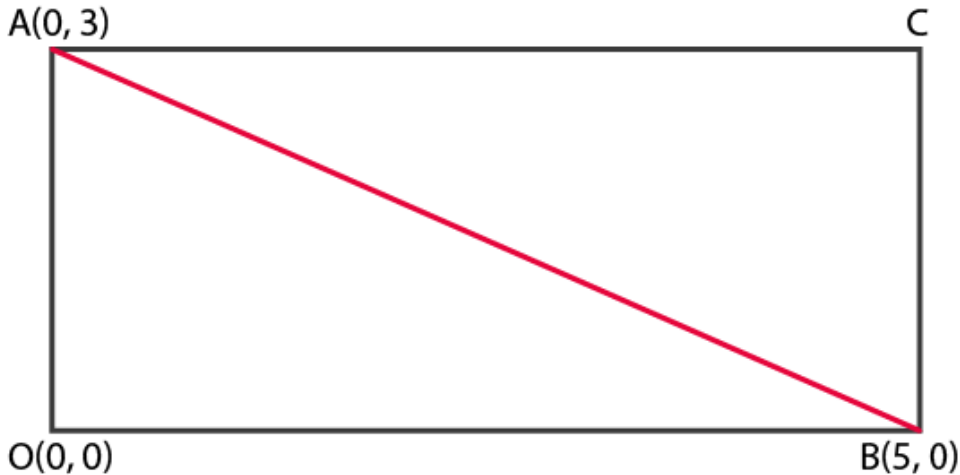
We know that, the diagonals of a rectangle are of equal length,

Length of the diagonal AB = Distance between the points A and B

Distance formula: $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

According to the question,

We have;



$$x_1 = 0, x_2 = 5$$

$$y_1 = 3, y_2 = 0$$

$$d^2 = (5 - 0)^2 + (0 - 3)^2$$

$$d = \sqrt{(5-0)^2 + (0-3)^2}$$

$$d = \sqrt{25 + 9} = \sqrt{34}$$

Distance between $A(0, 3)$ and $B(5, 0)$ is $\sqrt{34}$

Therefore, the length of its diagonal is $\sqrt{34}$

6. The perimeter of a triangle with vertices $(0, 4)$, $(0, 0)$ and $(3, 0)$ is

- (A) 5 (B) 12 (C) 11 (D) $7 + \sqrt{5}$

Solution:

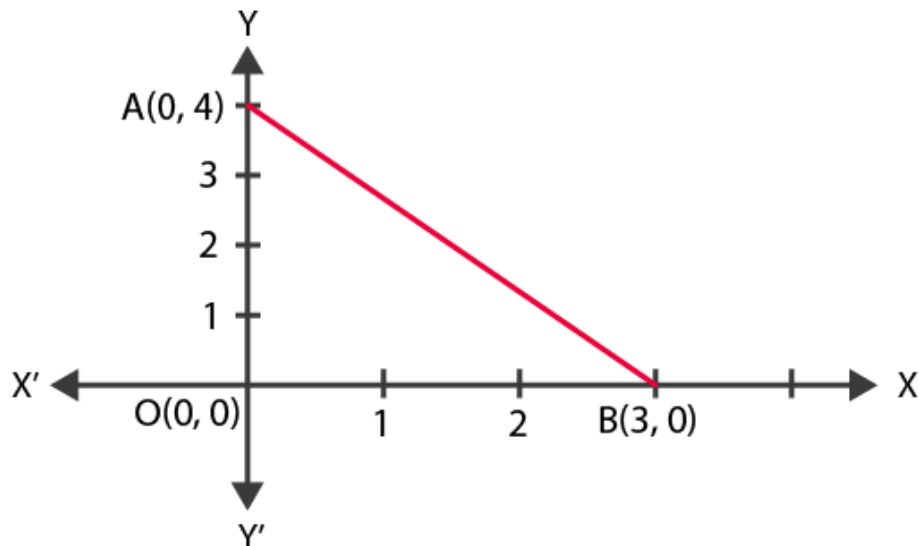
(B) 12

The vertices of a triangle are $(0, 4)$, $(0, 0)$ and $(3, 0)$.

Now, perimeter of $\triangle AOB$ = Sum of the length of all its sides:
= distance between $(OA+OB+AB)$

Distance between the points (x_1, y_1) and (x_2, y_2) is given by,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



To find:

Distance between A(0, 4) and O(0, 0) + Distance between O(0, 0) and B(3, 0) +
Distance between A(0, 4) and B(3, 0)

$$\begin{aligned} &= \sqrt{(0-0)^2 + (0-4)^2} + \sqrt{(3-0)^2 + (0-0)^2} \\ &\quad + \sqrt{(3-0)^2 + (0-4)^2} \\ &= \sqrt{0+16} + \sqrt{9+0} + \sqrt{(3)^2 + (4)^2} \\ &= 4 + 3 + \sqrt{9+16} \\ &= 7 + \sqrt{25} = 7 + 5 = 12 \end{aligned}$$

Therefore, the required perimeter of triangle is 12

7. The area of a triangle with vertices A (3, 0), B (7, 0) and C (8, 4) is

(A) 14 (B) 28 (C) 8 (D) 6

Solution:

(c) 8

Vertices of the triangle are,

$$A(x_1, y_1) = (3, 0)$$

$$B(x_2, y_2) = (7, 0)$$

$$C(x_3, y_3) = (8, 4)$$

$$\text{Area of triangle} = \left| \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \right|$$

$$= \left| \frac{1}{2} [3(0 - 4) + 7(4 - 0) + 8(0 - 0)] \right|$$

$$= \left| \frac{1}{2} [-12 + 28 + 0] \right|$$

$$= \left| \frac{1}{2} [16] \right|$$

$$= 8$$

Therefore, the area of $\triangle ABC$ is 8.

EXERCISE 7.2

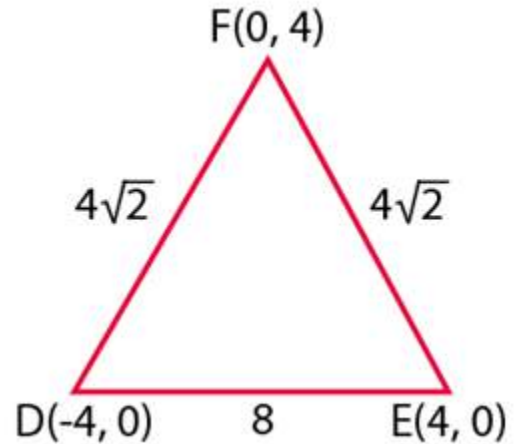
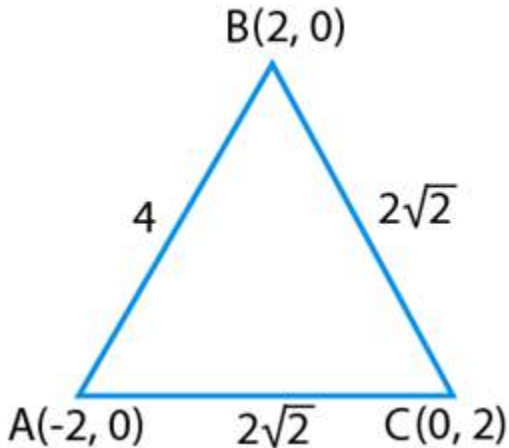
State whether the following statements are true or false. Justify your answer.

1. $\triangle ABC$ with vertices A (-2, 0), B (2, 0) and C (0, 2) is similar to $\triangle DEF$ with vertices D (-4, 0) E (4, 0) and F (0, 4).

Solution:

True.

Justification:



Using distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

We can find,

$$AB = \sqrt{(2 + 2)^2 + 0} = \sqrt{16} = 4$$

$$BC = \sqrt{(0 - 2)^2 + (2 - 0)^2} = \sqrt{8} = 2\sqrt{2}$$

$$CA = \sqrt{(-2 - 0)^2 + (0 - 2)^2} = \sqrt{8} = 2\sqrt{2}$$

$$DE = \sqrt{(4 + 4)^2 + 0} = \sqrt{64} = 8$$

$$EF = \sqrt{(0 - 4)^2 + (4 - 0)^2} = \sqrt{32} = 4\sqrt{2}$$

$$FD = \sqrt{(-4 - 0)^2 + (0 - 4)^2} = \sqrt{32} = 4\sqrt{2}$$

$$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{1}{2} \quad \Rightarrow \quad \triangle ABC \sim \triangle DEF$$

Hence, triangle ABC and DEF are similar.

2. Point P (-4, 2) lies on the line segment joining the points A (-4, 6) and B (-4, -6).

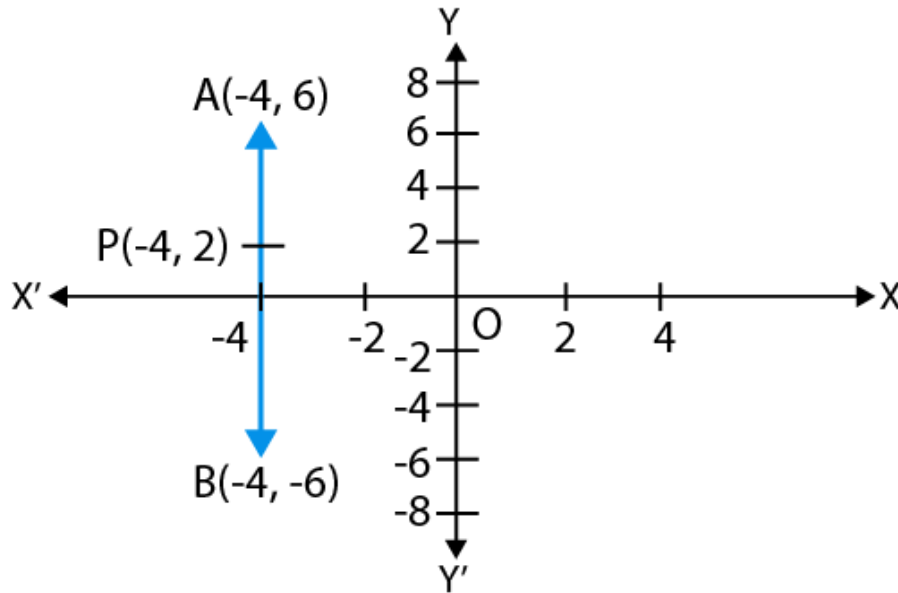
Solution:

True.

Justification:

Plotting the points P(-4, 2), A (-4, 6) and B (-4, -6) on a graph paper and connecting

the points we get the graph,



Hence, from the graph it is clear that, point P (- 4, 2) lies on the line segment joining the points A (- 4, 6) and B (- 4, - 6),

3. The points (0, 5), (0, -9) and (3, 6) are collinear.

Solution:

False

Justification:

The points are collinear if area of a triangle formed by its points is equals to the zero.

Given,

$$x_1 = 0, x_2 = 0, x_3 = 3 \text{ and}$$

$$y_1 = 5, y_2 = -9, y_3 = 6$$

$$\therefore \text{Area of triangle} = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$\Delta = \frac{1}{2}[0(-9 - 6) + 0(6 - 5) + 3(5 + 9)]$$

$$\Delta = \frac{1}{2}(0 + 0 + 3 \times 14)$$

$$\Delta = 42/2 = 21 \neq 0$$

From the above equation, it is clear that the points are not collinear.

4. Point P (0, 2) is the point of intersection of y-axis and perpendicular bisector of line segment joining the points A (-1, 1) and B (3, 3).

Solution:

False

Justification:

We know that, the points lying on perpendicular bisector of the line segment joining the two points is equidistant from the two points.
i.e., PA should be equals to the PB.

Using distance formula,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PA = \sqrt{[-1 - 0]^2 + [1 - 2]^2}$$

$$PA = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$$

$$PB = \sqrt{[3 - 0]^2 + [3 - 2]^2}$$

$$PB = \sqrt{9 + 1} = \sqrt{10}$$

$$\therefore PA \neq PB$$

5. Points A (3, 1), B (12, -2) and C (0, 2) cannot be the vertices of a triangle.

Solution:

True.

Justification:

Coordinates of A = $(x_1, y_1) = (3, 1)$

Coordinates of B = $(x_2, y_2) = (12, -2)$

Coordinates of C = $(x_3, y_3) = (0, 2)$

Area of $\Delta ABC = \Delta = \frac{1}{2} [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$

$$\Delta = \frac{1}{2} [3 - (2 - 2) + 12(2 - 1) + 0\{1 - (-2)\}]$$

$$\Delta = \frac{1}{2} [3(-4) + 12(1) + 0]$$

$$\Delta = \frac{1}{2} (-12 + 12) = 0$$

Area of $\Delta ABC = 0$

Since, the points A (3, 1), B (12, -2) and C (0, 2) are collinear.

Therefore, the points A (3, 1), B (12, -2) and C (0, 2) can't be the vertices of a triangle.

6. Points A (4, 3), B (6, 4), C (5, -6) and D (-3, 5) are the vertices of a parallelogram.

Solution:

False

Justification:

The given points are A (4, 3), B (6, 4), C (5, -6) and D (-3, 5)

Finding the distance between A and B

$$AB = \sqrt{(6 - 4)^2 + (4 - 3)^2} = \sqrt{2^2 + 1^2} = \sqrt{5}$$

Finding the distance between B and C

$$BC = \sqrt{(5 - 6)^2 + (-6 - 4)^2}$$

$$BC = \sqrt{(-1)^2 + (-10)^2}$$

$$BC = \sqrt{1 + 100} = \sqrt{101}$$

Finding the distance between C and D

$$CD = \sqrt{(-3 - 5)^2 + (5 + 6)^2}$$

$$CD = \sqrt{(-8)^2 + (11)^2}$$

$$CD = \sqrt{64 + 121}$$

$$CD = \sqrt{185}$$

Finding the distance between D and A

$$DA = \sqrt{(4 + 3)^2 + (3 - 5)^2}$$

$$DA = \sqrt{7^2 + (-2)^2}$$

$$DA = \sqrt{49 + 4} = \sqrt{53}$$

Since the distances are different, we can conclude that the points are not the vertices of a parallelogram.

EXERCISE 7.3

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1. Name the type of triangle formed by the points A (-5, 6), B (-4, -2) and C (7, 5).

Solution:

The points are A (-5, 6), B (-4, -2) and C (7, 5)

Using distance formula,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{((-4+5)^2 + (-2-6)^2)}$$

$$= \sqrt{1+64}$$

$$= \sqrt{65}$$

$$BC = \sqrt{((7+4)^2 + (5+2)^2)}$$

$$= \sqrt{121 + 49}$$

$$= \sqrt{170}$$

$$AC = \sqrt{((7+5)^2 + (5-6)^2)}$$

$$= \sqrt{144 + 1}$$

$$= \sqrt{145}$$

Since all sides are of different length, ABC is a scalene triangle.

2. Find the points on the x-axis which are at a distance of $2\sqrt{5}$ from the point (7, -4). How many such points are there?

Solution:

Let coordinates of the point = (x, 0) (given that the point lies on x axis)

$$x_1 = 7, y_1 = -4$$

$$x_2 = x, y_2 = 0$$

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

According to the question,

$$2\sqrt{5} = \sqrt{(x-7)^2 + (0-(-4))^2}$$

Squaring L.H.S and R.H.S

$$20 = x^2 + 49 - 14x + 16$$

$$20 = x^2 + 65 - 14x$$

$$0 = x^2 - 14x + 45$$

$$0 = x^2 - 9x - 5x + 45$$

$$0 = x(x-9) - 5(x-9)$$

$$0 = (x-9)(x-5)$$

$$x-9 = 0, x-5 = 0$$

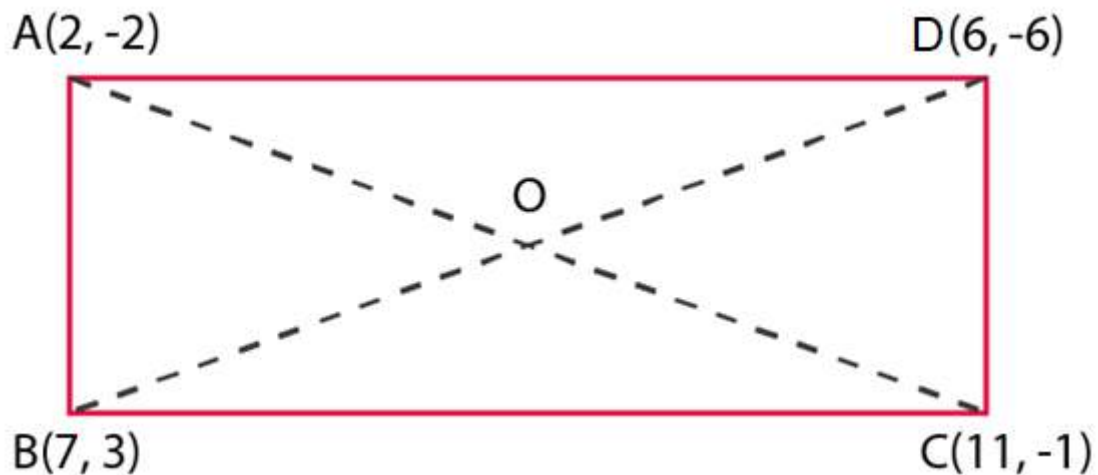
$$x = 9 \text{ or } x = 5$$

Therefore, coordinates of points.....(9,0) or (5,0)

3. What type of a quadrilateral do the points A (2, -2), B (7, 3), C (11, -1) and D (6, -6) taken in that order, form?

Solution:

The points are A (2, -2), B (7, 3), C (11, -1) and D (6, -6)



Using distance formula,
 $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
 $AB = \sqrt{(7 - 2)^2 + (3 + 2)^2}$

$$= \sqrt{(5)^2 + (5)^2}$$

$$= \sqrt{25 + 25}$$

$$= \sqrt{50}$$

$$= 5\sqrt{2}$$

$$BC = \sqrt{(11 - 7)^2 + (-1 - 3)^2}$$

$$= \sqrt{(4)^2 + (-4)^2}$$

$$= \sqrt{16 + 16}$$

$$= \sqrt{32}$$

$$= 4\sqrt{2}$$

$$CD = \sqrt{(6 - 11)^2 + (-6 + 1)^2}$$

$$= \sqrt{(-5)^2 + (-5)^2}$$

$$= \sqrt{25 + 25}$$

$$= \sqrt{50}$$

$$= 5\sqrt{2}$$

$$DA = \sqrt{(2 - 6)^2 + (-2 + 6)^2}$$

$$= \sqrt{(-4)^2 + (4)^2}$$

$$= \sqrt{16 + 16}$$

$$= \sqrt{32}$$

$$= 4\sqrt{2}$$

Finding diagonals AC and BD, we get,

$$AC = \sqrt{(11 - 2)^2 + (-1 + 2)^2}$$

$$= \sqrt{(9)^2 + (1)^2}$$

$$= \sqrt{81 + 1}$$

$$= \sqrt{82}$$

$$\text{And } BD = \sqrt{(6 - 7)^2 + (-6 - 3)^2}$$

$$= \sqrt{(-1)^2 + (-9)^2}$$

$$= \sqrt{1 + 81}$$

$$= \sqrt{82}$$

The Quadrilateral formed is rectangle.

4. Find the value of a , if the distance between the points A $(-3, -14)$ and B $(a, -5)$ is 9 units.

Solution:

Distance between two points (x_1, y_1) (x_2, y_2) is :

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance between A $(-3, -14)$ and B $(a, -5)$ is :

$$= \sqrt{[(a+3)^2 + (-5+14)^2]} = 9$$

Squaring on L.H.S and R.H.S.

$$(a+3)^2 + 81 = 81$$

$$(a+3)^2 = 0$$

$$(a+3)(a+3) = 0$$

$$a+3 = 0$$

$$a = -3$$

5. Find a point which is equidistant from the points A $(-5, 4)$ and B $(-1, 6)$? How many such points are there?

Solution:

Let the point be P

According to the question,

P is equidistant from A $(-5, 4)$ and B $(-1, 6)$

Then the point P $= ((x_1+x_2)/2, (y_1+y_2)/2)$

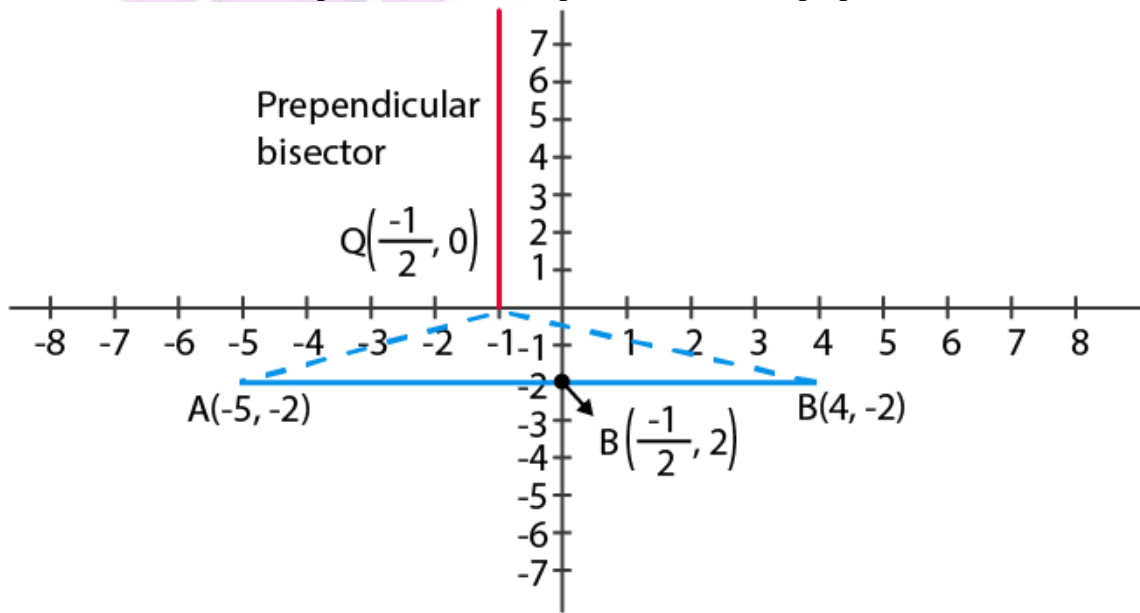
$$= ((-5-1)/2, (6+4)/2)$$

$$= (-3, 5)$$

6. Find the coordinates of the point Q on the x-axis which lies on the perpendicular bisector of the line segment joining the points A $(-5, -2)$ and B $(4, -2)$. Name the type of triangle formed by the points Q, A and B.

Solution:

Point Q is the midpoint of AB as the point P lies on the perpendicular bisector of AB.



By mid point formula:

$$\begin{aligned}(x_1 + x_2)/2 &= (-5+4)/2 \\ &= -1/2 \\ x &= -1/2\end{aligned}$$

Given that, P lies on x axis, so $y=0$

$$P(x,y) = (-1/2, 0)$$

Therefore, it is an isosceles triangle

7. Find the value of m if the points (5, 1), (-2, -3) and (8, 2m) are collinear.

Solution:

The points A(5, 1), B(-2, -3) and C(8, 2m) are collinear.

i.e., Area of $\Delta ABC = 0$

$$\frac{1}{2} [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)] = 0$$

$$\frac{1}{2} [5(-3 - 2m) + (-2)(2m - 1) + 8(1 - (-3))] = 0$$

$$\frac{1}{2} (-15 - 10m - 4m + 2 + 32) = 0$$

$$\frac{1}{2} (-14m + 19) = 0$$

$$m = 19/14$$

8. If the point A (2, -4) is equidistant from P (3, 8) and Q (-10, y), find the values of y. Also find distance PQ.

Solution:

Given points are A(2, -4), P(3, 8) and Q(-10, y)

According to the question,

$$\begin{aligned}PA &= QA \\ \sqrt{(2-3)^2 + (-4-8)^2} &= \sqrt{(2+10)^2 + (-4-y)^2} \\ \sqrt{(-1)^2 + (-12)^2} &= \sqrt{(12)^2 + (4+y)^2} \\ \sqrt{1+144} &= \sqrt{144+16+y^2+8y} \\ \sqrt{145} &= \sqrt{160+y^2+8y}\end{aligned}$$

On squaring both sides, we get

$$145 = 160 + y^2 + 8y$$

$$y^2 + 8y + 160 - 145 = 0$$

$$y^2 + 8y + 15 = 0$$

$$y^2 + 5y + 3y + 15 = 0$$

$$y(y+5) + 3(y+5) = 0$$

$$\Rightarrow (y+5)(y+3) = 0$$

$$\Rightarrow y+5 = 0 \quad \Rightarrow y = -5$$

$$\text{and } y+3 = 0 \quad \Rightarrow y = -3$$

$$\begin{aligned} \therefore y &= -3, -5 \\ \text{Now, } PQ &= \sqrt{(-10-3)^2 + (y-8)^2} \\ \text{For } y &= -3 \quad PQ = \sqrt{(-13)^2 + (-3-8)^2} = \sqrt{169+121} = \sqrt{290} \text{ units} \\ \text{and for } y &= -5 \quad PQ = \sqrt{(-13)^2 + (-5-8)^2} = \sqrt{169+169} = \sqrt{338} \text{ units} \\ \text{Hence, values of } y &\text{ are } -3 \text{ and } -5, PQ = \sqrt{290} \text{ and } \sqrt{338} \end{aligned}$$

9. Find the area of the triangle whose vertices are $(-8, 4)$, $(-6, 6)$ and $(-3, 9)$.

Solution:

Given vertices are:

$$(x_1, y_1) = (-8, 4)$$

$$(x_2, y_2) = (-6, 6)$$

$$(x_3, y_3) = (-3, 9)$$

$$\begin{aligned} \text{Area of triangle} &= \frac{1}{2} (x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)) \\ &= \frac{1}{2} (-8(6 - 9) + -6(9 - 4) + -3(4 - 6)) \\ &= \frac{1}{2} (-8(-3) + -6(5) + -3(-2)) \\ &= \frac{1}{2} (24 - 30 + 6) \\ &= \frac{1}{2} (30 - 30) \\ &= 0 \text{ units.} \end{aligned}$$

10. In what ratio does the x-axis divide the line segment joining the points $(-4, -6)$ and $(-1, 7)$? Find the coordinates of the point of division.

Solution:

Let the ratio in which x-axis divides the line segment joining $(-4, -6)$ and $(-1, 7) = 1: k$.

Then,

$$\text{x-coordinate becomes } (-1 - 4k) / (k + 1)$$

$$\text{y-coordinate becomes } (7 - 6k) / (k + 1)$$

Since P lies on x-axis, y coordinate = 0

$$(7 - 6k) / (k + 1) = 0$$

$$7 - 6k = 0$$

$$k = 6/7$$

$$\text{Now, } m_1 = 6 \text{ and } m_2 = 7$$

By using section formula,

$$x = (m_1x_2 + m_2x_1) / (m_1 + m_2)$$

$$= (6(-1) + 7(-4)) / (6 + 7)$$

$$= (-6 - 28) / 13$$

$$= -34/13$$

So, now

$$y = (6(7) + 7(-6)) / (6 + 7)$$

$$= (42 - 42) / 13$$

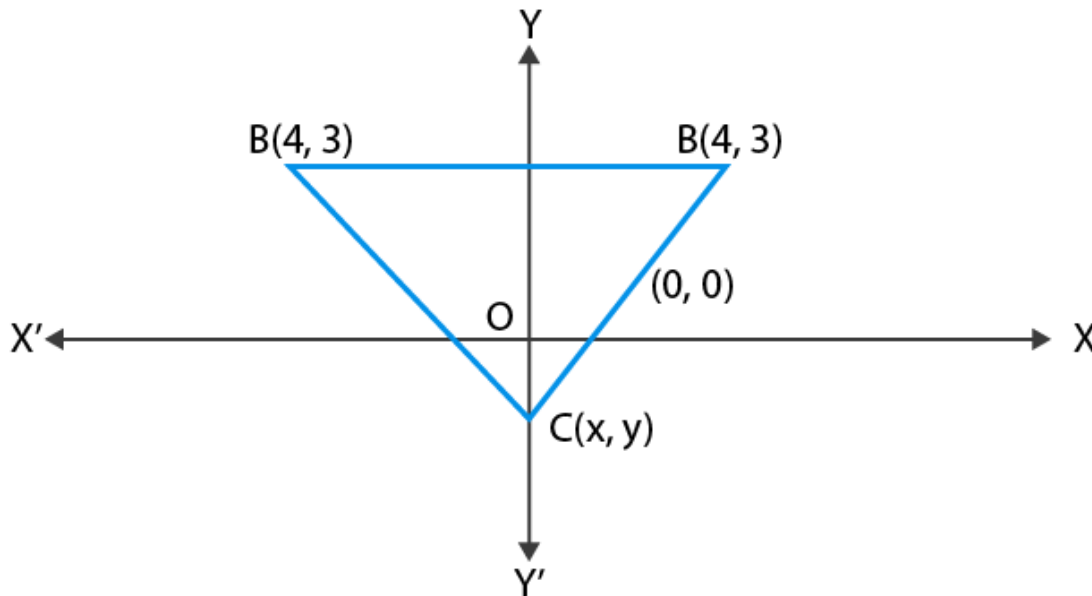
$$= 0$$

Hence, the coordinates of P are $(-34/13, 0)$

EXERCISE 7.4

1. If $(-4, 3)$ and $(4, 3)$ are two vertices of an equilateral triangle, find the coordinates of the third vertex, given that the origin lies in the interior of the triangle.

Solution:



Let the vertices be (x, y)

Distance between (x, y) & $(4, 3)$ is $= \sqrt{(x-4)^2 + (y-3)^2}$(1)

Distance between (x, y) & $(-4, 3)$ is $= \sqrt{(x+4)^2 + (y-3)^2}$(2)

Distance between $(4, 3)$ & $(-4, 3)$ is $= \sqrt{(4+4)^2 + (3-3)^2} = \sqrt{8^2} = 8$

According to the question,

Equation (1)=(2)

$$(x-4)^2 = (x+4)^2$$

$$x^2 - 8x + 16 = x^2 + 8x + 16$$

$$16x = 0$$

$$x = 0$$

Also, equation (1)=8

$$(x-4)^2 + (y-3)^2 = 64$$
..... (3)

Substituting the value of x in (3)

$$\text{Then } (0-4)^2 + (y-3)^2 = 64$$

$$(y-3)^2 = 64 - 16$$

$$(y-3)^2 = 48$$

$$y-3 = (\pm) 4\sqrt{3}$$

$$y = 3(\pm) 4\sqrt{3}$$

Neglect $y = 3 + 4\sqrt{3}$ as if $y = 3 + 4\sqrt{3}$ then origin cannot interior of triangle

Therefore, the third vertex = $(0, 3 - 4\sqrt{3})$

2. A $(6, 1)$, B $(8, 2)$ and C $(9, 4)$ are three vertices of a parallelogram ABCD. If E is the midpoint of DC, find the area of $\triangle ADE$.

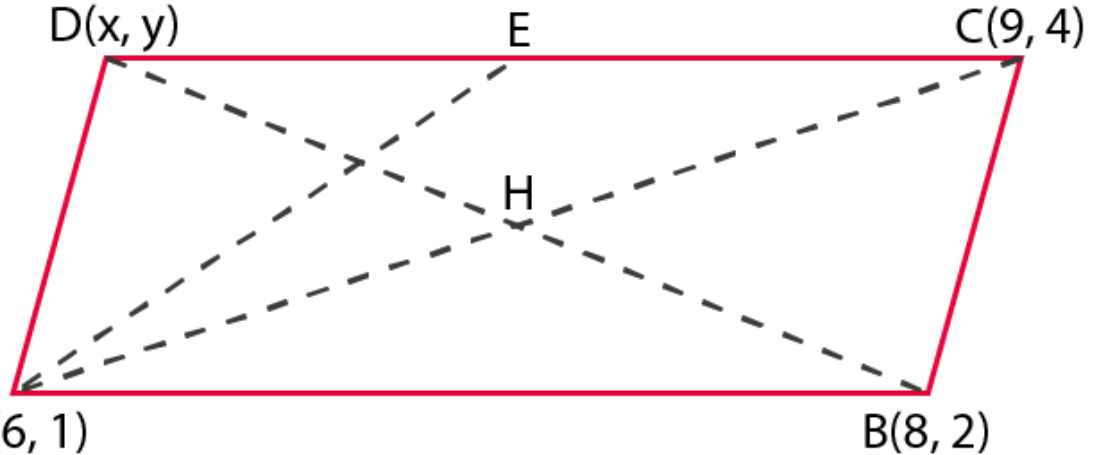
Solution:

According to the question,

The three vertices of a parallelogram ABCD are A (6, 1), B (8, 2) and C (9, 4)

Let the fourth vertex of parallelogram = (x, y),

We know that, diagonals of a parallelogram bisect each other



A(6, 1)

B(8, 2)

Since, mid - point of a line segment joining the points (x_1, y_1) and (x_2, y_2) is given by,

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Mid - point of BD = Mid - point of AC

$$\left(\frac{8+x}{2}, \frac{2+y}{2} \right) = \left(\frac{6+9}{2}, \frac{1+4}{2} \right)$$

$$\left(\frac{8+x}{2}, \frac{2+y}{2} \right) = \left(\frac{15}{2}, \frac{5}{2} \right)$$

So, we have,

$$\frac{8+x}{2} = \frac{15}{2}$$

$$8+x = 15$$

$$x = 7$$

And,

$$\frac{2+y}{2} = \frac{5}{2}$$

$$2+y = 5 \rightarrow y = 3$$

So, fourth vertex of a parallelogram is D (7, 3)

Now,

Mid - point of side

$$DC = \left(\frac{7+9}{2}, \frac{3+4}{2} \right)$$

$$E = \left(8, \frac{7}{2} \right)$$

∴ Area of ΔABC with vertices (x_1, y_1) , (x_2, y_2) and (x_3, y_3) ;

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

∴ Area of ΔADE with vertices A (6, 1), D (7, 3) and E (8, (7/2))

$$\begin{aligned}\Delta &= \frac{1}{2} \left[6 \left(3 - \frac{7}{2} \right) + 7 \left(\frac{7}{2} - 1 \right) + 8(1 - 3) \right] \\ &= \frac{1}{2} \left[6 \times \left(\frac{-1}{2} \right) + 7 \left(\frac{5}{2} \right) + 8(-2) \right] \\ &= \frac{1}{2} \left(\frac{35}{2} - 19 \right) \\ &= \frac{1}{2} \left(\frac{-3}{2} \right)\end{aligned}$$

= - 3/4 but area can't be negative

Hence, the required area of ΔADE is $3/4$ sq. units

3. The points A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are the vertices of ABC.

(i) The median from A meets BC at D. Find the coordinates of the point D.

(ii) Find the coordinates of the point P on AD such that AP : PD = 2 : 1

(iii) Find the coordinates of points Q and R on medians BE and CF, respectively such that BQ : QE = 2 : 1 and CR : RF = 2 : 1

(iv) What are the coordinates of the centroid of the triangle ABC?

Solution:

According to the question,

The vertices of $\Delta ABC = A, B$ and C

Coordinates of A, B and C = A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)

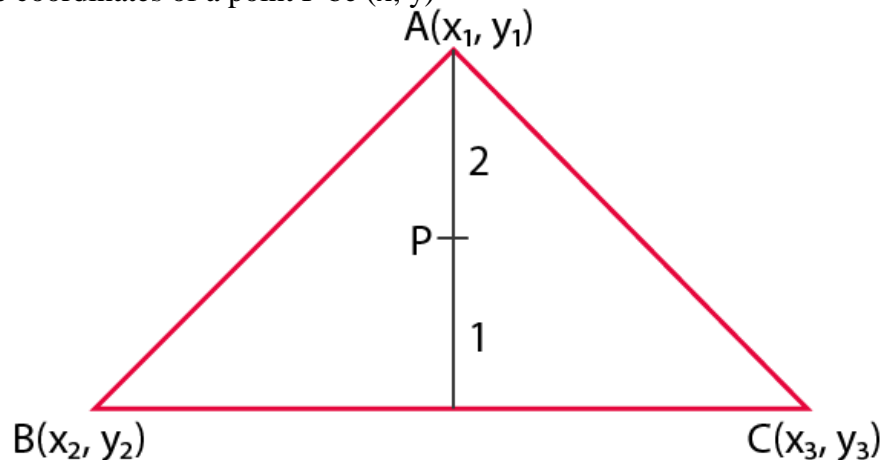
(i) As per given information D is the mid - point of BC and it bisect the line into two equal parts.

Coordinates of the mid - point of BC;

$$BC = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2} \right)$$

$$\Rightarrow D = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2} \right)$$

(ii) Let the coordinates of a point P be (x, y)



Given,

The ratio in which the point P(x, y), divide the line joining,

A(x_1, y_1) and D($(x_2+x_3)/2, (y_2+y_3)/2$) = 2:1

Then,

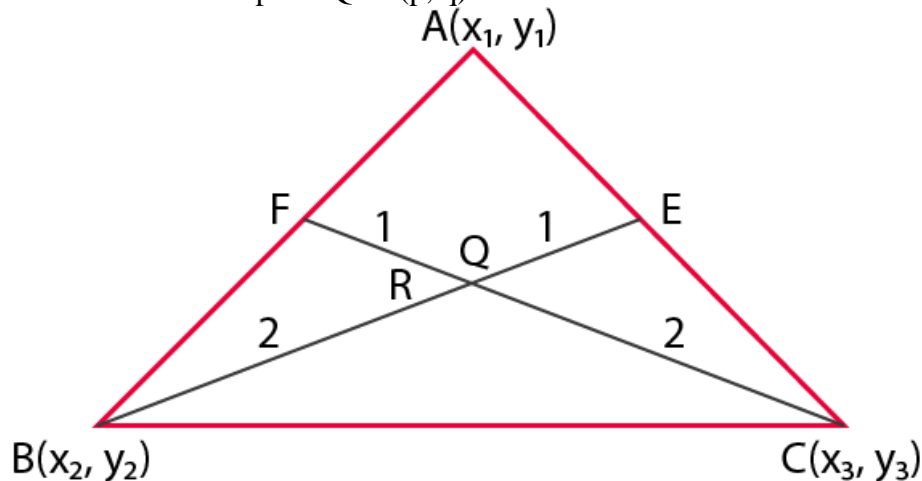
Coordinates of P =

$$\left[\frac{2 \times \left(\frac{x_2 + x_3}{2} \right) + 1 \times x_1}{2 + 1}, \frac{2 \times \left(\frac{y_2 + y_3}{2} \right) + 1 \times y_1}{2 + 1} \right]$$

By using internal section formula;

$$\begin{aligned} &= \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \right) \\ &= \left(\frac{x_2 + x_3 + x_1}{3}, \frac{y_2 + y_3 + y_1}{3} \right) \end{aligned}$$

(iii) Let the coordinates of a point Q be (p, q)



Given,

The point Q (p, q),

Divide the line joining B(x₂, y₂) and E $\left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2} \right)$ in the ratio 2:1,

Then,

Coordinates of Q =

$$\begin{aligned} &\left[\frac{2 \times \left(\frac{x_1 + x_3}{2} \right) + 1 \times x_2}{2 + 1}, \frac{2 \times \left(\frac{y_1 + y_3}{2} \right) + 1 \times y_2}{2 + 1} \right] \\ &= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right) \end{aligned}$$

Since, BE is the median of side CA, So BE divides AC in to two equal parts.

∴ mid - point of AC = Coordinate of E;

$$E = \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2} \right)$$

So, the required coordinate of point Q;

$$Q = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

Now,

Let the coordinates of a point E be (α, β)

Given,

Point R (α , β) divide the line joining C(x_3 , y_3) and F $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ in the ratio 2:1,

Then the coordinates of R;

$$= \left[\frac{2 \times \left(\frac{x_1 + x_2}{2}\right) + 1 \times x_3}{2 + 1}, \frac{2 \times \left(\frac{y_1 + y_2}{2}\right) + 1 \times y_3}{2 + 1} \right]$$

$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

Since, CF is the median of side AB.

So, CF divides AB in to two equal parts.

\therefore mid - point of AB = Coordinate of F;

$$F = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

So, the required coordinate of point R;

$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

(iv) Coordinate of the centroid of the ΔABC ;

$$= \left(\frac{\text{Sum of all coordinates of all vertices}}{3}, \frac{\text{Sum of all coordinates of all vertices}}{3} \right)$$

$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$