EXERCISE 9.1 PAGE NO: 102

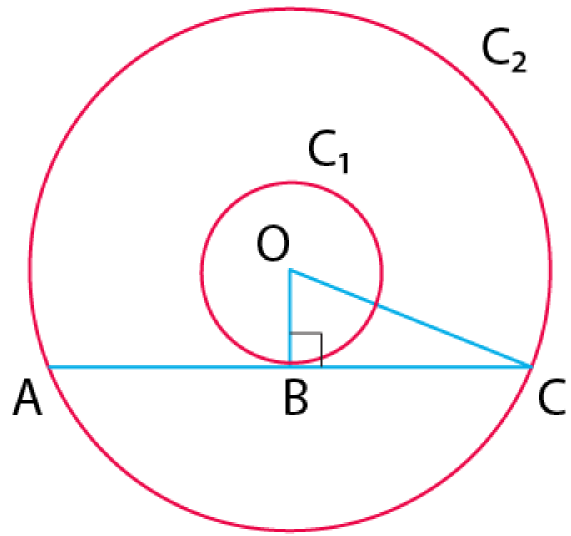
Choose the correct answer from the given four options in the following questions:

1. If radii of two concentric circles are 4 cm and 5 cm, then the length of each chord of one circle which is tangent to the other circle is

(A) 3 cm (B) 6 cm

(C) 9 cm (D) 1 cm

Solution:



According to the question,

OA = 4cm, OB = 5cm

And, OA ⊥ BC

Therefore, $OB^2 = OA^2 + AB^2$

⇒ $5^2 = 4^2 + AB^2$

⇒ $AB = \sqrt{(25 - 16)} = 3\text{cm}$

⇒ $BC = 2AB = 2 \times 3\text{cm} = 6\text{cm}$

2. In Fig. 9.3, if AOB = 125°, then COD is equal to

(A) 62.5° (B) 45°

(C) 35° (D) 55°

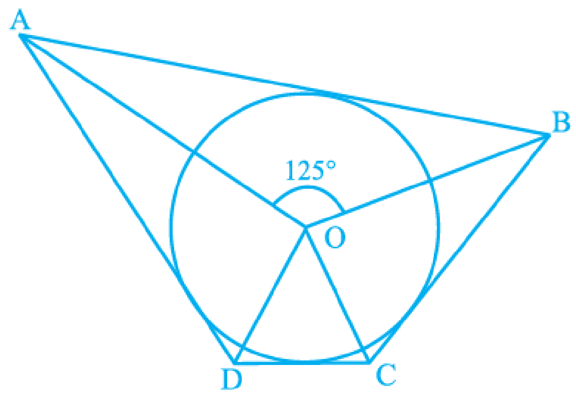


Fig. 9.3

Solution:

ABCD is a quadrilateral circumscribing the circle
We know that, the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the center of the circle.
So, we have
$\angle AOB + \angle COD = 180°$
$125° + \angle COD = 180°$
$\angle COD = 55°$

3. In Fig. 9.4, AB is a chord of the circle and AOC is its diameter such that ACB = 50°. If AT is the tangent to the circle at the point A, then BAT is equal to

(A) 65° **(B) 60°**
(C) 50° **(D) 40°**

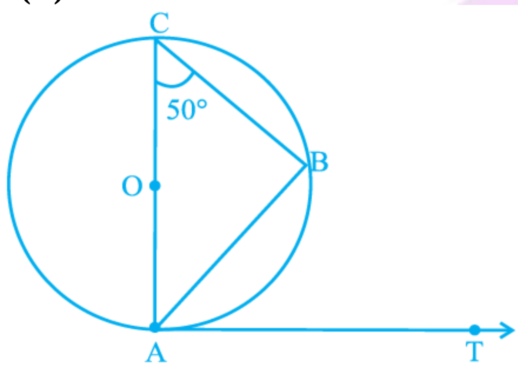


Fig. 9.4

Solution:

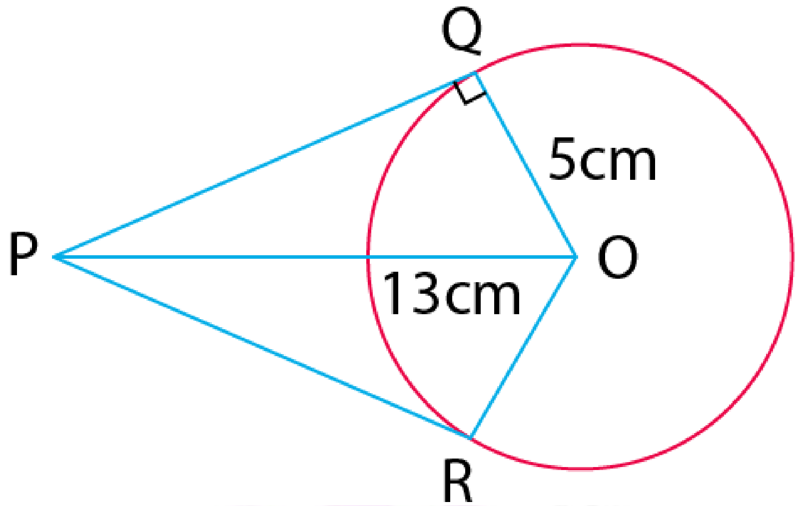
According to the question,
A circle with centre O, diameter AC and $\angle ACB = 50°$
AT is a tangent to the circle at point A
Since, angle in a semicircle is a right angle
$\angle CBA = 90°$
By angle sum property of a triangle,
$\angle ACB + \angle CAB + \angle CBA = 180°$
$50° + \angle CAB + 90° = 180°$
$\angle CAB = 40°$ … (1)
Since tangent to at any point on the circle is perpendicular to the radius through point of contact,
We get,
$OA \perp AT$
$\angle OAT = 90°$
$\angle OAT + \angle BAT = 90°$
$\angle CAT + \angle BAT = 90°$
$40° + \angle BAT = 90°$ [from equation (1)]
$\angle BAT = 50°$

4. From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral PQOR is

(A) 60 cm^2 **(B) 65 cm^2**
(C) 30 cm^2 **(D) 32.5 cm^2**

Solution:

Construction: Draw a circle of radius 5 cm with center O.
Let P be a point at a distance of 13 cm from O.
Draw a pair of tangents, PQ and PR.
OQ = OR = radius = 5cm …equation (1)
And OP = 13 cm

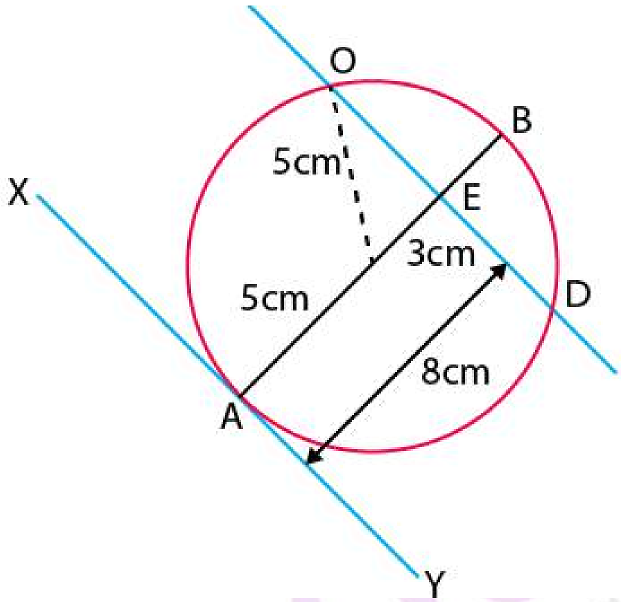


We know that, tangent to at any point on the circle is perpendicular to the radius through point of contact,
Hence, we get,
OQ ⊥ PQ and OR ⊥PR
△POQ and △POR are right-angled triangles.
Using Pythagoras Theorem in △PQO,
(Base)^2+ (Perpendicular)^2= (Hypotenuse)^2
$(PQ)^2 + (OQ)^2 = (OP)^2$
$(PQ)^2 + (5)^2 = (13)^2$
$(PQ)^2 + 25 = 169$
$(PQ)^2 = 144$
PQ = 12 cm
Tangents through an external point to a circle are equal.
So,
PQ = PR = 12 cm … (2)
Therefore, Area of quadrilateral PQRS, A = area of △POQ + area of △POR
Area of right angled triangle = ½ x base x perpendicular
A = (½ x OQ x PQ) + (½ x OR x PR)
A = (½ x 5 x 12) + (½ x 5 x 12)
A = 30 + 30 = 60 cm^2

5. At one end A of a diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY and at a distance 8 cm from A is

(A) 4 cm **(B) 5 cm**
(C) 6 cm **(D) 8 cm**

Solution:

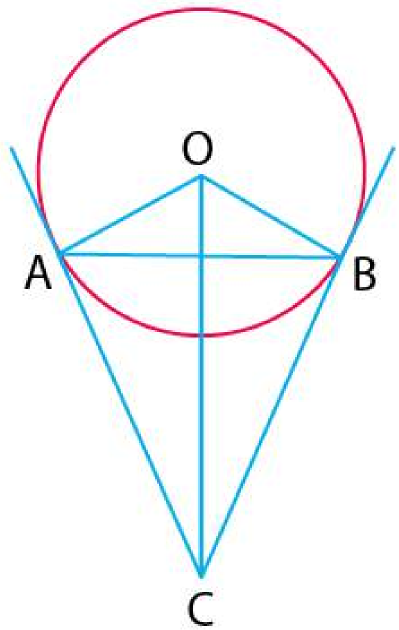


According to the question,
Radius of circle, AO=OC = 5cm
AM=8CM
AM=OM+AO
OM =AM-AO
Substituting these values in the equation,
OM= (8-5) =3CM
OM is perpendicular to the chord CD.
In ΔOCM $\angle OMC=90°$
By Pythagoras theorem,
$OC^2=OM^2+MC^2$
Therefore,
CD= 2 ×CM = 8 cm

EXERCISE 9.2 PAGE NO: 105

Write 'True' or 'False' and justify your answer in each of the following:
1. If a chord AB subtends an angle of 60° at the centre of a circle, then angle between the tangents at A and B is also 60°.
Solution:

False
Justification:
For example,
Consider the given figure. In which we have a circle with centre O and AB a chord with
$\angle AOB = 60°$



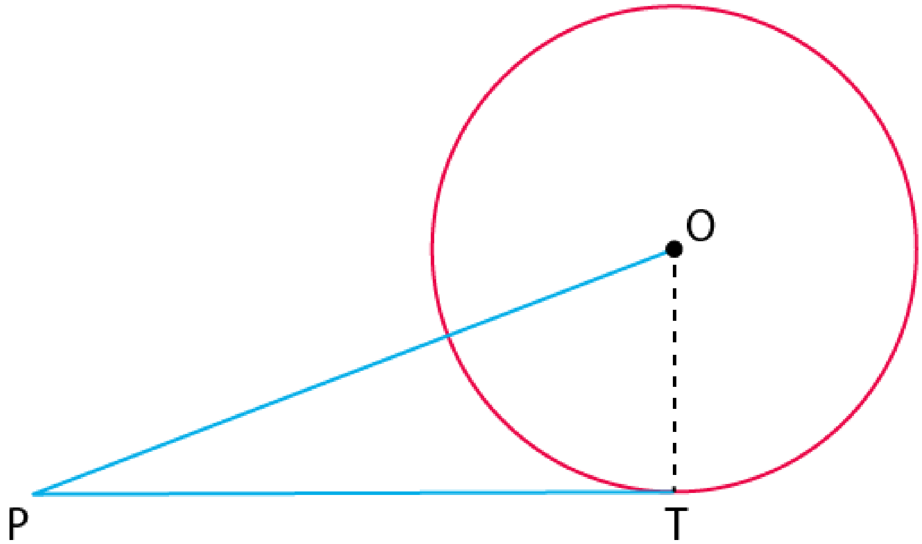
Since, tangent to any point on the circle is perpendicular to the radius through point of contact,
We get,
$OA \perp AC$ and $OB \perp CB$
$\angle OBC = \angle OAC = 90°$ … eq(1)
Using angle sum property of quadrilateral in Quadrilateral AOBC,
We get,
$\angle OBC + \angle OAC + \angle AOB + \angle ACB = 360°$
$90° + 90° + 60° + \angle ACB = 360°$
$\angle ACB = 120°$
Hence, the angle between two tangents is 120°.
Therefore, we can conclude that,
the given statement is false.

2. The length of tangent from an external point on a circle is always greater than the radius of the circle.
Solution:

False
Justification:
Length of tangent from an external point P on a circle may or may not be greater than the radius of the circle.

3. The length of tangent from an external point P on a circle with centre O is always less than OP.
Solution:

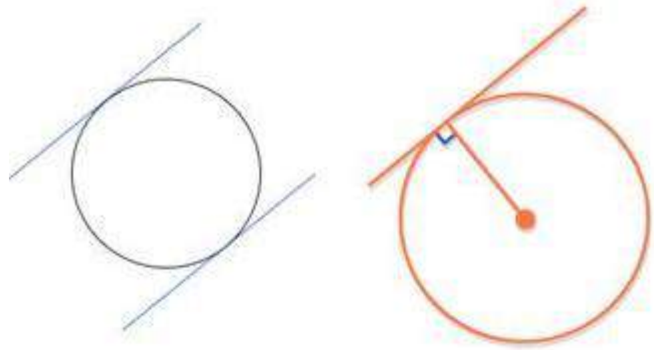
True
Justification:
Consider the figure of a circle with centre O.
Let PT be a tangent drawn from external point P.
Now, Joint OT.
OT ⊥ PT



We know that,
Tangent at any point on the circle is perpendicular to the radius through point of contact
Hence, OPT is a right-angled triangle formed.
We also know that,
In a right angled triangle, hypotenuse is always greater than any of the two sides of the triangle.
Hence,
OP > PT or PT < OP
Hence, length of tangent from an external point P on a circle with center O is always less than OP.

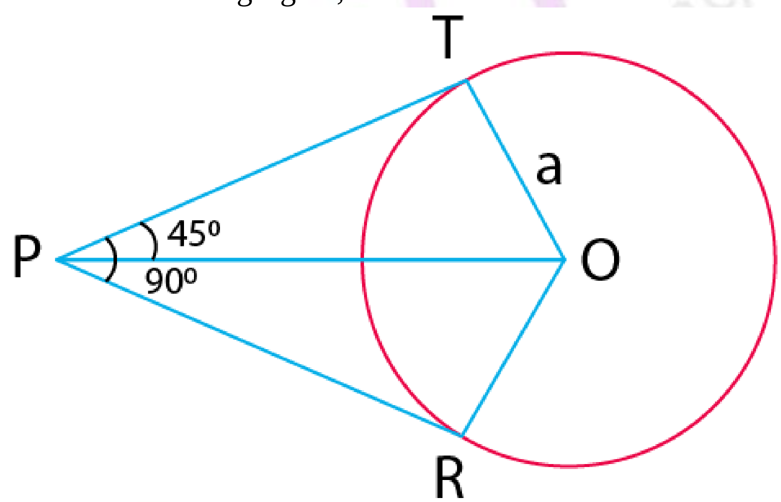
4. The angle between two tangents to a circle may be 0°.
Solution:

True
Justification:
The angle between two tangents to a circle may be 0°only when both tangent lines coincide or are parallel to each other.



5. If angle between two tangents drawn from a point P to a circle of radius a and centre O is 90°, then OP = a√2.
Solution:

Tangent is always perpendicular to the radius at the point of contact.
Hence, ∠RPT = 90
If 2 tangents are drawn from an external point, then they are equally inclined to the line segment joining the centre to that point.
Consider the following figure,

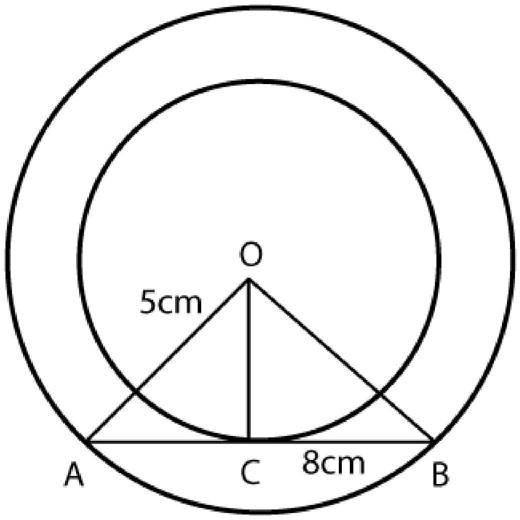


From point P, two tangents are drawn.
It is given that, OT = a
And line OP bisects ∠RPT.
So,
∠TPO = ∠RPO =45°
We know that, OT ⊥ PT
In right - angled triangle OTP,
Sin 45° = OT/OP
= 1/√2 = a/OP
Hence, OP = a√ 2

EXERCISE 9.3 PAGE NO: 107

1. Out of the two concentric circles, the radius of the outer circle is 5 cm and the chord AC of length 8 cm is a tangent to the inner circle. Find the radius of the inner circle.

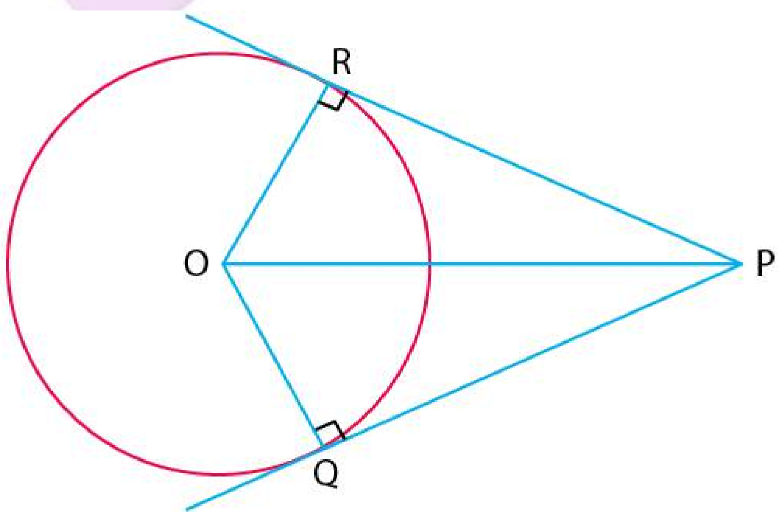
Solution:



From the figure,
Chord AB = 8 cm
OC is perpendicular to the chord AB
AC = CB = 4 cm
In right triangle OCA
$OC^2 + CA^2 = OA^2$
$OC^2 = 5^2 - 4^2 = 25 - 16 = 9$
OC = 3 cm

2. Two tangents PQ and PR are drawn from an external point to a circle with centre O. Prove that QORP is a cyclic quadrilateral.

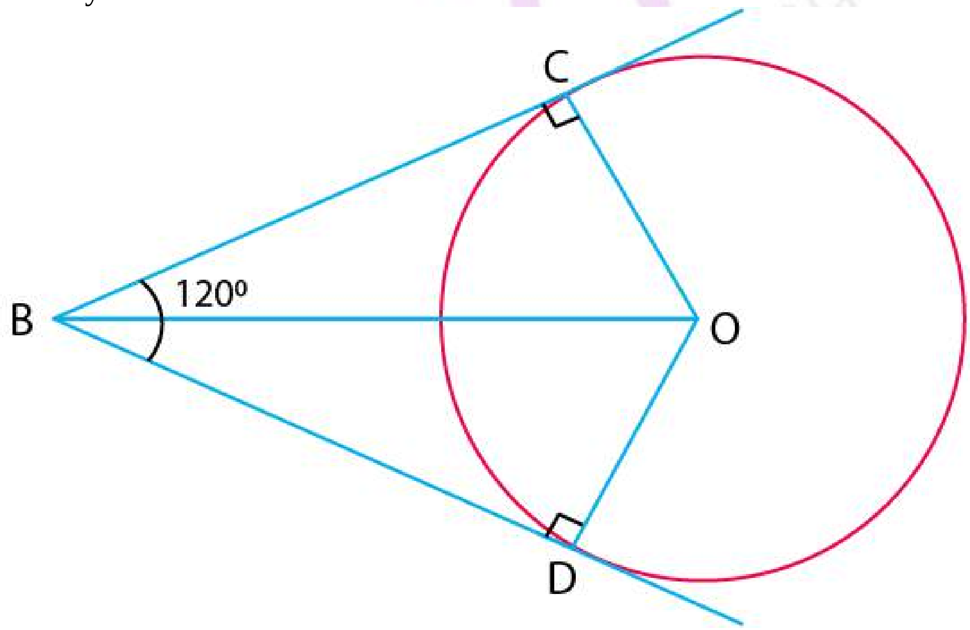
Solution:



We know that,
Radius ⊥ Tangent = OR ⊥ PR
i.e., ∠ORP = 90°
Likewise,
Radius ⊥ Tangent = OQ ⊥PQ
∠OQP = 90°
In quadrilateral ORPQ,
Sum of all interior angles = 360º
∠ORP + ∠RPQ+ ∠PQO + ∠QOR = 360º
90º + ∠RPQ + 90º + ∠QOR = 360º
Hence, ∠O + ∠P = 180º
PROQ is a cyclic quadrilateral.

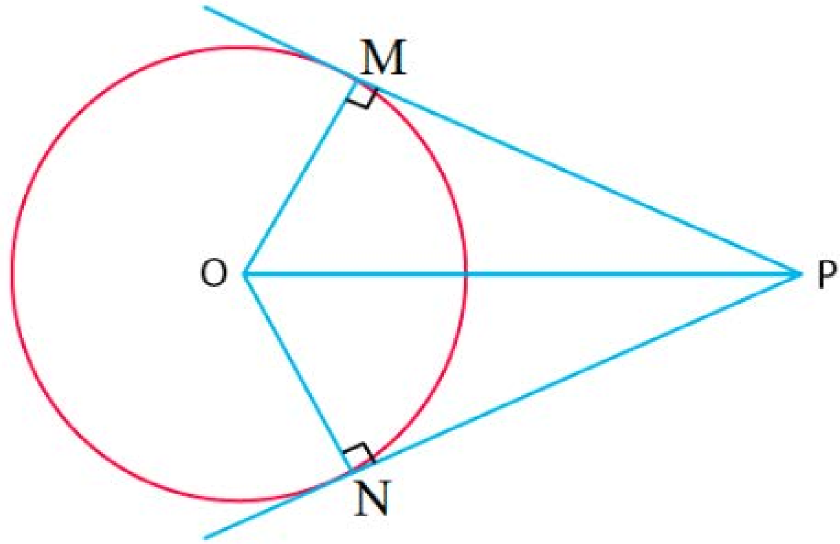
3. If from an external point B of a circle with centre O, two tangents BC and BD are drawn such that angle DBC = 120°, prove that BC + BD = BO, i.e., BO = 2BC.
Solution:

According to the question,
By RHS rule,
ΔOBC and ΔOBD are congruent
By CPCT



∠OBC and ∠ OBD are equal
Therefore,
∠OBC = ∠OBD =60°
In triangle OBC,
cos 60°=BC/OB
½ =BC/OB
OB=2BC
Hence proved

4. Prove that the centre of a circle touching two intersecting lines lies on the angle bisector of the lines.
Solution:



Let the lines be l_1 and l_2.
Assume that O touches l_1 and l_2 at M and N,
We get,
OM = ON (Radius of the circle)
Therefore,
From the centre "O" of the circle, it has equal distance from l_1 & l_2.
In Δ OPM & OPN,
OM = ON (Radius of the circle)
∠OMP = ∠ONP (As, Radius is perpendicular to its tangent)
OP = OP (Common sides)

Therefore,
Δ OPM = ΔOPN (SSS congruence rule)
By C.P.C.T,
∠MPO = ∠NPO
So, l bisects ∠MPN.
Therefore, O lies on the bisector of the angle between l_1 & l_2 .
Hence, we prove that the centre of a circle touching two intersecting lines lies on the angle bisector of the lines.

5. In Fig. 9.13, AB and CD are common tangents to two circles of unequal radii. Prove that AB = CD.

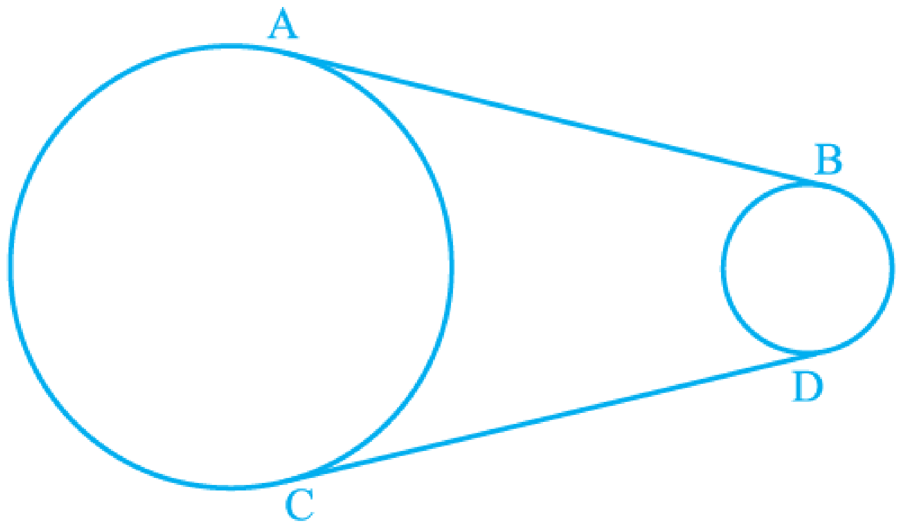
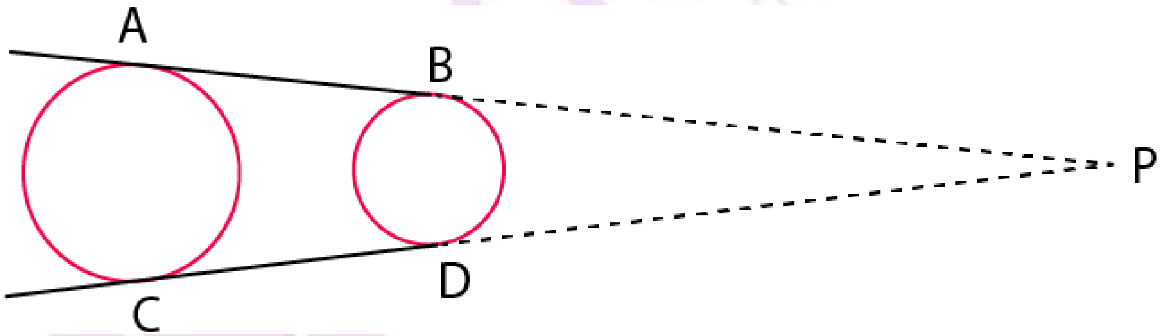


Fig. 9.13

Solution:

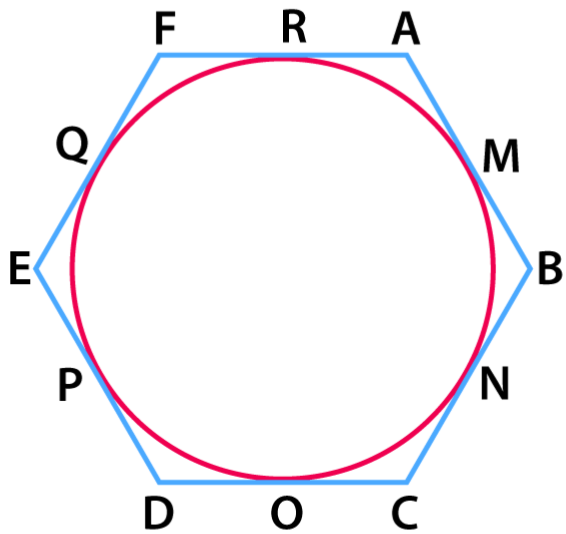
According to the question,
AB = CD



Construction: Produce AB and CD, to intersect at P.
Proof:
Consider the circle with greater radius.
Tangents drawn from an external point to a circle are equal
AP = CP …(1)
Also,
Consider the circle with smaller radius.
Tangents drawn from an external point to a circle are equal
BP = BD …(2)
Subtract Equation (2) from (1). We Get
AP - BP = CP – BD
AB = CD
Hence Proved.

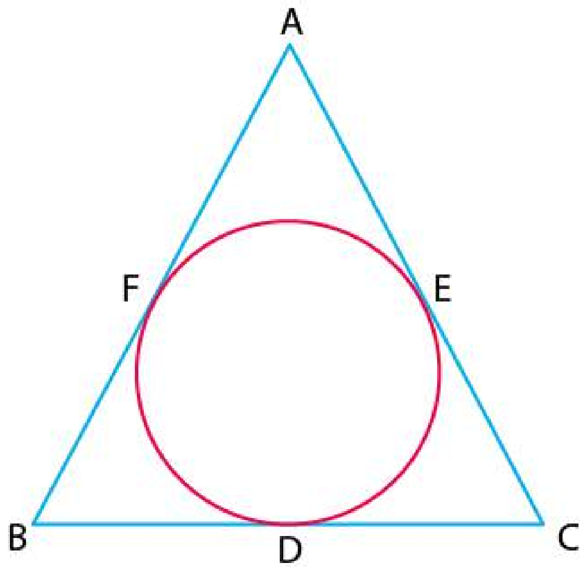
EXERCISE 9.4

1. If a hexagon ABCDEF circumscribe a circle, prove that AB + CD + EF = BC + DE + FA.
Solution:



According to the question,
A Hexagon ABCDEF circumscribe a circle.
To prove:
AB + CD + EF = BC + DE + FA
Proof:
Tangents drawn from an external point to a circle are equal.
Hence, we have
AM = RA … eq 1 [tangents from point A]
BM = BN … eq 2 [tangents from point B]
CO = NC … eq 3 [tangents from point C]
OD = DP … eq 4 [tangents from point D]
EQ = PE … eq 5 [tangents from point E]
QF = FR … eq 6 [tangents from point F]
[eq 1]+[eq 2]+[eq 3]+[eq 4]+[eq 5]+[eq 6]
AM + BM + CO + OD + EQ + QF = RA + BN + NC + DP + PE + FR
On rearranging, we get,
(AM + BM) + (CO + OD) + (EQ + QF) = (BN + NC) + (DP + PE) + (FR + RA)
AB + CD + EF = BC + DE + FA
Hence Proved!

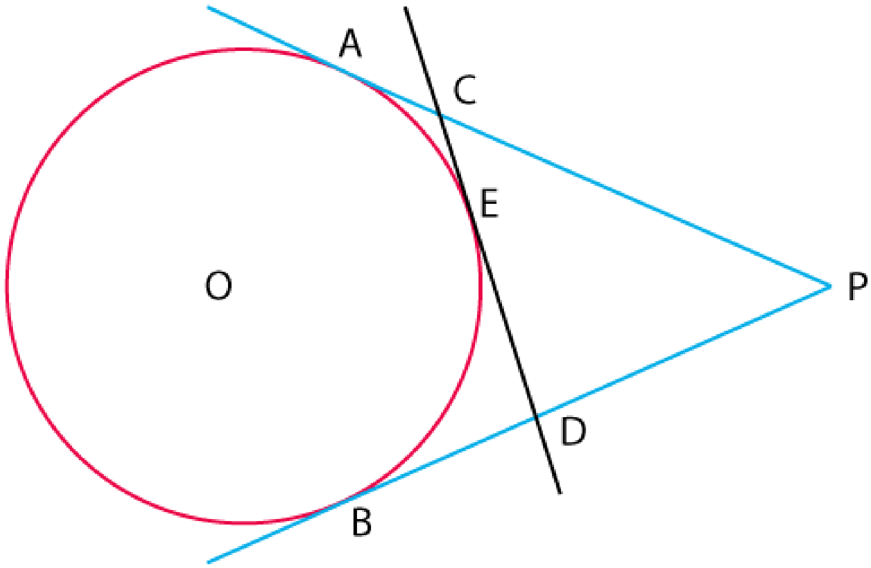
2. Let s denote the semi-perimeter of a triangle ABC in which BC = a, CA = b, AB = c. If a circle touches the sides BC, CA, AB at D, E, F, respectively, prove that BD = $s - b$.
Solution:



According to the question,
A triangle ABC with BC = a , CA = b and AB = c . Also, a circle is inscribed which touches the sides BC, CA and AB at D, E and F respectively and s is semi- perimeter of the triangle
To Prove: BD = s – b
Proof:
According to the question,
We have,
Semi Perimeter = s
Perimeter = 2s
2s = AB + BC + AC [1]
As we know,
Tangents drawn from an external point to a circle are equal
So we have
AF = AE [2] [Tangents from point A]
BF = BD [3] [Tangents From point B]
CD = CE [4] [Tangents From point C]
Adding [2] [3] and [4]
AF + BF + CD = AE + BD + CE
AB + CD = AC + BD
Adding BD both side
AB + CD + BD = AC + BD + BD
AB + BC - AC = 2BD
AB + BC + AC - AC - AC = 2BD
2s - 2AC = 2BD [From 1]
2BD = 2s - 2b [as AC = b]
BD = s – b
Hence Proved.

3. From an external point P, two tangents, PA and PB are drawn to a circle with centre O. At one point E on the circle tangent is drawn which intersects PA and PB at C and D, respectively. If PA = 10 cm, find the perimeter of the triangle PCD.

Solution:



According to the question,
From an external point P, two tangents, PA and PB are drawn to a circle with center O.
At a point E on the circle tangent is drawn which intersects PA and PB at C and D, respectively. And PA = 10 cm
To Find : Perimeter of △PCD
As we know that, Tangents drawn from an external point to a circle are equal.
So we have
AC = CE [1] [Tangents from point C]
ED = DB [2] [Tangents from point D]
Now Perimeter of Triangle PCD
= PC + CD + DP
= PC + CE + ED + DP
= PC + AC + DB + DP [From 1 and 2]
= PA + PB
Now,
PA = PB = 10 cm as tangents drawn from an external point to a circle are equal
So we have
Perimeter = PA + PB = 10 + 10 = 20 cm

4. If AB is a chord of a circle with centre O, AOC is a diameter and AT is the tangent at A as shown in Fig. 9.17. Prove that ∠BAT = ∠ACB

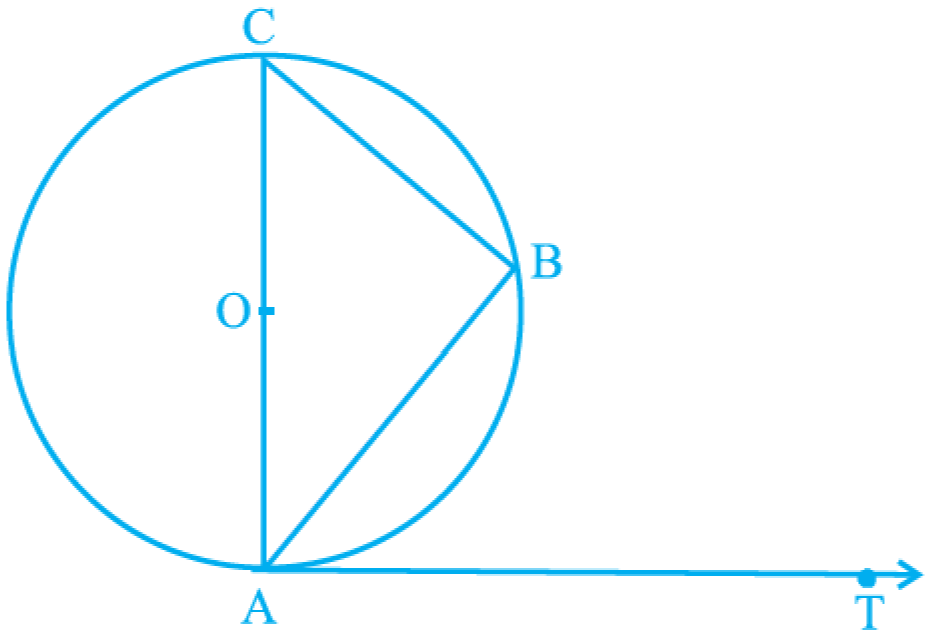
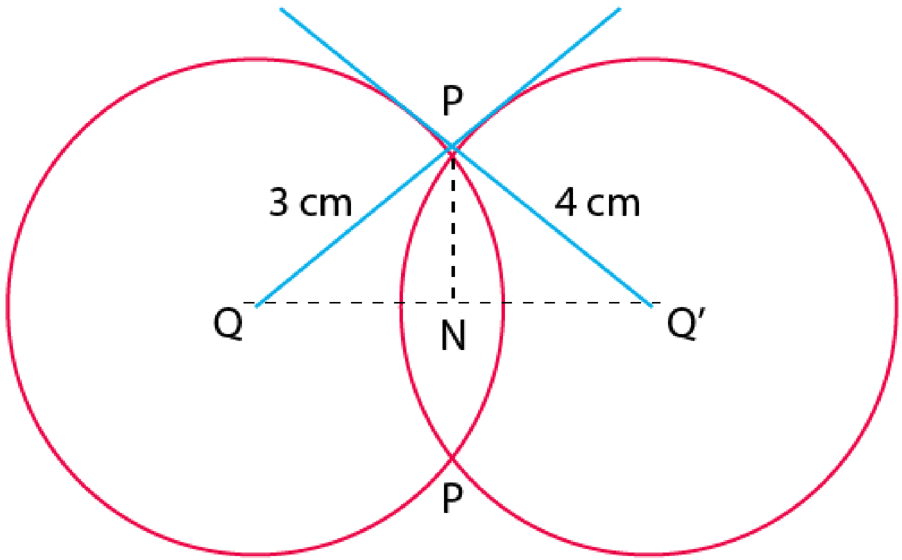


Fig. 9.17

Solution:

According to the question,
A circle with center O and AC as a diameter and AB and BC as two chords also AT is a tangent at point A
To Prove : ∠BAT = ∠ACB
Proof :
∠ABC = 90° [Angle in a semicircle is a right angle]
In △ABC By angle sum property of triangle
∠ABC + ∠ BAC + ∠ACB = 180 °
∠ACB + 90° = 180° - ∠BAC
∠ACB = 90 - ∠BAC [1]
Now,
OA ⊥ AT [Tangent at a point on the circle is perpendicular to the radius through point of contact]
∠OAT = ∠CAT = 90°
∠BAC + ∠BAT = 90°
∠BAT = 90° - ∠BAC [2]
From [1] and [2]
∠BAT = ∠ACB [Proved]

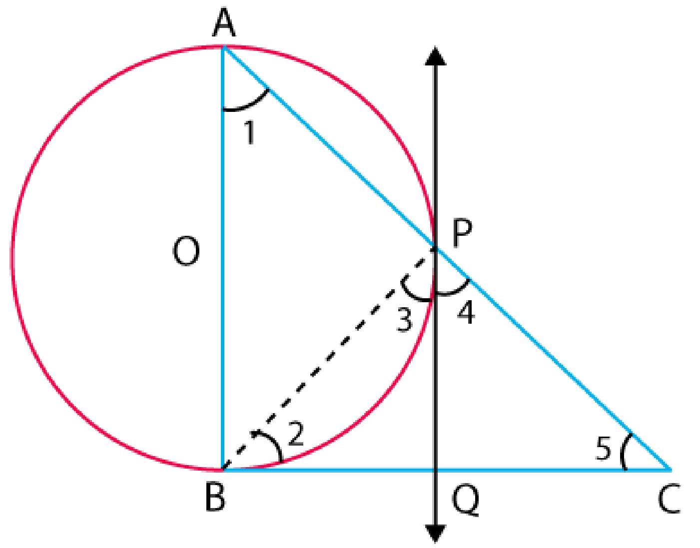
5. Two circles with centres O and O′ of radii 3 cm and 4 cm, respectively intersect at two points P and Q such that OP and O′P are tangents to the two circles. Find the length of the common chord PQ.
Solution:



According to the question,
Two circles with centers O and O' of radii 3 cm and 4 cm, respectively intersect at two points P and Q, such that OP and O'P are tangents to the two circles and PQ is a common chord.
To Find: Length of common chord PQ
∠OPO' = 90° [Tangent at a point on the circle is perpendicular to the radius through point of contact]
So OPO is a right-angled triangle at P
Using Pythagoras in △ OPO', we have
$(OO')^2 = (O'P)^2 + (OP)^2$
$(OO')^2 = (4)^2 + (3)^2$
$(OO')^2 = 25$
OO' = 5 cm
Let ON = x cm and NO' = 5 - x cm
In right angled triangle ONP
$(ON)^2 + (PN)^2 = (OP)^2$
$x^2 + (PN)^2 = (3)^2$
$(PN)^2 = 9 - x^2$ [1]
In right angled triangle O'NP
$(O'N)^2 + (PN)^2 = (O'P)^2$
$(5 - x)^2 + (PN)^2 = (4)^2$
$25 - 10x + x^2 + (PN)^2 = 16$
$(PN)^2 = -x^2 + 10x - 9$[2]
From [1] and [2]
$9 - x^2 = -x^2 + 10x – 9$
$10x = 18$
$x = 1.8$
From (1) we have
$(PN)^2 = 9 - (1.8)^2$

$= 9 - 3.24 = 5.76$
PN = 2.4 cm
PQ = 2PN = 2(2.4) = 4.8 cm

6. In a right triangle ABC in which ∠B = 90°, a circle is drawn with AB as diameter intersecting the hypotenuse AC and P. Prove that the tangent to the circle at P bisects BC.
Solution:



According to the question,
In a right angle ΔABC is which ∠B = 90°, a circle is drawn with AB as diameter intersecting the hypotenuse AC at P. Also PQ is a tangent at P
To Prove: PQ bisects BC i.e. BQ = QC
Proof:
∠APB = 90° [Angle in a semicircle is a right-angle]
∠BPC = 90° [Linear Pair]
∠3 + ∠4 = 90 [1]
Now, ∠ABC = 90°
So in △ABC
∠ABC + ∠BAC + ∠ACB = 180°
90 + ∠1 + ∠5 = 180
∠1 + ∠5 = 90 [2]
Now,
∠ 1 = ∠ 3[angle between tangent and the chord equals angle made by the chord in alternate segment]
Using this in [2] we have
∠3 + ∠5 = 90 [3]
From [1] and [3] we have
∠3 + ∠4 = ∠3 + ∠5
∠4 = ∠5
QC = PQ [Sides opposite to equal angles are equal]

But Also PQ = BQ [Tangents drawn from an external point to a circle are equal]
So, BQ = QC
i.e. PQ bisects BC .

7. In Fig. 9.18, tangents PQ and PR are drawn to a circle such that ∠RPQ = 30°. A chord RS is drawn parallel to the tangent PQ. Find the ∠RQS.
[Hint: Draw a line through Q and perpendicular to QP.]

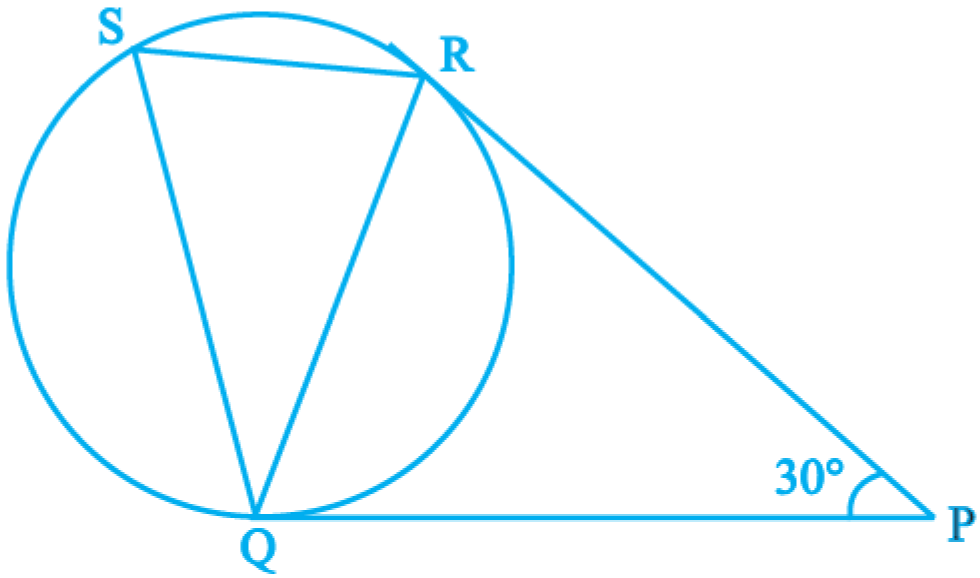
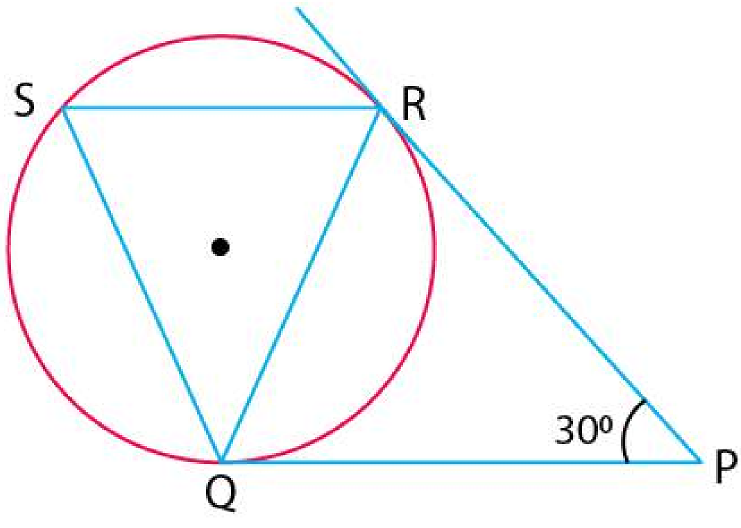


Fig. 9.18

Solution:



According to the question,
Tangents PQ and PR are drawn to a circle such that ∠RPQ = 30°. A chord RS is drawn parallel to the tangent PQ.
To Find : ∠RQS

PQ = PR [Tangents drawn from an external point to a circle are equal]
$\angle PRQ = \angle PQR$ [Angles opposite to equal sides are equal] [1]
In $\triangle PQR$
$\angle PRQ + \angle PQR + \angle QPR = 180°$
$\angle PQR + \angle PQR + \angle QPR = 180°$ [Using 1]
$2\angle PQR + \angle RPQ = 180°$
$2\angle PQR + 30 = 180$
$2\angle PQR = 150$
$\angle PQR = 75°$
$\angle QRS = \angle PQR = 75°$ [Alternate interior angles]
$\angle QSR = \angle PQR = 75°$ [angle between tangent and the chord equals angle made by the chord in alternate segment]
Now In $\triangle RQS$
$\angle RQS + \angle QRS + \angle QSR = 180$
$\angle RQS + 75 + 75 = 180$
$\angle RQS = 30°$