EXERCISE

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SHORT ANSWER TYPE:

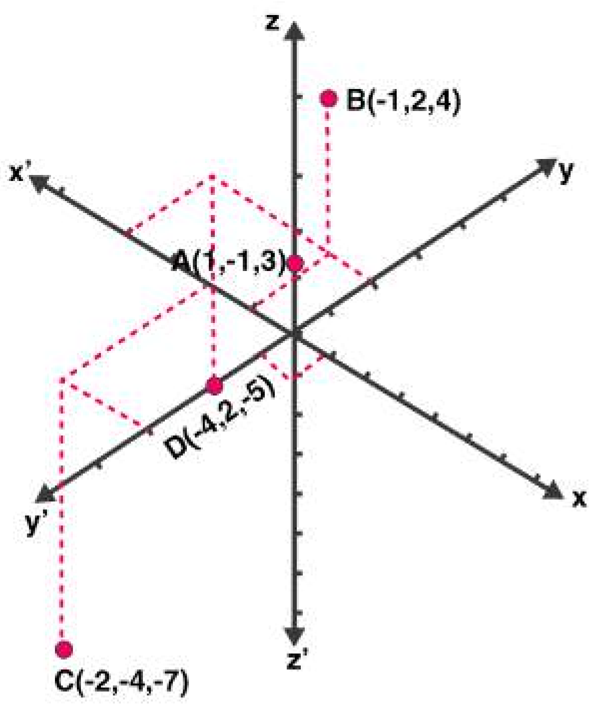
1. Locate the following points:
(i) (1, – 1, 3),
(ii) (– 1, 2, 4)
(iii) (– 2, – 4, –7)
(iv) (– 4, 2, – 5).

Solution:
(i) (1, – 1, 3):- 4th octant,

(ii) (– 1, 2, 4):- 2nd octant,

(iii) (– 2, – 4, –7):- 7th octant,

(iv) (– 4, 2, – 5):- 6th octant.



2. Name the octant in which each of the following points lies.

(i) (1, 2, 3),
(ii) (4, – 2, 3),
(iii) (4, –2, –5)
(iv) (4, 2, –5)
(v) (– 4, 2, 5)
(vi) (–3, –1, 6)
(vii) (2, – 4, – 7)
(viii) (– 4, 2, – 5).

Solution:
(i) (1, 2, 3):- 1st Octant,

(ii) (4, – 2, 3):- 4th Octant,

(iii) (4, –2, –5):- 8th Octant,

(iv) (4, 2, –5):- 5th Octant,

(v) (– 4, 2, 5):- 2nd Octant,

(vi) (–3, –1, 6):- 3rd Octant,

(vii) (2, – 4, – 7):- 8th Octant,

(viii) (– 4, 2, – 5):- 6th Octant.

3. Let A, B, C be the feet of perpendiculars from a point P on the x, y, z-axis respectively. Find the coordinates of A, B and C in each of the following where the point P is :
(i) A = (3, 4, 2)
(ii) (–5, 3, 7)
(iii) (4, – 3, – 5)

Solution:
(i) (3, 4, 2):- A (3, 0, 0), B (0, 4, 0), C (0, 0, 2)

(ii) (–5, 3, 7):- A (–5, 0, 0), B (0, 3, 0), C (0, 0, 7)

(iii) (4, – 3, – 5):- A (4, 0, 0), B (0, −3, 0), C (0, 0, −5)

4. Let A, B, C be the feet of perpendiculars from a point P on the xy, yz and zx planes respectively. Find the coordinates of A, B, C in each of the following where the point P is
(i) (3, 4, 5)
(ii) (–5, 3, 7)
(iii) (4, – 3, – 5).

Solution:
(i) (3, 4, 5):- A (3, 4, 0), B (0, 4, 5), C (3, 0, 5)

(ii) (–5, 3, 7):- A (−5, 3, 0), B (0, 3, 7), C (−5, 0, 7)

(iii) (4, – 3, – 5):- A (4, −3, 0), B (0, −3, −5), C (4, 0, −5)

5. How far apart are the points (2, 0, 0) and (–3, 0, 0)?

Solution:
The points (2, 0, 0) and (–3, 0, 0) are at a distance of =
|2 − (−3)| = 5 units.

6. Find the distance from the origin to (6, 6, 7).

Solution:
The distance from the origin to (6, 6, 7) =
$\sqrt{6^2 + 6^2 + 7^2}$
$= \sqrt{36 + 36 + 49}$
$= \sqrt{121}$
=11 units.

7. Show that if $x^2 + y^2 = 1$, then the point $\left(x,\ y\ \sqrt{1 - x^2 - y^2}\right)$ is at a distance 1 unit from the origin.
Solution:

Given $x^2 + y^2 = 1 \Rightarrow 1 - x^2 - y^2 = 0$

Distance of the point $\left(x, y\sqrt{1-x^2-y^2}\right)$ from origin is =

$\sqrt{x^2+y^2+\left(\sqrt{1-x^2-y^2}\right)^2}$

$= \sqrt{x^2+y^2+(1-x^2-y^2)}$

$= \sqrt{x^2+y^2+1-x^2-y^2}$

$=\sqrt{1}$

= 1 unit.

When x = 1 the distance of that point from origin will be 1 unit.

8. Show that the point A (1, – 1, 3), B (2, – 4, 5) and (5, – 13, 11) are collinear.

Solution:

Given points are A (1, – 1, 3), B (2, – 4, 5) and (5, – 13, 11).

To prove collinear,

$AB = \sqrt{(1-2)^2+(-1+4)^2+(3-5)^2} = \sqrt{1+9+4} = \sqrt{14}$

$BC = \sqrt{(2-5)^2+(-4+13)^2+(5-11)^2} = \sqrt{9+81+36} = 3\sqrt{14}$

$AC = \sqrt{(1-5)^2+(-1+13)^2+(3-11)^2} = \sqrt{16+144+64} = 4\sqrt{14}$

$\therefore\ AB + BC = \sqrt{14} + 3\sqrt{14}$

$=4\sqrt{14}$

=AC

∴ Points A, B and C are collinear.

9. Three consecutive vertices of a parallelogram ABCD are A (6, – 2, 4), B (2, 4, – 8), C (–2, 2, 4). Find the coordinates of the fourth vertex.

Solution:

Given three consecutive vertices of a parallelogram ABCD are A (6, – 2, 4), B (2, 4, – 8), C (–2, 2, 4).

Let the forth vertex be D(x, y, z).

Midpoint of diagonal AC = $P\left(\frac{6-2}{2}, \frac{-2+2}{2}, \frac{4+4}{2}\right) = P(2,0,4)$

Midpoint of diagonal BD $= P\left(\frac{x+2}{2}, \frac{y+4}{2}, \frac{z-8}{2}\right) = P(2,0,4)$

$$\Rightarrow \frac{x+2}{2} = 2 \Rightarrow x = 2$$
$$\Rightarrow \frac{y+4}{2} = 0 \Rightarrow y = -4$$
$$\Rightarrow \frac{z-8}{2} = 4 \Rightarrow z = 16$$

∴ D (2, −4, 16) is the fourth vertex.

10. Show that the triangle ABC with vertices A (0, 4, 1), B (2, 3, – 1) and C (4, 5, 0) is right angled.

Solution:

Given vertices are A (0, 4, 1), B (2, 3, – 1) and C (4, 5, 0).

To prove right angled triangle, consider

$$AB = \sqrt{(0-2)^2 + (4-3)^2 + (1+1)^2} = \sqrt{4+1+4} = \sqrt{9} = 3$$
$$BC = \sqrt{(2-4)^2 + (3-5)^2 + (-1-0)^2} = \sqrt{4+4+1} = \sqrt{9} = 3$$
$$AC = \sqrt{(0-4)^2 + (4-5)^2 + (1-0)^2} = \sqrt{16+1+1} = \sqrt{18} = 3\sqrt{2}$$

$\Rightarrow AC^2 = AB^2 + BC^2$

∴ Triangle ABC a right angled.

11. Find the third vertex of triangle whose centroid is origin and two vertices are (2, 4, 6) and (0, –2, –5).

Solution:

Given the centroid is origin and two vertices are (2, 4, 6) and (0, –2, –5).

Let the third vertex be (x, y, z)

For a triangle the coordinates of the centroid is given by the average of the coordinates of its vertices.

$$\Rightarrow (0,0,0) = \left(\frac{2+0+x}{3}, \frac{4+(-2)+y}{3}, \frac{6+(-5)+z}{3}\right)$$
$$\Rightarrow \frac{2+x}{3} = 0, \therefore x = -2$$
$$\Rightarrow \frac{2+y}{3} = 0, \therefore y = -2$$

$$\Rightarrow \frac{1+x}{3} = 0, \therefore x = -1$$

Therefore the third vertex is (−2, −2, −1).

12. Find the centroid of a triangle, the mid-point of whose sides are D (1, 2, – 3), E (3, 0, 1) and F (– 1, 1, – 4).

Solution:

Mid - points of sides of triangle DEF are:
D(1, 2, -3), E(3, 0, 1) and F(-1, 1, -4).
By using the geometry of centroid,
We know that the centroid of triangle DEF is given as:
G = [(1 + 3 - 1)/3 , (2 + 0 + 1)/3, (-3 + 1 - 4)/3]
= (1, 1, -2)
Hence, the centroid if triangle DEF is (1, 1, -2).

13. The mid-points of the sides of a triangle are (5, 7, 11), (0, 8, 5) and (2, 3, – 1). Find its vertices.

Solution:

Given the mid-points of the sides of a triangle are (5, 7, 11), (0, 8, 5) and (2, 3, – 1).
Let he vertices be A (x_1, y_1, z_1), B (x_2, y_2, z_2) and A (x_3, y_3, z_3) respectively.
Using midpoint formula,

$$\Rightarrow \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right) = (5, 7, 11)$$

$$\Rightarrow x_1 = 10 - x_2, y_1 = 14 - y_2, z_1 = 22 - z_2$$

$$\Rightarrow \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}, \frac{z_2+z_3}{2}\right) = (0, 8, 5)$$

Now consider,

$\Rightarrow x_3 = -x_2, y_3 = 16 - y_2, z_3 = 10 - z_2$

$\Rightarrow \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}, \frac{z_1 + z_3}{2}\right) = (0, 8, 5)$

$\Rightarrow \left(\frac{10 - x_2 - x_2}{2}, \frac{14 - y_2 + 16 - y_2}{2}, \frac{22 - z_2 + 10 - z_2}{2}\right) = (2, 3, -1)$

$x_2 = 3$, $y_2 = 12$, $z_2 = 17$.

$x_1 = 10 - x_2 = 7$, $y_1 = 14 - y_2 = 2$, $z_1 = 22 - z_2 = 5$.

∴ $x_3 = -x_2 = -3$, $y_3 = 16 - y_2 = 4$, $z_3 = 10 - z_2 = -7$.

∴ A (7, 2, 5), B (3, 12, 17), C (−3, 4, −7) are the required vertices.

14. Three vertices of a Parallelogram ABCD are A (1, 2, 3), B (– 1, – 2, – 1) and C (2, 3, 2). Find the fourth vertex D.

Solution:

Given three consecutive vertices of a parallelogram ABCD are A (1, 2, 3), B (– 1, – 2, – 1) and C (2, 3, 2)

Let the fourth vertex be D(x, y, z).

Now by using midpoint formula,

Midpoint of diagonal AC = $P\left(\frac{1+2}{2}, \frac{2+3}{2}, \frac{3+2}{2}\right) = P\left(\frac{3}{2}, \frac{5}{2}, \frac{5}{2}\right)$

Midpoint of diagonal BD = $P\left(\frac{x+(-1)}{2}, \frac{y+(-2)}{2}, \frac{z+(-1)}{2}\right) = P\left(\frac{3}{2}, \frac{5}{2}, \frac{5}{2}\right)$

On comparing we get

$\Rightarrow \frac{x-1}{2} = \frac{3}{2}$

$\Rightarrow x = 4$

$\Rightarrow \frac{y-2}{2} = \frac{5}{2}$

$\Rightarrow y = 7$

$\Rightarrow \frac{z-1}{2} = \frac{5}{2}$

$\Rightarrow z = 6$

∴ D (4, 7, 6) is the fourth vertex.

15. Find the coordinate of the points which trisect the line segment joining the points A (2, 1, – 3) and B (5, – 8, 3).

Solution:

Given the line segment joining the points are A (2, 1, – 3) and B (5, – 8, 3).
Now let P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) be the points which trisects the line segment.
⇒ P divides AB in the ratio 2:1

$$\Rightarrow\ x_1 = \frac{2+2\times 5}{1+2} = 4$$

$$\Rightarrow\ y_1 = \frac{1+2\times(-8)}{1+2} = -5$$

$$\Rightarrow\ z_1 = \frac{-3+2\times 3}{1+2} = 1$$

⇒ Q divides AP in the ratio 1:1

$$\Rightarrow\ x_2 = \frac{2+4}{2} = 3$$

$$\Rightarrow\ y_2 = \frac{1+(-5)}{2} = -2$$

$$\Rightarrow\ z_2 = \frac{-3+1}{2} = -1$$

∴ (4, −5, 1) and (3, −2, −1) are the coordinate of the points which trisect the line segment joining the points A (2, 1, – 3) and B (5, – 8, 3).

16. If the origin is the centroid of a triangle ABC having vertices A (a, 1, 3), B (– 2, b, – 5) and C (4, 7, c), find the values of a, b, c.

Solution:
Given triangle ABC having vertices A (a, 1, 3), B (– 2, b, – 5) and C (4, 7, c) and origin is the centroid.
For a triangle the coordinates of the centroid is given by the average of the coordinates of its vertices.
Therefore,

$$\Rightarrow\ (0,0,0) = \left(\frac{a+(-2)+4}{3}, \frac{1+b+7}{3}, \frac{3+(-5)+c}{3}\right)$$

Now by comparing the each point we get

$$\Rightarrow\ \frac{a+2}{3} = 0, \therefore a = -2$$

$$\Rightarrow\ \frac{b+8}{3} = 0, \therefore b = -8$$

$\Rightarrow \frac{c-2}{3} = 0, \therefore c = 2$

17. Let A (2, 2, – 3), B (5, 6, 9) and C (2, 7, 9) be the vertices of a triangle. The internal bisector of the angle A meets BC at the point D. Find the coordinates of D.

Solution:

Given A (2, 2, – 3), B (5, 6, 9) and C (2, 7, 9) are the vertices of a triangle.

And also given that the internal bisector of the angle A meets BC at the point D.

$$AB = \sqrt{(5-2)^2 + (6-2)^2 + (9-(-3))^2} = \sqrt{9+16+144} = \sqrt{169} = 13$$

Now,

$$AC = \sqrt{(2-2)^2 + (7-2)^2 + (9-(-3))^2} = \sqrt{0+25+144} = \sqrt{169} = 13$$

⇒ ABC is an isosceles triangle and thus the internal bisector of the angle A meets BC at its midpoint.

$\Rightarrow \; D\left(\frac{5+2}{2}, \frac{6+7}{2}, \frac{9+9}{2}\right)$

∴ The coordinates of D is $\left(\frac{7}{2}, \frac{13}{2}, 9\right)$

18. Show that the three points A (2, 3, 4), B (–1, 2, – 3) and C (– 4, 1, – 10) are collinear and find the ratio in which C divides AB.

Solution:

Given three points are A (2, 3, 4), B (–1, 2, – 3) and C (– 4, 1, – 10)

To find collinear points,

$AB = \sqrt{(2+1)^2 + (3-2)^2 + (4+3)^2}$

$= \sqrt{9+1+49}$

$=\sqrt{59}$

Now consider,

$BC = \sqrt{(-1+4)^2 + (2-1)^2 + (-3+10)^2}$

$= \sqrt{9+1+49}$

$=\sqrt{59}$

Again we have,

$AC = \sqrt{(2+4)^2 + (3-1)^2 + (4+10)^2}$

$= \sqrt{36 + 4 + 196}$

$=\sqrt{236}$

$=2\sqrt{59}$

⇒ AB + BC = AC; Points A, B and C are collinear.

AC: BC = $2\sqrt{59}$: $\sqrt{59}$ = 2: 1

From the lengths of AB, BC and AC we can say that C divides AB in the ratio 2:1 externally.

19. The mid-point of the sides of a triangle are (1, 5, – 1), (0, 4, – 2) and (2, 3, 4). Find its vertices. Also find the centroid of the triangle.

Solution:

Given the mid-point of the sides of a triangle are (1, 5, – 1), (0, 4, – 2) and (2, 3, 4).

Let the vertices be A (x_1, y_1, z_1), B (x_2, y_2, z_2) and C (x_3, y_3, z_3) respectively.

Now by using midpoint formula we get

$\Rightarrow \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right) = (1, 5, -1)$

$\Rightarrow x_1 = 2 - x_2,\ y_1 = 10 - y_2,\ z_1 = -2 - z_2$

$\Rightarrow \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}, \frac{z_2 + z_3}{2}\right) = (0, 4, -2)$

$\Rightarrow x_3 = -x_2,\ y_3 = 8 - y_2,\ z_3 = -4 - z_2$

$\Rightarrow \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}, \frac{z_1 + z_3}{2}\right) = (2, 3, 4)$

$\Rightarrow \left(\frac{2 - x_2 - x_2}{2}, \frac{10 - y_2 + 8 - y_2}{2}, \frac{-2 - z_2 - 4 - z_2}{2}\right) = (2, 3, 4)$

Now by comparing the equations to their RHS we get

$\therefore x_2 = -1,$

$y_2 = 6,$

$z_2 = -7.$

$\therefore x_1 = 2 - x_2 = 3,$

$y_1 = 10 - y_2 = 4,$

$z_1 = -4 - z_2 = 5.$

$\therefore x_3 = -x_2 = 1$,

$y_3 = 8 - y_2 = 2$,

$z_3 = -4 - z_2 = 3$.

$\therefore$ A (−1, 6, −7), B (3, 4, 5), C (1, 2, 3) are the required vertices.

Centroid of a triangle is given by the average of the coordinates of its vertices or midpoint of sides.

Centroid is $\left(\frac{1+0+2}{3}, \frac{5+4+3}{3}, \frac{-1-2+4}{3}\right) = \left(1, 4, \frac{1}{3}\right)$

20. Prove that the points (0, – 1, – 7), (2, 1, – 9) and (6, 5, – 13) are collinear. Find the ratio in which the first point divides the join of the other two.

Solution:

Given that three points A (0, – 1, – 7), B (2, 1, – 9) and C (6, 5, – 13) are collinear

Therefore we can write as

$$AB = \sqrt{(2-0)^2 + (1-(-1))^2 + \left((-9)-(-7)\right)^2} = \sqrt{4+4+4} = 2\sqrt{3}$$

$$BC = \sqrt{(6-2)^2 + (5-1)^2 + \left((-13)-(-9)\right)^2} = \sqrt{16+16+16} = 4\sqrt{3}$$

$$AC = \sqrt{(6-0)^2 + (5-(-1))^2 + \left((-13)-(-7)\right)^2} = \sqrt{36+36+36} = 6\sqrt{3}$$

$\Rightarrow$ AB + BC = AC

Since points A, B and C are collinear.

AB: AC = $2\sqrt{3}:6\sqrt{3}$ = 1:3

Hence from the lengths of AB, BC and AC we can say that the first point divides the join of the other two in the ratio 1:3 externally.

21. What are the coordinates of the vertices of a cube whose edge is 2 units, one of whose vertices coincides with the origin and the three edges passing through the origin, coincides with the positive direction of the axes through the origin?

Solution:

Given that cube with 2 units edge, one of whose vertices coincides with the origin and the three edges passing through the origin, coincides with the positive direction of the axes through the origin.

The Coordinates of the vertices are;
(0, 0, 0), (2, 0, 0), (0, 2, 0), (0, 0, 2), (2, 2, 0), (0, 2, 2), (2, 0, 2) and (2, 2, 2).

OBJECTIVE TYPE QUESTIONS:

Choose the correct answer from the given four options indicated against each of the Exercises from 22 (M.C.Q.).

22. The distance of point P (3, 4, 5) from the y z-plane is
(A) 3 units (B) 4 units (C) 5 units (D) 550

Solution:
(A) 3 units

Explanation:
From basic ides of three-dimensional geometry, we know that x-coordinate of a point is its distance from y z plane.
∴ Distance of Point P (3, 4, 5) from y z plane is given by its x coordinate.
∵ x-coordinate of point P = 3
∴ Distance of (3, 4, 5) from y z plane is 3 units
Hence, option (A) is the only correct choice.

23. What is the length of foot of perpendicular drawn from the point P (3, 4, 5) on y-axis
(A) √41 (B) √34 (C) 5 (D) none of these

Solution:
(B) √34

Explanation:
As we know that y-axis lies on x y plane and y z.
So, its distance from x y and y z plane is 0.
∴ By basic definition of three-dimension coordinate we can say that x-coordinate and z–coordinate are 0.
As, perpendicular is drawn from point P to y-axis, so distance of point of intersection of this line from x z plane remains the same.
∴ y-coordinate of the new point say Q = 4

Or we can say that corresponding point on y-axis is (0, 4, 0)

∴ Length of perpendicular = distance between P and Q

From distance formula-

PQ = $\sqrt{(3-0)^2+(4-4)^2+(5-0)^2} = \sqrt{9+25} = \sqrt{34}$

∴ Length of foot of perpendicular drawn from the point P (3, 4, 5) on y-axis is √34 units.

Hence, option (B) is the only correct choice.

24. Distance of the point (3, 4, 5) from the origin (0, 0, 0) is

(A) √50 (B) 3 (C) 4 (D) 5

Solution:

(A) √50

Explanation:

Let P be the point whose coordinate is (3, 4, 5) and Q represents the origin.

From distance formula we can write as

PQ = $\sqrt{(3-0)^2+(4-0)^2+(5-0)^2}$

$= \sqrt{9+16+25}$

=√50

∴ Distance of the point (3, 4, 5) from the origin (0, 0, 0) is √50 units.

Hence, option (A) is the only correct choice.

25. If the distance between the points (a, 0, 1) and (0, 1, 2) is √27, then the value of a is

(A) 5 (B) ± 5 (C) – 5 (D) none of these

Solution:

(B) ± 5

Explanation:

Let P be the point whose coordinate is (a, 0, 1) and Q represents the point (0, 1, 2).

Given, PQ = √27

From distance formula we have

PQ = $\sqrt{(a-0)^2+(0-1)^2+(1-2)^2} = \sqrt{a^2+2}$

$\Rightarrow \sqrt{27} = \sqrt{a^2+2}$

Squaring on both sides

$a^2 + 2 = 27$

$\Rightarrow a^2 = 25$

$\Rightarrow a = \pm\sqrt{25} = \pm 5$

$\therefore a = 5$ or $a = -5$

Option (A) and (B) both have one correct answer but only option (C) contains both the answers

Hence, option (B) is the most suitable choice.