EXERCISE

SHORT ANSWER TYPE:

1. Find the mean deviation about the mean of the distribution:

Size	20	21	22	23	24
Frequency	6	4	5	1	4

Solution:

Given data distribution

Now we have to find the mean deviation about the mean of the distribution

Construct a table of the given data

Size (x_i)	Frequency (f_i)	f_ix_i
20	6	20 × 6 = 120
21	4	21 × 4 = 84
22	5	22 × 5 = 110
23	1	23 × 1 = 23
24	4	24 × 4 = 96
Total	20	433

We know that mean, $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{433}{20} = 21.65$

To find mean deviation we have to construct another table

Size (x_i)	Frequency (f_i)	f_ix_i	$d_i = \lvert x_i - \text{mean} \rvert$	f_id_i
20	6	120	1.65	9.90
21	4	84	0.65	2.60
22	5	110	0.35	1.75
23	1	23	1.35	1.35
24	4	96	2.35	9.40
Total	20	433	6.35	25.00

Hence Mean Deviation becomes,

$$M.D = \frac{\sum f_i d_i}{\sum f_i} = \frac{25}{20} = 1.25$$

Therefore, the mean deviation about the mean of the distribution is 1.25

2. Find the mean deviation about the median of the following distribution:

Marks obtained	10	11	12	14	15
No. of Students	2	3	8	3	4

Solution:

Given data distribution

Now we have to find the mean deviation about the median

Let us make a table of the given data and append other columns after calculations

Marks Obtained (x_i)	Number of Students (f_i)	Cumulative frequency (c. f)
10	2	2
11	3	2 + 3 = 5
12	8	5 + 8 = 13
14	3	13 + 3 = 16
15	4	16 + 4 = 20
Total	20	

Now, here N=20, which is even.

Here median,

$$M = \frac{1}{2}\left[\left(\frac{N}{2}\right)^{th} \text{observation} + \left(\frac{N}{2}+1\right)^{th} \text{observation}\right]$$

$$M = \frac{1}{2}\left[\left(\frac{20}{2}\right)^{th} \text{observation} + \left(\frac{20}{2}+1\right)^{th} \text{observation}\right]$$

$$M = \frac{1}{2}\left[10^{th}\text{observation} + 11^{th}\text{observation}\right]$$

Both these observations lie in cumulative frequency 13, for which corresponding observation is 12.

$$M = \frac{1}{2}[12 + 12] = 12$$

So the above table with more columns is as shown below,

Marks Obtained	Number of Students	Cumulative frequency	$d_i = \lvert x_i - M \rvert$	$f_i d_i$
10	2	2	2	4
11	3	2 + 3 = 5	1	3
12	8	5 + 8 = 13	0	0
14	3	13 + 3 = 16	2	6
15	4	16 + 4 = 20	3	12
Total	20		6.35	25

Hence Mean Deviation becomes,

$$M.D = \frac{\sum f_i d_i}{\sum f_i} = \frac{25}{20} = 1.25$$

Therefore, the mean deviation about the median of the distribution is 1.25

3. Calculate the mean deviation about the mean of the set of first n natural numbers when n is an odd number.

Solution:

Given set of first n natural numbers when n is an odd number

Now we have to find the mean deviation about the mean

We know first n natural numbers are 1, 2, 3……, n.

And given n is odd number.

So mean is,

$$\bar{x} = \frac{1+2+3+\cdots+n}{n} = \frac{\frac{n(n+1)}{2}}{n} = \frac{(n+1)}{2}$$

The deviations of numbers from the mean are as shown below,

$$1-\frac{(n+1)}{2}, 2-\frac{(n+1)}{2}, 3-\frac{(n+1)}{2}, \ldots\ldots, (n-2)-\frac{(n+1)}{2}, (n-1)-\frac{(n+1)}{2}, n-\frac{(n+1)}{2}$$

Or,

$$\frac{2-(n+1)}{2}, \frac{4-(n+1)}{2}, \frac{6-(n+1)}{2}, \ldots \frac{2(n-2)-(n+1)}{2}, \frac{2(n-1)-(n+1)}{2}, \frac{2n-(n+1)}{2}$$

$$\frac{2-(n+1)}{2}, \frac{4-(n+1)}{2}, \frac{6-(n+1)}{2}, \ldots\ldots \frac{2n-4-(n+1)}{2}, \frac{2n-2-(n+1)}{2}, \frac{2n-(n+1)}{2}$$

Or,

$$\frac{1-n}{2}, \frac{3-n}{2}, \frac{5-n}{2}, \ldots\ldots \frac{n-5}{2}, \frac{n-3}{2}, \frac{n-1}{2}$$

By taking negative common

$$\frac{-(n-1)}{2}, \frac{-(n-3)}{2}, \frac{-(n-5)}{2}, \ldots\ldots, \frac{n-5}{2}, \frac{n-3}{2}, \frac{n-1}{2}$$

So the absolute values of deviation from the mean is

$$|x_i - \bar{x}| = \frac{(n-1)}{2}, \frac{(n-3)}{2}, \frac{(n-5)}{2}, \ldots\ldots, \frac{n-5}{2}, \frac{n-3}{2}, \frac{n-1}{2}$$

The sum of absolute values of deviations from the mean, is

$$\sum|x_i - \bar{x}| = \frac{(n-1)}{2} + \frac{(n-3)}{2} + \frac{(n-5)}{2} + \cdots + \frac{n-5}{2} + \frac{n-3}{2} + \frac{n-1}{2}$$

$$\sum|x_i - \bar{x}| = 2\left(1 + 2 + 3 + \cdots + \frac{(n-5)}{2} + \frac{(n-3)}{2} + \frac{(n-1)}{2} +\right)$$

$$\sum|x_i - \bar{x}| = 2\left(1 + 2 + 3 + \cdots + \frac{(n-5)}{2} + \frac{(n-3)}{2} + \frac{(n-1)}{2}\right)$$

That is 2 times sum of (n – 1)2 terms, so it can be written as

$$\sum|x_i - \bar{x}| = 2\left(\frac{\frac{(n-1)}{2}\left(\frac{(n-1)}{2} + 1\right)}{2}\right)$$

Taking LCM we get

$$\sum|x_i - \bar{x}| = 2\left(\frac{\frac{(n-1)}{2}\left(\frac{(n-1)+2}{2}\right)}{2}\right)$$

The above equation can be written as

$$\sum|x_i - \bar{x}| = \left(\frac{(n-1)}{2}\left(\frac{n+1}{2}\right)\right)$$

Multiplying the above equation we get

$$\sum|x_i - \bar{x}| = \left(\frac{n^2-1}{4}\right)$$

Therefore, mean deviation about the mean is

$$M.D = \frac{\sum|x_i - \bar{x}|}{n} = \frac{\left(\frac{n^2-1}{4}\right)}{n} = \left(\frac{n^2-1}{4n}\right)$$

Hence the mean deviation about the mean of the set of first n natural numbers when n is an odd number is

$$\left(\frac{n^2-1}{4n}\right)$$

4. Calculate the mean deviation about the mean of the set of first n natural numbers when n is an even number.

Solution:

Given set of first n natural numbers when n is an even number.

Now we have to find the mean deviation about the mean

We know first n natural numbers are 1, 2, 3……, n. And given n is even number.

So mean is,

$$\bar{x} = \frac{1+2+3+\cdots+n}{n} = \frac{\frac{n(n+1)}{2}}{n} = \frac{(n+1)}{2}$$

The deviations of numbers from the mean are as shown below,

$$1-\frac{(n+1)}{2}, 2-\frac{(n+1)}{2}, 3-\frac{(n+1)}{2}, \dots, \frac{(n-2)}{2}-\frac{(n+1)}{2}, \frac{(n)}{2} - \frac{(n+1)}{2}, \frac{(n+2)}{2}-\frac{(n+1)}{2}, \dots, n-\frac{(n+1)}{2}$$

Or,

$$\frac{2-(n+1)}{2}, \frac{4-(n+1)}{2}, \frac{6-(n+1)}{2}, \dots, \frac{n-2-(n+1)}{2}, \frac{n-(n+1)}{2}, \frac{(n+2)-(n+1)}{2}, \dots, \frac{2n-(n+1)}{2}$$

The above equation can be written as

$$\frac{2-(n+1)}{2}, \frac{4-(n+1)}{2}, \frac{6-(n+1)}{2}, \dots\dots, \frac{-3}{2}, \frac{-1}{2}, \frac{1}{2} \dots, \frac{2n-(n+1)}{2}$$

Or,

$$\frac{1-n}{2}, \frac{3-n}{2}, \frac{5-n}{2}, \dots\dots, \frac{-3}{2}, \frac{-1}{2}, \frac{1}{2}, \dots\dots, \frac{n-1}{2}$$

So the absolute values of deviation from the mean is

$$|x_i - \bar{x}| = \frac{(n-1)}{2}, \frac{(n-3)}{2}, \frac{(n-5)}{2}, \dots\dots, \frac{3}{2}, \frac{1}{2}, \frac{1}{2}, \dots\dots, \frac{n-1}{2}$$

The sum of absolute values of deviations from the mean, is

$$\sum|x_i - \bar{x}| = \frac{(n-1)}{2} + \frac{(n-3)}{2} + \frac{(n-5)}{2} + \cdots + \frac{3}{2} + \frac{1}{2} + \frac{1}{2} + \cdots + \frac{n-1}{2}$$

We can write as

$$\sum|x_i - \bar{x}| = \left(\frac{1}{2} + \frac{3}{2} + \cdots + \frac{(n-1)}{2}\right)\left(\frac{n}{2}\right)$$

Now we know sum of first n natural numbers = n^2

Therefore, mean deviation about the mean is

$$M.D = \frac{\sum|x_i - \bar{x}|}{n} = \frac{\left(\frac{1}{2} + \frac{3}{2} + \cdots + \frac{(n-1)}{2}\right)\left(\frac{n}{2}\right)}{n}$$

$$\text{M.D} = \frac{\sum|x_i - \bar{x}|}{n} = \frac{\left(\frac{n}{2}\right)^2}{n}$$

$$\text{M.D} = \frac{\sum|x_i - \bar{x}|}{n} = \frac{n^2}{4n} = \frac{n}{4}$$

Hence the mean deviation about the mean of the set of first n natural numbers when n is an even number is n/4

5. Find the standard deviation of the first n natural numbers.

Solution:

Given set of first n natural numbers

Now we have to find the standard deviation

Given first n natural numbers, we can write in table as shown below

x_i	1	2	3	4	5	...	....	n
x_i^2	1	4	9	16	25	....	....	n^2

So, the sums of these are

$$\sum x_i = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

And

$$\sum x_i^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Therefore, the standard deviation can be written as,

$$\sigma = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

Substituting the values we get

$$\sigma = \sqrt{\frac{\frac{n(n+1)(2n+1)}{6}}{n} - \left(\frac{\frac{n(n+1)}{2}}{n}\right)^2}$$

On simplifying

$$\sigma = \sqrt{\frac{n(n+1)(2n+1)}{6n} - \frac{n^2(n+1)^2}{4n^2}}$$

$$\sigma = \sqrt{\frac{n(2n+1)+1(2n+1)}{6} - \frac{(n^2+2n+1)}{4}}$$

Multiplying the numerator we get

$$\sigma = \sqrt{\frac{2n^2+n+2n+1}{6} - \frac{n^2+2n+1}{4}}$$

Taking LCM and simplifying we get

$$\sigma = \sqrt{\frac{2(2n^2+3n+1)-3(n^2+2n+1)}{12}}$$

$$\sigma = \sqrt{\frac{4n^2+6n+2-3n^2-6n-3}{12}}$$

$$\sigma = \sqrt{\frac{n^2-1}{12}}$$

Hence the standard deviation of the first n natural numbers is $\sqrt{\frac{n^2-1}{12}}$

6. The mean and standard deviation of some data for the time taken to complete a test are calculated with the following results:

Number of observations = 25, mean = 18.2 seconds, standard deviation = 3.25 seconds.

Further, another set of 15 observations $x_1, x_2, \ldots, x_{15}$, also in seconds, is now available and we have

$\sum_{i=1}^{15} x_1 = 279$ **and** $\sum_{i=1}^{15} x_i^2 = 5524$ **.**

Calculate the standard derivation based on all 40 observations.

Solution:

Given: Number of observations = 25, mean = 18.2 seconds, standard deviation = 3.25 seconds. Another set of 15 observations $x_1, x_2, \ldots, x_{15}$, also in seconds,

is $\sum_{i=1}^{15} x_1 = 279$ and $\sum_{i=1}^{15} x_i^2 = 5524$

Now we have to find the standard derivation based on all 40 observations

As per the given criteria,

In first set,

Number of observations, n_1=25

Mean, $\overline{x_1} = 18.2$

And standard deviation, $\sigma_1 = 3.25$

And

In second set,

Number of observations, $n_2=15$

$\sum_{i=1}^{15} x_i = 279$ and $\sum_{i=1}^{15} x_i^2 = 5524$

For the first set we have

$$\overline{x_1} = 18.2 = \frac{\sum x_i}{25}$$

$\sum x_i = 25 \times 18.2 = 455$

Therefore the standard deviation becomes,

$$\sigma_1^2 = \frac{\sum x_i^2}{25} - (18.2)^2$$

Substituting the values, we get

$$(3.25)^2 = \frac{\sum x_i^2}{25} - 331.24$$

$$\Rightarrow 10.5625 + 331.24 = \frac{\sum x_i^2}{25}$$

On rearranging we get

$$\Rightarrow \frac{\sum x_i^2}{25} = 341.8025$$

On cross multiplication we get

$$\Rightarrow \sum x_i^2 = 25 \times 341.8025 = 8545.06$$

For the combined standard deviation of the 40 observation, n = 40

And

$$\Rightarrow \sum x_i^2 = 8545.06 + 5524 = 14069.69$$

$$\Rightarrow \sum x_i = 455 + 279 = 734$$

Therefore the standard deviation can be written as,

$$\sigma = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

Substituting the values, we get

Therefore the standard deviation can be written as,

$$\sigma = \sqrt{\frac{14069.69}{40} - \left(\frac{734}{40}\right)^2}$$

On simplifying we get

$\sigma = \sqrt{351.7265 - (18.35)^2}$

$\sigma = \sqrt{351.7265 - 336.7225}$

$\sigma = \sqrt{15.004}$

σ=3.87

Hence the standard derivation based on all 40 observations is 3.87.

7. The mean and standard deviation of a set of n_1 observations are $\overline{x}_1$ and s_1, respectively while the mean and standard deviation of another set of n_2 observations are $\overline{x}_2$ and s_2, respectively. Show that the standard deviation of the combined set of $(n_1 + n_2)$ observations is given by

$$\text{S.D.} = \sqrt{\frac{n_1(s_1)^2 + n_2(s_2)^2}{n_1 + n_2} + \frac{n_1 n_2(\overline{x}_1 - \overline{x}_2)^2}{(n_1 + n_2)^2}}$$

Solution:

Given the mean and standard deviation of a set of n_1 observations are $\overline{x}_1$ and s_1, respectively while the mean and standard deviation of another set of n_2 observations are $\overline{x}_2$ and s_2, respectively

To show that the standard deviation of the combined set of $(n_1 + n_2)$ observations is given by

$$\text{S. D} = \sqrt{\frac{n_1(s_1)^2+n_2(s_2)^2}{n_1+n_2} + \frac{n_1 n_2(\overline{x}_1-\overline{x}_2)^2}{(n_1+n_2)^2}}$$

As per given criteria,

For first set

Let x_i where i=1, 2, 3, 4, ..., n_1

For second set

And y_j where j=1, 2, 3, 4, ..., n_2

And the means are

$$\overline{x_1} = \frac{1}{n_1}\sum_{i=1}^{n} x_i, \overline{x_2} = \frac{1}{n_2}\sum_{j=1}^{n} y_j$$

Now mean of the combined series is given by

$$\bar{x} = \frac{1}{n_1 + n_2}\left[\sum_{i=1}^{n} x_i + \sum_{j=1}^{n} y_j\right] = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} \ldots\ldots (i)$$

And the corresponding square of standard deviation is

$$\sigma_1^2 = \frac{1}{n_1}\sum_{i=1}^{n}(x_i - \bar{x})^2, \sigma_2^2 = \frac{1}{n_2}\sum_{j=1}^{n}(y_j - \bar{x})^2$$

Therefore, square of standard deviation becomes,

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 = \frac{1}{n_1 + n_2}\left[\sum_{i=1}^{n}(x_i - \bar{x})^2 + \sum_{j=1}^{n}(y_j - \bar{x})^2\right] \ldots\ldots (ii)$$

Now,

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = \sum_{i=1}^{n}(x_i - \bar{x}_j + \bar{x}_j - \bar{x})^2$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = \sum_{i=1}^{n}(x_i - \bar{x}_j)^2 + n_1(\bar{x}_j - \bar{x})^2 + 2(\bar{x}_j - \bar{x})\sum_{i=1}^{n}(x_i - \bar{x}_j)^2$$

But the algebraic sum of the deviation of values of first series from their mean is zero.

$$\sum_{i=1}^{n}(x_i - \bar{x}_j)^2 = 0$$

Also,

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = n_1 s_1^2 + n_1(\bar{x}_1 - \bar{x})^2 \ldots (iii)$$

But

$$d_1 = \bar{x}_1 - \bar{x}$$

Substituting value from equation (i), we get

$$\bar{x}_1 - \bar{x} = \bar{x}_1 - \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$\bar{x}_1 - \bar{x} = \frac{(\bar{x}_1)(n_1 + n_2) - (n_1\bar{x}_1 + n_2\bar{x}_2)}{n_1 + n_2}$$

$$\bar{x}_1 - \bar{x} = \frac{(n_1\bar{x}_1 + n_2\bar{x}_1) - (n_1\bar{x}_1 + n_2\bar{x}_2)}{n_1 + n_2}$$

$$\bar{x}_1 - \bar{x} = \frac{(n_2\bar{x}_1) - (n_2\bar{x}_2)}{n_1 + n_2}$$

Substituting this value in equation (iii), we get

$$\sum_{i=1}^{n}(x_i-\bar{x})^2 = n_1 s_1^2 + n_1\left(\frac{n_2(\bar{x}_1-\bar{x}_2)}{n_1+n_2}\right)^2$$

$$\sum_{i=1}^{n}(x_i-\bar{x})^2 = n_1 s_1^2 + \frac{n_1 n_2^2(\bar{x}_1-\bar{x}_2)^2}{(n_1+n_2)^2} \dots \text{(iv)}$$

Similarly, we have

$$\sum_{j=1}^{n}(y_i-\bar{x})^2 = \sum_{j=1}^{n}\left(y_j-\bar{x}_i+\bar{x}_i-\bar{x}\right)^2$$

$$\sum_{j=1}^{n}(y_i-\bar{x})^2 = \sum_{j=1}^{n}\left(y_j-\bar{x}_j\right)^2 + n_2\left(\bar{x}_j-\bar{x}\right)^2 + 2(\bar{x}_j-\bar{x})\sum_{j=1}^{n}\left(y_j-\bar{x}_j\right)^2$$

But the algebraic sum of the deviation of values of second series from their mean is zero.

$$\sum_{j=1}^{n}\left(y_j-\bar{x}_i\right)^2 = 0$$

Also,

$$\sum_{j=1}^{n}(y_i-\bar{x})^2 = n_2 s_2^2 + n_2(\bar{x}_2-\bar{x})^2 \dots \text{(v)}$$

But $d_2 = \bar{x}_2 - \bar{x}$

Substituting value from equation (i), we get

$$\bar{x}_2-\bar{x} = \bar{x}_2 - \frac{n_1\bar{x}_1+n_2\bar{x}_2}{n_1+n_2}$$

$$\bar{x}_2-\bar{x} = \frac{(\bar{x}_2)(n_1+n_2)-(n_1\bar{x}_1+n_2\bar{x}_2)}{n_1+n_2}$$

$$\bar{x}_2-\bar{x} = \frac{(n_1\bar{x}_2+n_2\bar{x}_2)-(n_1\bar{x}_1+n_2\bar{x}_2)}{n_1+n_2}$$

$$\bar{x}_2-\bar{x} = \frac{(n_1\bar{x}_2)-(n_1\bar{x}_2)}{n_1+n_2}$$

$$\bar{x}_2-\bar{x} = \frac{n_1(\bar{x}_2-\bar{x}_1)}{n_1+n_2}$$

Substituting this value in equation (v), we get

$$\sum_{j=1}^{n}(y_j-\bar{x})^2 = n_2 s_2^2 + n_2\left(\frac{n_1(\bar{x}_2-\bar{x}_1)}{n_1+n_2}\right)^2$$

$$\sum_{j=1}^{n}(y_j-\bar{x})^2 = n_2 s_2^2 + \frac{n_2 n_1^2(\bar{x}_2-\bar{x}_1)^2}{(n_1+n_2)^2} \dots \text{(vi)}$$

Substituting equation (iv) and (vi) in equation (ii), we get

$$\sigma^2 = \sigma_1^2+\sigma_2^2 = \frac{1}{n_1+n_2}\left[\sum_{i=1}^{n}(x_i-\bar{x})^2+\sum_{j=1}^{n}(y_j-\bar{x})^2\right]$$

$$\sigma^2 = \frac{1}{n_1+n_2}\left[n_1 s_1^2 + \frac{n_1 n_2^2(\bar{x}_1-\bar{x}_2)^2}{(n_1+n_2)^2} + n_2 s_2^2 + \frac{n_2 n_1^2(\bar{x}_2-\bar{x}_1)^2}{(n_1+n_2)^2}\right]$$

$$\sigma^2 = \frac{1}{n_1+n_2}\left[n_1 s_1^2 + n_2 s_2^2 + \frac{n_1 n_2^2(\bar{x}_1-\bar{x}_2)^2}{(n_1+n_2)^2} + \frac{n_2 n_1^2(-(\bar{x}_1-\bar{x}_2))^2}{(n_1+n_2)^2}\right]$$

$$\sigma^2 = \frac{1}{n_1+n_2}\left[n_1 s_1^2 + n_2 s_2^2 + \frac{n_1 n_2^2(\bar{x}_1-\bar{x}_2)^2}{(n_1+n_2)^2} + \frac{n_2 n_1^2(\bar{x}_1-\bar{x}_2)^2}{(n_1+n_2)^2}\right]$$

$$\sigma^2 = \frac{1}{n_1+n_2}\left[n_1 s_1^2 + n_2 s_2^2 + \frac{n_1 n_2(\bar{x}_1-\bar{x}_2)^2}{(n_1+n_2)^2}(n_2+n_1)\right]$$

$$\sigma^2 = \left[\frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2} + \frac{\frac{n_1 n_2(\bar{x}_1-\bar{x}_2)^2}{(n_1+n_2)^2}(n_2+n_1)}{n_1+n_2}\right]$$

$$\sigma^2 = \left[\frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2} + \frac{n_1 n_2(\bar{x}_1-\bar{x}_2)^2}{(n_1+n_2)^2}\right]$$

So the combined standard deviation

$$\text{S.D}\,(\sigma) = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2} + \frac{n_1 n_2(\bar{x}_1-\bar{x}_2)^2}{(n_1+n_2)^2}}$$

Hence proved

8. Two sets each of 20 observations have the same standard derivation 5. The first set has a mean 17 and the second a mean 22. Determine the standard deviation of the set obtained by combining the given two sets.

Solution:

Given two sets each of 20 observations, have the same standard derivation 5. The first set has a mean 17 and the second a mean 22.

Now we have to show that the standard deviation of the set obtained by combining the given two sets

As per given criteria, for first set

Number of observations, $n_1=20$

Standard deviation, $s_1=5$

And mean, $\bar{x}_1 = 17$

For second set, number of observations, $n_2=20$

Standard deviation, $s_2=5$

And mean, $\bar{x}_2 = 22$

We know the standard deviation for combined two series is

$$S.D\ (\sigma) = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2} + \frac{n_1 n_2 (\bar{x}_1 - \bar{x}_2)^2}{(n_1 + n_2)^2}}$$

Substituting the corresponding values, we get

$$S.D\ (\sigma) = \sqrt{\frac{(20)(5)^2 + (20)(5)^2}{20 + 20} + \frac{(20 \times 20)(17 - 22)^2}{(20 + 20)^2}}$$

On simplifying we get

$$S.D\ (\sigma) = \sqrt{\frac{(20)25 + (20)25}{40} + \frac{(400)(-5)^2}{(40)^2}}$$

$$S.D\ (\sigma) = \sqrt{\frac{2(500)}{40} + \frac{(400)(25)}{1600}}$$

$$S.D\ (\sigma) = \sqrt{\frac{1000}{40} + \frac{10000}{1600}}$$

$$S.D\ (\sigma) = \sqrt{25 + \frac{25}{4}}$$

Taking LCM and simplifying,

$$S.D\ (\sigma) = \sqrt{\frac{100 + 25}{4}}$$

$$S.D\ (\sigma) = \sqrt{\frac{125}{4}}$$

Or, σ=5.59

Hence the standard deviation of the set obtained by combining the given two sets is 5.59

9. The frequency distribution:

x	A	2A	3A	4A	5A	6A
F	2	1	1	1	1	1

Where A is a positive integer, has a variance of 160. Determine the value of A.

Solution:

Given frequency distribution table, where variance =160

Now we have to find the value of A, where A is a positive number

Now we have to construct a table of the given data

Size (x_i)	Frequency (f_i)	f_ix_i	$f_ix_i^2$
A	2	2A	$2A^2$
2A	1	2A	$4A^2$
3A	1	3A	$9A^2$
4A	1	4A	$16A^2$
5A	1	5A	$25A^2$
6A	1	6A	$36A^2$
Total	7	22A	$92A^2$

And we know variance is

$$\sigma^2 = \frac{\sum f_i x_i^2}{n} - \left(\frac{\sum f_i x_i}{n}\right)^2$$

Substituting values from above table and also given variance = 160, we get

$$160 = \frac{92A^2}{7} - \left(\frac{22A}{7}\right)^2$$

$$160 = \frac{92A^2}{7} - \frac{484A^2}{49}$$

$$160 = \frac{7 \times 92A^2 - 484A^2}{49}$$

$$160 = \frac{644A^2 - 484A^2}{49}$$

$$160 = \frac{160A^2}{49}$$

$\Rightarrow A^2=49$

$\Rightarrow A=7$

Hence the value of A is 7.

10. For the frequency distribution:

x	2	3	4	5	6	7
f	4	9	16	14	11	6

Find the standard distribution.

Solution:

Given frequency distribution table

Now we have to find the standard deviation

Let us make a table of the given data and append other columns after calculations

Size (x_i)	Frequency (f_i)	f_ix_i	$f_ix_i^2$
2	4	8	16
3	9	27	81
4	16	64	256
5	14	70	350
6	11	6	396
7	6	42	294
Total	60	277	1393

And we know standard deviation is

$$\sigma = \sqrt{\frac{\sum f_ix_i^2}{n} - \left(\frac{\sum f_ix_i}{n}\right)^2}$$

Substituting values from above table, we get

$$\sigma = \sqrt{\frac{1393}{60} - \left(\frac{277}{60}\right)^2}$$

$$\sigma = \sqrt{23.23 - (4.62)^2}$$

$$\sigma = \sqrt{23.23 - 21.34}$$

$\Rightarrow \sigma=1.37$

Hence the standard deviation is 1.37.

11. There are 60 students in a class. The following is the frequency distribution of the marks obtained by the students in a test:

Marks	**0**	**1**	**2**	**3**	**4**	**5**
Frequency	**$x - 2$**	**x**	**x^2**	**$(x + 1)^2$**	**$2x$**	**$x + 1$**

Where x is a positive integer. Determine the mean and standard deviation of the marks.

Solution:

Given there are 60 students in a class. The frequency distribution of the marks obtained by the students in a test is also given.

Now we have to find the mean and standard deviation of the marks.

It is given there are 60 students in the class, so

$\sum f_i=60$

$\Rightarrow (x - 2) + x + x^2 + (x + 1)^2 + 2x + x + 1 = 60$

$\Rightarrow 5x - 1 + x^2 + x^2 + 2x + 1 = 60$

$\Rightarrow 2x^2 +7x = 60$

$\Rightarrow 2x^2 +7x - 60 = 0$

Splitting the middle term, we get

$\Rightarrow 2x^2 + 15x - 8x - 60 = 0$

$\Rightarrow x (2x + 15) - 4(2x + 15) = 0$

$\Rightarrow (2x + 15) (x - 4) = 0$

$\Rightarrow 2x + 15 = 0$ or $x - 4 = 0$

$\Rightarrow 2x = -15$ or $x = 4$

Given x is a positive number, so x can take 4 as the only value.

And let assumed mean, a=3.

Now put x = 4 and a = 3 in the frequency distribution table and add other columns after calculations, we get

Marks (x_i)	Frequency (f_i)	$d_i = x_i - a$	$f_i d_i$	$f_i d_i^2$
0	$x - 2 = 4 - 2 = 2$	-3	-6	18
1	$x = 4$	-2	-8	16
2	$x^2 = 4^2 = 16$	-1	-16	16
3	$(x + 1)^2 = 25$	0	0	0
4	$2x = 8$	1	4	8
5	$x + 1 = 5$	2	10	20
Total	60		-12	78

And we know standard deviation is

$$\sigma = \sqrt{\frac{\sum f_i d_i^2}{n} - \left(\frac{\sum f_i d_i}{n}\right)^2}$$

Substituting values from above table, we get

$$\sigma = \sqrt{\frac{78}{60} - \left(\frac{-12}{60}\right)^2}$$

$$\sigma = \sqrt{1.3 - (0.2)^2}$$

$$\sigma = \sqrt{1.3 - 0.04}$$

$\Rightarrow \sigma = 1.12$

Hence the standard deviation is 1.12

Now mean is

$$\overline{x} = A + \frac{\sum f_i d_i}{N}$$

$$= 3 + \left(-\frac{12}{60}\right)$$

$$= 3 - \frac{1}{5} = \frac{14}{5}$$

=2.8

Hence the mean and standard deviation of the marks are 2.8 and 1.12 respectively.

12. The mean life of a sample of 60 bulbs was 650 hours and the standard deviation was 8 hours. A second sample of 80 bulbs has a mean life of 660 hours and standard deviation 7 hours. Find the overall standard deviation.

Solution:

Given the mean life of a sample of 60 bulbs was 650 hours and the standard deviation was 8 hours. A second sample of 80 bulbs has a mean life of 660 hours and standard deviation 7 hours

Now we have to find the overall standard deviation

As per given criteria, in first set of samples,

Number of sample bulbs, n_1=60

Standard deviation, s_1=8hrs

Mean life, $\bar{x}_1 = 650$

And in second set of samples,

Number of sample bulbs, n_2=80

Standard deviation, s_2=7hrs

Mean life, $\bar{x}_2 = 660$

We know the standard deviation for combined two series is

$$S.D\ (\sigma) = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2} + \frac{n_1 n_2 (\bar{x}_1 - \bar{x}_2)^2}{(n_1 + n_2)^2}}$$

Substituting the corresponding values, we get

$$S.D\ (\sigma) = \sqrt{\frac{(60)(8)^2 + (80)(7)^2}{60 + 80} + \frac{(60 \times 80)(650 - 660)^2}{(60 + 80)^2}}$$

By adding the denominator

$$S.D\ (\sigma) = \sqrt{\frac{(60)64 + (80)49}{140} + \frac{(4800)(10)^2}{(140)^2}}$$

$$S.D\ (\sigma) = \sqrt{\frac{(60)64 + (80)49}{140} + \frac{(4800)100}{19600}}$$

On simplifying we get

$$S.D\ (\sigma) = \sqrt{\frac{(6)64 + (8)49}{14} + \frac{(4800)1}{196}}$$

$$S.D\ (\sigma) = \sqrt{\frac{384 + 392}{14} + \frac{4800}{196}}$$

$$S.D\ (\sigma) = \sqrt{\frac{388}{7} + \frac{1200}{49}}$$

$$S.D\ (\sigma) = \sqrt{\frac{388 \times 7 + 1200}{49}}$$

$$S.D\ (\sigma) = \sqrt{\frac{2716 + 1200}{49}}$$

$$S.D\ (\sigma) = \sqrt{\frac{3916}{49}}$$

Or, $\sigma=8.9$

Hence the standard deviation of the set obtained by combining the given two sets is 8.9

13. Mean and standard deviation of 100 items are 50 and 4, respectively. Find the sum of all items and the sum of the squares of the items.

Solution:

Given mean and standard deviation of 100 items are 50 and 4, respectively

Now we have to find the sum of all items and the sum of the squares of the items

As per given criteria,

Number of items, n=100

Mean of the given items, $\overline{x} = 50$

But we know,

$$\overline{x} = \frac{\sum x_i}{n}$$

Substituting the corresponding values, we get

$$50 = \frac{\sum x_i}{100}$$

$\Rightarrow \sum x_i=50\times 100=5000$

Hence the sum of all the 100 items = 5000

Also given the standard deviation of the 100 items is 4

i.e., $\sigma=4$

But we know

$$\sigma = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

Substituting the corresponding values, we get

$$4 = \sqrt{\frac{\sum x_i^2}{100} - \left(\frac{5000}{100}\right)^2}$$

Now taking square on both sides, we get

$$4^2 = \frac{\sum x_i^2}{100} - (50)^2$$

$$\Rightarrow 16 = \frac{\sum x_i^2}{100} - 2500$$

$$\Rightarrow 16 + 2500 = \frac{\sum x_i^2}{100}$$

On rearranging we get

$$\Rightarrow \frac{\sum x_i^2}{100} = 2516$$

$$\Rightarrow \sum x_i^2 = 2516 \times 100$$

$$\Rightarrow \sum x_i^2 = 251600$$

And the sum of the squares of all the 100 items is 251600.

14. If for a distribution ∑ (x −5)=3, ∑ (x −5)^2 = 43 and the total number of item is 18, find the mean and standard deviation.

Solution:

Given for a distribution ∑ (x −5) = 3, ∑ (x −5)^2 = 43 and the total number of item is 18

Now we have to find the mean and standard deviation.

As per given criteria,

Number of items, n=18

And given ∑(x - 5) = 3,

And also given, ∑(x −5)^2 = 43

But we know mean can be written as,

$$\bar{x} = A + \frac{\sum(x-5)}{n}$$

Here assumed mean is 5, so substituting the corresponding values in above equation, we get

$$\bar{x} = 5 + \frac{3}{18} = \frac{18 \times 5 + 3}{18} = \frac{93}{18} = 5.17$$

And we know the standard deviation can be written as,

$$\sigma = \sqrt{\frac{\sum(x-5)^2}{n} - \left(\frac{\sum(x-5)}{n}\right)^2}$$

Substituting the corresponding values, we get

$$\sigma = \sqrt{\frac{43}{18} - \left(\frac{3}{18}\right)^2}$$

$$\sigma = \sqrt{2.39 - (0.166)^2}$$

$$\sigma = \sqrt{2.39 - 0.027} = \sqrt{2.363}$$

Hence σ=1.54

So the mean and standard deviation of given items is 5.17 and 1.54 respectively.

15. Find the mean and variance of the frequency distribution given below:

x	$1 \le x < 3$	$3 \le x < 5$	$5 \le x < 7$	$7 \le x < 10$
f	6	4	5	1

Solution:

Given the frequency distribution

Now we have to find the mean and variance

Converting the ranges of x to groups, the given table can be rewritten as shown below,

x (Class)	f_i	x_i	f_ix_i	$f_ix_i^2$
1 – 3	6	2	12	24
3 - 5	4	4	16	64
5 - 7	5	6	30	180
7 - 10	1	8.5	8.5	72.25
Total	16		66.5	340.25

And we know variance can be written as

$$\sigma^2 = \frac{\sum f_i x_i^2}{n} - \left(\frac{\sum f_i x_i}{n}\right)^2$$

Substituting values from above table, we get

$$\sigma^2 = \frac{340.25}{16} - \left(\frac{66.5}{16}\right)^2$$

On simplifying we get

$$\sigma^2 = 21.265 - (4.16)^2$$

$$\sigma^2 = 21.265 - 17.305 = 3.96$$

We also know mean can be written as

$$\bar{x} = \frac{\sum f_i x_i}{n}$$

Substituting values from above table, we get

$$\bar{x} = \frac{66.5}{16} = 4.16$$

Hence the mean and variance of the given frequency distribution is 4.16 and 3.96 respectively.

16. Calculate the mean deviation about the mean for the following frequency distribution:

Class interval	0 - 4	4 - 8	8 -12	12 - 16	16 - 20
Frequency	4	6	8	5	2

Solution:

Given the frequency distribution

Now we have to find the mean deviation about the mean

Let us make a table of the given data and append other columns after calculations

Class interval	Mid – Value (x_i)	Frequency (f_i)	f_ix_i
0 - 4	2	4	8
4 – 8	6	6	36
8 – 12	10	8	80
12 – 16	14	5	70
16 - 20	18	2	36
	total	25	230

Here mean, $\overline{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{230}{25} = 9.2$

Now we have to find mean deviation

Class interval	Mid – Value (x_i)	Frequency (f_i)	f_ix_i	$d_i = \|x_i - \text{mean}\|$	f_id_i
0 - 4	2	4	8	7.2	28.8
4 – 8	6	6	36	3.2	19.2
8 – 12	10	8	80	0.8	6.4
12 – 16	14	5	70	1.8	24.4
16 - 20	18	2	36	8.8	17.6
	total	25	230		96

Hence Mean Deviation becomes,

$$\mathrm{M.D} = \frac{\sum f_i d_i}{\sum f_i} = \frac{96}{25} = 3.84$$

Therefore, the mean deviation about the mean of the distribution is 3.84

17. Calculate the mean deviation from the median of the following data:

Class interval	**0 - 6**	**6 - 12**	**12 - 18**	**18 - 24**	**24 - 30**
Frequency	**4**	**5**	**3**	**6**	**2**

Solution:

Given the frequency distribution

Now we have to find the mean deviation from the median

Let us make a table of the given data and append other columns after calculations

Class interval	Mid – Value (x_i)	Frequency (f_i)	CF
0 – 6	3	4	4
6 – 12	9	5	9
12 – 18	15	3	12
18 – 24	21	6	18
24 - 30	27	2	20
	total	20	

Now, here N=20, which is even.

Now, here N=20, which is even.

Here median class= $\frac{N}{2} = 10^{th}$term ,

This observation lie in the class interval 12-18, so median can be written as,

$$M = l + \frac{\frac{N}{2} - cf}{f} \times h$$

Here l = 12, c f = 9, f = 3, h = 6 and N = 20, substituting these values, the above equation becomes,

$$M = 12 + \frac{\frac{20}{2} - 9}{3} \times 6$$

$$\Rightarrow M = 12 + \frac{10 - 9}{3} \times 6$$

$$\Rightarrow M = 12 + \frac{1 \times 6}{3}$$

⇒M = 12 + 2 = 14

Class interval	Mid – Value (x_i)	Frequency (f_i)	f_ix_i	d_i=|x_i – mean|	f_id_i
0 – 6	3	4	4	11	44
6 – 12	9	5	9	5	25
12 – 18	15	3	12	1	3
18 – 24	21	6	18	7	42
24 - 30	27	2	20	13	26
	total	20			140

Hence Mean Deviation becomes,

$$M.D = \frac{\sum f_i d_i}{\sum f_i} = \frac{140}{20} = 7$$

Therefore, the mean deviation about the median of the distribution is 7

18. Determine the mean and standard deviation for the following distribution:

Marks	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Frequency	1	6	6	6	8	2	2	3	0	2	1	0	0	0	1

Solution:

Given the frequency distribution

Now we have to find the mean and standard deviation

Let us make a table of the given data and append other columns after calculations

Marks (x_i)	Frequency (f_i)	f_ix_i
2	1	2
3	6	18
4	6	24
5	6	30
6	8	48
7	2	14
8	2	16
9	3	27
10	0	0
11	2	22
12	1	12
13	0	0
14	0	0
15	0	0
16	1	16
Total	N=38	$\sum f_ix_i = 229$

Here mean, $\bar{x} = \frac{\sum f_i x_i}{N} = \frac{229}{40} = 6.02 = 6$

So the above table with more columns is as shown below,

Marks (x_i)	Frequency (f_i)	f_ix_i	$d_i = x_i - \bar{x}$	f_id_i	$f_id_i^2$
2	1	2	2-6=-4	1×-4=-4	$1\times-4^2=16$
3	6	18	3-6=-3	6×-3=-18	$6\times-3^2=54$
4	6	24	4-6=-2	6×-2=-12	$6\times-2^2=24$
5	6	30	5-6=-1	6×-1=-6	$6\times-1^2=6$
6	8	48	6-6=0	8× 0=0	$8\times 0^2=0$
7	2	14	7-6=1	2×1=2	$2\times1^2=2$
8	2	16	8-6=2	2×2=4	$2\times2^2=8$
9	3	27	9-6=3	3×3=9	$3\times3^2=27$
10	0	0	10-6=4	0×4=0	$0\times4^2=0$
11	2	22	11-6=5	2×5=10	$2\times5^2=50$
12	1	12	12-6=6	1×6=6	$1\times6^2=36$
13	0	0	13-6=7	0×7=0	$0\times7^2=0$
14	0	0	14-6=8	0×8=0	$0\times8^2=0$
15	0	0	15-6=9	0×9=0	$0\times9^2=0$
16	1	16	16-6=10	1×10=10	$1\times10^2=100$
Total	N=38	Σf_ix_i =229		Σf_id_i =1	$\Sigma f_id_i^2$ =323

And we know standard deviation is

$$\sigma = \sqrt{\frac{\sum f_i d_i^2}{n} - \left(\frac{\sum f_i d_i}{n}\right)^2}$$

Substituting values from above table, we get

$$\sigma = \sqrt{\frac{323}{38} - \left(\frac{1}{38}\right)^2}$$

$$\sigma = \sqrt{8.5 - (0.026)^2}$$

$$\sigma = \sqrt{8.5 - 0.000676} = \sqrt{8.5}$$

$\Rightarrow \sigma=2.9$

Hence the mean and standard deviation of the marks are 6 and 2.9 respectively.

19. The weights of coffee in 70 jars are shown in the following table:

Weight (in grams)	Frequency
200 - 201	13
201 - 202	27
202 - 203	18
203 - 204	10
204 - 205	1
205 - 206	1

Determine variance and standard deviation of the above distribution.

Solution:

Given the weights of coffee in 70 jars

Now we have to find the variance and standard deviation of the distribution

Let us make a table of the given data and append other columns after calculations

Weight (in grams)	Mid-Value (x_i)	Frequency (f_i)	f_ix_i
200 - 201	200.5	13	13×200.5=2606.5
201 - 202	201.5	27	27×201.5=5440.5
202 - 203	202.5	18	18×202.5=3645
203 - 204	203.5	10	10×203.5=2035
204 - 205	204.5	1	1×204.5=204.5
205 - 206	205.5	1	1×205.5=205.5
	Total	N=70	$\sum f_ix_i = 14137$

Here mean, $\overline{x} = \frac{\sum f_i x_i}{N} = \frac{14137}{70} = 201.9$

So the above table with more columns is as shown below,

Weight (in grams)	Mid-Value (x_i)	Frequency (f_i)	$d_i = x_i - \bar{x}$	f_id_i	$f_id_i^2$
200 - 201	200.5	13	200.5-201.9= -1.4	13×-1.4= -18.2	13×-1.4² =25.48
201 - 202	201.5	27	201.5-201.9= -0.4	27×-0.4= -10.8	27×-0.4² =4.32
202 - 203	202.5	18	202.5-201.9= 0.6	18×0.6= 10.8	18×0.6²= 6.48
203 - 204	203.5	10	203.5-201.9= 1.6	10×1.6= 16	10×1.6²= 25.6
204 - 205	204.5	1	204.5-201.9= 2.6	1×2.6= 2.6	1×2.6² =6.76
205 - 206	205.5	1	205.5-201.9= 3.6	1×3.6= 3.6	1×3.6² =12.96
	Total	N=70		Σf_id_i=4	$\Sigma f_id_i^2$ =81.6

And we know standard deviation is

$$\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{n} - \left(\frac{\Sigma f_i d_i}{n}\right)^2}$$

Substituting values from above table, we get

$$\sigma = \sqrt{\frac{81.6}{70} - \left(\frac{4}{70}\right)^2}$$

$$\sigma = \sqrt{1.17 - (0.057)^2}$$

$$\sigma = \sqrt{1.17 - 0.003249} = \sqrt{1.17}$$

$\Rightarrow \sigma$=1.08g

And σ^2=1.08^2=1.17g

Hence the variance and standard deviation of the distribution are 1.166g and 1.08 respectively.

20. Determine mean and standard deviation of first n terms of an A.P. whose first term is a and common difference is d.

Solution:

Given first n terms of an A.P. whose first term is a and common difference is d

Now we have to find mean and standard deviation

The given AP in tabular form is as shown below,

x_i	$d_i=x_i-a$	d_i^2
a	0	0
a+d	d	d^2
a+2d	2d	$4d^2$
a+3d	3d	$9d^2$
-	-	-
a+(n-1)d	(n-1)d	$(n-1)^2d^2$

Here we have assumed a as mean.

Given the AP have n terms. And we know the sum of all the terms of AP can be written as,

$$\sum x_i = \frac{n}{2}[2a + (n-1)d]$$

Now we will calculate the actual mean,

$$\bar{x} = \frac{\sum x_i}{n}$$

Substituting the corresponding values, we get

$$\bar{x} = \frac{\frac{n}{2}[2a + (n-1)d]}{n}$$

The above equation can be written as

The above equation can be written as

$$\bar{x} = \frac{[2a + (n-1)d]}{2}$$

$$\bar{x} = a + \frac{[(n-1)d]}{2}$$

$$\bar{x} = a + \frac{(n-1)}{2}d$$

Now we other two columns sums, i.e.,

$$\sum d_i = \sum (x_i - a) = d[1 + 2 + 3 + \cdots + (n-1)] = d\left(\frac{n(n-1)}{2}\right)$$

$$\sum d_i^2 = \sum (x_i - a)^2 = d^2[1^2 + 2^2 + 3^2 + \cdots + (n-1)^2] = d^2\left(\frac{n(n-1)(2n-1)}{6}\right)$$

Now we know standard deviation is given by,

$$\sigma = \sqrt{\frac{\sum(x_i - a)^2}{n} - \left(\frac{\sum(x_i - a)}{n}\right)^2}$$

Substituting the corresponding values, we get

$$\sigma = \sqrt{\frac{d^2\left(\frac{n(n-1)(2n-1)}{6}\right)}{n} - \left(\frac{d\left(\frac{n(n-1)}{2}\right)}{n}\right)^2}$$

$$\sigma = \sqrt{d^2\left(\frac{n(n-1)(2n-1)}{6n}\right) - d^2\left(\frac{n^2(n-1)^2}{4n^2}\right)}$$

Cancelling the like terms, we get

$$\sigma = \sqrt{d^2\left(\frac{(n-1)(2n-1)}{6}\right) - d^2\left(\frac{(n-1)^2}{4}\right)}$$

Taking out common terms we get

$$\sigma = \sqrt{\left(\frac{d^2(n-1)}{2}\right)\left(\frac{(2n-1)}{3} - \frac{n-1}{2}\right)}$$

By taking the LCM, we get

$$\sigma = \sqrt{\left(\frac{d^2(n-1)}{2}\right)\left(\frac{2(2n-1) - 3(n-1)}{6}\right)}$$

$$\sigma = \sqrt{\left(\frac{d^2(n-1)}{2}\right)\left(\frac{4n - 2 - 3n + 3}{6}\right)}$$

$$\sigma = \sqrt{\left(\frac{d^2(n-1)}{2}\right)\left(\frac{n+1}{6}\right)}$$

$$\sigma = d\sqrt{\frac{(n^2 - 1)}{12}}$$

Hence the mean and standard deviation of the given AP is

$a + \frac{(n-1)}{2}d$ and $d\sqrt{\frac{(n^2-1)}{12}}$ respectively.

21. Following are the marks obtained, out of 100, by two students Ravi and Hashina in 10 tests.

Ravi	25	50	45	30	70	42	36	48	35	60
Hashina	10	70	50	20	95	55	42	60	48	80

Who is more intelligent and who is more consistent?

Solution:

Given the marks obtained, out of 100, by two students Ravi and Hashina in 10 tests

Now we have to find who is more intelligent and who is more consistent

Case 1: For Ravi

The marks of Ravi being taken separately and finding other values can be tabulated as shown below,

x_i	$d_i=x_i-45$	d_i^2
25	25-45=-20	$-20^2=400$
50	50-45=5	$5^2=25$
45	45-45=0	0
30	30-45=-15	$-15^2=225$
70	70-45=25	$25^2=625$
42	42-45=-3	$-3^2=9$
36	36-45=-9	$-9^2=81$
48	48-45=3	$3^2=9$
35	35-45=-10	$-10^2=100$
60	60-45=15	$15^2=225$
$\Sigma x_i=441$	$\Sigma(x_i-45)=-9$	$\Sigma(x_i-45)^2=1699$

Here we have assumed 45 as mean.

Total there are marks of 10 subjects.

Now we know standard deviation is given by,

$$\sigma = \sqrt{\frac{\sum(x_i - a)^2}{n} - \left(\frac{\sum(x_i - a)}{n}\right)^2}$$

Substituting the corresponding values, we get

$$\sigma = \sqrt{\frac{1699}{10} - \left(\frac{-9}{10}\right)^2}$$

$$\sigma = \sqrt{169.9 - 0.81}$$

$\sigma = \sqrt{169.09}$

σ=13

And mean is

$$\bar{x} = A + \frac{\sum d_i}{N} = 45 - \frac{9}{10} = 44.1$$

Hence the mean and standard deviation of the Ravi is 44.1 and 13 respectively.

Case 1: For Hashina

The marks of Hashina being taken separately and finding other values can be tabulated as shown below,

x_i	$d_i=x_i-53$	d_i^2
10	10-53=-43	$-43^2=1849$
70	70-53=17	$17^2=289$
50	50-53=-3	$-3^2=9$
20	20-53=-33	$-33^2=1089$
95	95-53=42	$42^2=1764$
55	55-53=2	$2^2=4$
42	42-53=-11	$-11^2=121$
60	60-53=7	$7^2=49$
48	48-53=-5	$-5^2=25$
80	80-53=27	$27^2=729$
$\Sigma x_i=530$	$\Sigma(x_i-53)=0$	$\Sigma(x_i-53)^2=5928$

Here as $\frac{530}{10} = 53,$ so 53 is mean.

Total there are marks of 10 subjects.

Now we know standard deviation is given by,

$$\sigma = \sqrt{\frac{\sum(x_i - a)^2}{n} - \left(\frac{\sum(x_i - a)}{n}\right)^2}$$

Substituting the corresponding values, we get

$$\sigma = \sqrt{\frac{5928}{10} - \left(\frac{0}{10}\right)^2}$$

$\sigma = \sqrt{592.8}$

σ=24.35

Hence the mean and standard deviation of the Hasina is 53 and 24.35 respectively.
Now we will analyze them,
For Ravi,
$C.V = \frac{\sigma}{\bar{x}} \times 100 = \frac{13}{44.1} \times 100 = 29.48$
For Hasina,
$C.V = \frac{\sigma}{\bar{x}} \times 100 = \frac{24.35}{53} \times 100 = 45.94$
Now as CV (of Ravi) < CV of Hashina
Hence Ravi is more consistent.
Mean of Hashina > Mean of Ravi,
Hence Hashina is more intelligent.

22. Mean and standard deviation of 100 observations were found to be 40 and 10, respectively. If at the time of calculation two observations were wrongly taken as 30 and 70 in place of 3 and 27 respectively, find the correct standard deviation.

Solution:

Given mean and standard deviation of 100 observations were found to be 40 and 10, respectively. If at the time of calculation two observations were wrongly taken as 30 and 70 in place of 3 and 27 respectively
Now we have to find the correct standard deviation.
As per given criteria,
Number of observations, n=100
Mean of the given observations before correction, $\bar{x} = 40$
But we know,
$$\bar{x} = \frac{\sum x_i}{n}$$
Substituting the corresponding values, we get
$$40 = \frac{\sum x_i}{100}$$
$\Rightarrow \sum x_i = 40 \times 100 = 4000$
It is said two observations were wrongly taken as 30 and 70 in place of 3 and 27 respectively,
So $\sum x_i = 4000 - 30 - 70 + 3 + 27 = 3930$
So the correct mean after correction is

$$\bar{x} = \frac{\sum x_i}{n} = \frac{3930}{100} = 39.3$$

Also given the standard deviation of the 100 observations is 10 before correctior

i.e., σ=10

But we know

$$\sigma = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

Substituting the corresponding values, we get

$$10 = \sqrt{\frac{\sum x_i^2}{100} - \left(\frac{4000}{100}\right)^2}$$

Now taking square on both sides, we get

$$10^2 = \frac{\sum x_i^2}{100} - (40)^2$$

$$\Rightarrow 100 = \frac{\sum x_i^2}{100} - 1600$$

$$\Rightarrow 100 + 1600 = \frac{\sum x_i^2}{100}$$

$$\Rightarrow \frac{\sum x_i^2}{100} = 1700$$

$$\Rightarrow \sum x_i^2 = 170000$$

It is said two observations were wrongly taken as 30 and 70 in place of 3 and 27 respectively, so correction is

$$\Rightarrow \sum x_i^2 = 170000 - (30)^2 - (70)^2 + 3^2 + (27)^2$$

$$\Rightarrow \sum x_i^2 = 170000 - 900 - 4900 + 9 + 729 = 164938$$

$$\Rightarrow \sum x_i^2 = 164938$$

So the correct standard deviation after correction is

$$\sigma = \sqrt{\frac{164938}{100} - \left(\frac{3930}{100}\right)^2}$$

$$\sigma = \sqrt{1649.38 - (39.3)^2}$$

$$\sigma = \sqrt{1649.38 - 1544.49} = \sqrt{104.89}$$

σ=10.24

Hence the corrected standard deviation is 10.24.

23. While calculating the mean and variance of 10 readings, a student wrongly used the reading 52 for the correct reading 25. He obtained the mean and variance as 45 and 16 respectively. Find the correct mean and the variance.

Solution:

Given while calculating the mean and variance of 10 readings, a student wrongly used the reading 52 for the correct reading 25. He obtained the mean and variance as 45 and 16 respectively

Now we have to find the correct mean and the variance.

As per given criteria,

Number of reading, n=10

Mean of the given readings before correction, $\overline{x} = 45$

But we know,

$$\overline{x} = \frac{\sum x_i}{n}$$

Substituting the corresponding values, we get

$$45 = \frac{\sum x_i}{10}$$

$\Rightarrow \sum x_i = 45 \times 10 = 450$

It is said one reading 25 was wrongly taken as 52,

So $\sum x_i = 450 - 52 + 25 = 423$

So the correct mean after correction is

$$\overline{x} = \frac{\sum x_i}{n} = \frac{423}{10} = 42.3$$

Also given the variance of the 10 readings is 16 before correction,

i.e., $\sigma^2 = 16$

But we know

$$\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$$

Substituting the corresponding values, we get

$$16 = \frac{\sum x_i^2}{10} - (45)^2$$

$$\Rightarrow 16 = \frac{\sum x_i^2}{10} - 2025$$

$$\Rightarrow 16 + 2025 = \frac{\sum x_i^2}{10}$$

$$\Rightarrow \frac{\sum x_i^2}{10} = 2041$$

$$\Rightarrow \sum x_i^2 = 20410$$

It is said one reading 25 was wrongly taken as 52, so

$$\Rightarrow \sum x_i^2 = 20410 - (52)^2 + (25)^2$$

$$\Rightarrow \sum x_i^2 = 20410 - 2704 + 625$$

$$\Rightarrow \sum x_i^2 = 18331$$

So the correct variance after correction is

$$\sigma^2 = \frac{18331}{10} - \left(\frac{423}{10}\right)^2$$

σ^2=1833.1-(42.3)^2=1833.1-1789.29

σ^2=43.81

Hence the corrected mean and variance is 42.3 and 43.81 respectively.