EXERCISE PAGE NO: 296

SHORT ANSWER TYPE:

1. If the letters of the word ALGORITHM are arranged at random in a row what is the probability the letters GOR must remain together as a unit?

Solution:

Given word is ALGORITHM

⇒ Total number of letters in algorithm = 9

∴ Total number of words = 9!

So, n(S) = 9!

If 'GOR' remain together, then we consider it as one group.

∴ Number of letters = 7

Number of words, if 'GOR' remain together in the order = 7!

So, n (E) = 7!

$$\text{Required Probability} = \frac{\text{Number of favourable outcome}}{\text{Total number of outcomes}}$$

$$= \frac{n(E)}{n(S)}$$

$$= \frac{7!}{9!} \; [\because n! = n \times (n-1) \times (n-2)\ldots1]$$

$$= \frac{7!}{9 \times 8 \times 7!} = \frac{1}{72}$$

2. Six new employees, two of whom are married to each other, are to be assigned six desks that are lined up in a row. If the assignment of employees to desks is made randomly, what is the probability that the married couple will have nonadjacent desks?

Solution:

Total new employees = 6

So, they can be arranged in 6! Ways

∴ n (S) = 6! = 6 × 5 × 4 × 3 × 2 × 1 = 720

Two adjacent desks for married couple can be selected in 5 ways i.e. (1, 2), (2, 3), (3, 4), (4, 5), (5, 6)

Married couple can be arranged in the two desks in 2! Ways

Other four persons can be arranged in 4! Ways

So, number of ways in which married couple occupy adjacent desks

= 5 × 2! × 4!

= 5 × 2 × 1 × 4 × 3 × 2 × 1

= 240

So, number of ways in which married couple occupy non – adjacent desks = 6! – 240

= (6 × 5 × 4 × 3 × 2 × 1) – 240

= 720 – 240

= 480 = n (E)

$$\text{Required Probability} = \frac{\text{Number of favourable outcome}}{\text{Total number of outcomes}}$$

$$= \frac{n(E)}{n(S)} = \frac{480}{720}$$

$$= \frac{2}{3}$$

3. Suppose an integer from 1 through 1000 is chosen at random, find the probability that the integer is a multiple of 2 or a multiple of 9.

Solution:

We have integers 1, 2, 3 … 1000

∴ Total number of outcomes, n(S) = 1000

Number of integers which are multiple of 2 are 2, 4, 6, 8, 10, … 1000

Let p be the number of terms

We know that, $a_p = a + (p – 1) d$

Here, a = 2, d = 2 and $a_p = 1000$

Putting the value, we get

2 + (p – 1)2 = 1000

⇒ 2 + 2p – 2 = 1000

p = 1000/2

⇒ p = 500

Total number of integers which are multiple of 2 = 500

Let the number of integers which are multiple of 9 be n.

Number which are multiples of 9 are 9, 18, 27, …999
∴ n^{th} term = 999
We know that, $a_n = a + (n – 1) d$
Here, a = 9, d = 9 and a_n = 999
Putting the value, we get
9 + (n – 1)9 = 999
⇒ 9 + 9n – 9 = 999
n = 999/9
⇒ n = 111
So, the number of multiples of 9 from 1 to 1000 is 111.
The multiple of 2 and 9 both are 18, 36, … 990.
Let m be the number of terms in above series.
∴ m^{th} term = 990
We know that, $a_m = a + (m – 1) d$
Here, a = 9 and d = 9
Putting the value, we get
18 + (m – 1)18 = 990
⇒ 18 + 18m – 18 = 990
m = 990/18
⇒ m = 55
Number of multiples of 2 or 9
= No. of multiples of 2 + no. of multiples of 9 – No. of multiples of 2 and 9 both
= 500 + 111 – 55
= 556 = n (E)

$$\text{Required Probability} = \frac{\text{Number of favourable outcome}}{\text{Total number of outcomes}}$$

$$= \frac{n(E)}{n(S)}$$

$$= \frac{556}{1000}$$

$$= 0.556$$

4. An experiment consists of rolling a die until a 2 appears.
(i) How many elements of the sample space correspond to the event that the 2 appears on the k^{th} roll of the die?
(ii) How many elements of the sample space correspond to the event that the 2 appears not later than the k^{th} roll of the die?

Solution:

Given number of outcomes when die is thrown = 6

(i) Given that 2 appears on the k^{th} roll of the die.

So, first $(k – 1)^{th}$ roll have 5 outcomes each and k^{th} roll results 2

∴ Number of outcomes = 5^{k-1}

(ii) If we consider that 2 appears not later than k^{th} roll of the die, then 2 comes before k^{th} roll.

If 2 appears in first roll, number of ways = 1 outcome

If 2 appears in second roll, number of ways = 5 x 1 (as first roll does not result in 2)

If 2 appears in third roll, number of ways = 5 x 5 x 1 (as first two rolls do not result in 2)

Similarly, if 2 appears in $(k – 1)^{th}$ roll, number of ways

= (5 x 5 x 5 … (k- 1) times) x 1

= 5^{k-1}

Possible outcomes if 2 appears before k^{th} roll

= $1 + 5 + 5^2 + 5^3 + … + 5^{k-1}$

Here, we get the series

We know that,

$$S_n = \begin{cases} \dfrac{a(r^n - 1)}{r - 1} & \text{if } r > 1 \\ \dfrac{a(1 - r^n)}{1 - r} & \text{if } r < 1 \end{cases}$$

So, here a = 1

and $r = \frac{5}{1} = 5 > 1$

Hence,

$$= \frac{1 \times (5^k - 1)}{5 - 1}$$

$$= \frac{5^k - 1}{4}$$

5. A die is loaded in such a way that each odd number is twice as likely to occur as each even number. Find P(G), where G is the event that a number greater than 3 occurs on a single roll of the die.

Solution:

Given that probability of odd numbers

= 2 × (Probability of even number)
⇒ P (Odd) = 2 × P (Even)
Now, P (Odd) + P (Even) = 1
⇒ 2 P (Even) + P (Even) = 1
⇒ 3 P (Even) = 1
P (Even) = 1/3
So,

$$P(\text{Odd}) = 1 - \frac{1}{3} = \frac{3-1}{3} = \frac{2}{3}$$

Now, Total number occurs on a single roll of die = 6
And the number greater than 3 = 4, 5 or 6
So, P (G) = P (number greater than 3)
= P (number is 4, 5 or 6)
Here, 4 and 6 are even numbers and 5 is odd
∴ P (G) = 2 × P (Even) × P (Odd)
= 2 × 1/3 × 2/3
= 4/9
Hence, the required probability is 4/9

6. In a large metropolitan area, the probabilities are .87, .36, .30 that a family (randomly chosen for a sample survey) owns a colour television set, a black and white television set, or both kinds of sets. What is the probability that a family owns either anyone or both kinds of sets?

Solution:
E_1 = Event that a family owns colour television
E_2 = Event that the family owns black and white television
Given that P (E_1) = 0.87
P (E_2) = 0.36 and P ($E_1 \cap E_2$) = 0.30
Now, we have to find the probability that a family owns either anyone or both kinds of sets.
By General Addition Rule, we have
P (A ∪ B) = P(A) + P(B) – P (A ∩ B)
∴ P ($E_1 \cup E_2$) = P (E_1) + P (E_2) – P ($E_1 \cap E_2$)
= 0.87 + 0.36 – 0.30
= 1.23 – 0.30
= 0.93
Hence, the required probability is 0.93

7. If A and B are mutually exclusive events, P (A) = 0.35 and P (B) = 0.45, find
(a) P (A')
(b) P (B')
(c) P (A ∪ B)
(d) P (A ∩ B)
(e) P (A ∩ B')
(f) P (A'∩ B')

Solution:
Given that P (A) = 0.35 and P (B) = 0.45
∵ The events A and B are mutually exclusive then P (A ⋂ B) = 0

(a) To find (a) P (A')
We know that,
P (A) + P (A') = 1
⇒ 0.35 + P(A') = 1 [given]
⇒ P (A') = 1 – 0.35
⇒ P (A') = 0.65

(b) To find (b) P (B')
We know that,
P (B) + P (B') = 1
⇒ 0.45 + P (B') = 1
⇒ P (B') = 1 – 0.45
⇒ P (B') = 0.55

(c) To find (c) P (A ⋃ B)
We know that,
P (A ∪ B) = P (A) + P (B) – P (A ∩ B)
⇒ P (A ⋃ B) = 0.35 + 0.45 – 0 [given]
⇒ P (A ⋃ B) = 0.80

(d) To find (d) P (A ⋂ B)
It is given that A and B are mutually exclusive events.
∴ P (A ⋂ B) = 0

(e) To find (e) P (A ⋂ B')
P (A ⋂ B') = P (A) – P (A ⋂ B)
= 0.35 – 0
= 0.35

(f) To find (f) P (A' ⋂ B')
P (A' ⋂ B') = P (A ⋃ B)'
= 1 – P (A ⋃ B)
= 1 – 0.8 [from part (c)]
= 0.2

8. A team of medical students doing their internship have to assist during surgeries at a city hospital. The probabilities of surgeries rated as very complex, complex, routine, simple or very simple are respectively, 0.15, 0.20, 0.31, 0.26, .08. Find the probabilities that a particular surgery will be rated.
(a) complex or very complex;
(b) neither very complex nor very simple;
(c) routine or complex
(d) routine or simple

Solution:
Let
E_1 = event that surgeries are rated as very complex
E_2 = event that surgeries are rated as complex
E_3 = event that surgeries are rated as routine
E_4 = event that surgeries are rated as simple
E_5 = event that surgeries are rated as very simple
Given: P (E_1) = 0.15, P (E_2) = 0.20, P (E_3) = 0.31, P (E_4) = 0.26, P (E_5) = 0.08
(a) P (complex or very complex) = P (E_1 or E_2) = P ($E_1 \cup E_2$)
By General Addition Rule:
P (A ∪ B) = P(A) + P(B) – P (A ∩ B)
⇒ P ($E_1 \cup E_2$) = P (E_1) + P (E_2) – P ($E_1 \cap E_2$)
= 0.15 + 0.20 – 0 [given]
[∵ All events are independent]
= 0.35

(b) P (neither very complex nor very simple) = P ($E_1' \cap E_5'$)

$= P(E_1 \cup E_5)'$
$= 1 - P(E_1 \cup E_5)$
[∵By Complement Rule]
$= 1 - [P(E_1) + P(E_5) - P(E_1 \cap E_5)]$
[∵ By General Addition Rule]
$= 1 - [0.15 + 0.08 - 0]$
$= 1 - 0.23$
$= 0.77$

(c) P (routine or complex) $= P(E_3 \cup E_2)$
$= P(E_3) + P(E_2) - P(E_3 \cap E_2)$
[∵ By General Addition Rule]
$= 0.31 + 0.20 - 0$ [given]
$= 0.51$

(d) P (routine or simple) $= P(E_3 \cup E_4)$
$= P(E_3) + P(E_4) - P(E_3 \cap E_4)$
[∵ By General Addition Rule]
$= 0.31 + 0.26 - 0$ [given]
$= 0.57$

9. Four candidates A, B, C, D have applied for the assignment to coach a school cricket team. If A is twice as likely to be selected as B, and B and C are given about the same chance of being selected, while C is twice as likely to be selected as D, what are the probabilities that
(a) C will be selected?
(b) A will not be selected?

Solution:
Given that A is twice as likely to be selected as B
i.e. P (A) = 2 P (B) ……1
and C is twice as likely to be selected as D
i.e. P (C) = 2 P (D) ……2
Now, B and C are given about the same chance
∴ P (B) = P (C) ……3
Since, sum of all probabilities = 1
∴ P (A) + P (B) + P (C) + P (D) = 1

⇒ P (A) + P (B) + P (B) + P (D) = 1 [from 3]

$\Rightarrow P(A) + \frac{P(A)}{2} + \frac{P(A)}{2} + \frac{P(C)}{2} = 1$[from 1 & 2]

$\Rightarrow \frac{2P(A)+P(A)+P(A)+P(B)}{2} = 1$[from 3]

$\Rightarrow 4P(A) + \frac{P(A)}{2} = 2$[from 1]

$\Rightarrow \frac{8P(A) + P(A)}{2} = 2$

⇒ 9P(A) = 4

$\Rightarrow P(A) = \frac{4}{9}$

(a) P (C will be selected) = P(C)

= P (B) [from 3]

$= \frac{P(A)}{2}$ [from 1]

$= \frac{4}{9} \times \frac{1}{2} = \frac{2}{9}$

(b) P (A will not be selected) = P(A')

By compliment rule

= 1 – P(A)

$= 1 - \frac{4}{9}$

= 5/9

10. One of the four persons John, Rita, Aslam or Gurpreet will be promoted next month. Consequently, the sample space consists of four elementary outcomes S = {John promoted, Rita promoted, Aslam promoted, Gurpreet promoted} You are told that the chances of John's promotion is same as that of Gurpreet, Rita's chances of promotion are twice as likely as Johns. Aslam's chances are four times that of John.

(a) Determine P (John promoted)

P (Rita promoted)

P (Aslam promoted)

P (Gurpreet promoted)

(b) If A = {John promoted or Gurpreet promoted}, find P (A).

Solution:

Given Sample Space, S = John promoted, Rita promoted, Aslam promoted, Gurpreet promoted

Let E_1 = events that John promoted

E_2 = events that Rita promoted

E_3 = events that Aslam promoted

E_4 = events that Gurpreet promoted

It is given that chances of John's promotion is same as that of Gurpreet

$P(E_1) = P(E_4)$ ……1

It is given that Rita's chances of promotion are twice as likely as John

$P(E_2) = 2P(E_1)$ ……2

and Aslam's chances of promotion are four times that of John

$P(E_3) = 4P(E_1)$ ……3

Since, sum of all probabilities = 1

$\Rightarrow P(E_1) + P(E_2) + P(E_3) + P(E_4) = 1$

$\Rightarrow P(E_1) + 2P(E_1) + 4P(E_1) + P(E_1) = 1$

$\Rightarrow 8P(E_1) = 1$

$\Rightarrow P(E_1) = 1/8$ ….4

(a) P (John promoted) = $P(E_1)$

$= \frac{1}{8}$[from (iv)]

P (Rita promoted) = $P(E_2)$

From 2 we have

$= 2P(E_1)$

From 4

$= 2 \times \frac{1}{8}$

$= \frac{1}{4}$

P (Aslam promoted) = $P(E_3)$

From 3 we have

$= 4P(E_1)$

From 4 we can write as

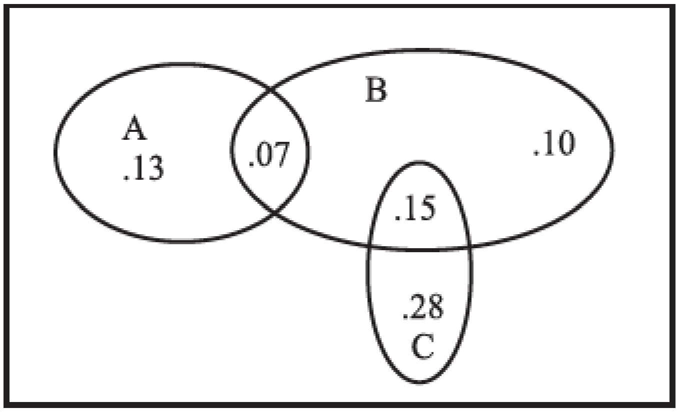
$= 4 \times \frac{1}{8}$

= ½

P (Gurpreet promoted) = P(E_4)
From 1
= P(E_1)
$= \frac{1}{8}$

(b) Given A = (John promoted or Gurpreet promoted)
∴, A = $E_1 \cup E_4$
P (A) = P ($E_1 \cup E_4$)
By general rule of addition, we have
= P (E_1) + P (E_4) – P ($E_1 \cap E_4$)
= P(E_1) + P(E_1) – 0 [from (i)]
$= \frac{1}{8} + \frac{1}{8}$
$= \frac{2}{8}$
$= \frac{1}{4}$

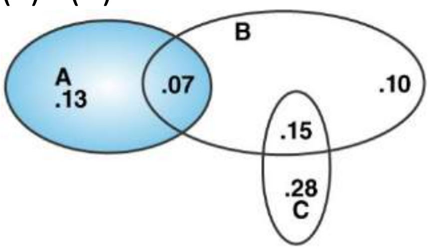
11. The accompanying Venn diagram shows three events, A, B, and C, and also the probabilities of the various intersections (for instance, P (A ∩ B) = .07. Determine



(a) P (A)
(b) $P(B \cap \bar{C})$
(c) P (A ∪ B)
(d) P (A ∩ B)
(e) P (B ∩ C)

(f) Probability of exactly one of the three occurs.

Solution:
Given P (A ⋂ B) = 0.07
From the given Venn Diagram
(a) P(A)

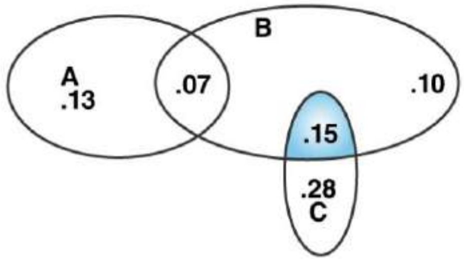


(b) $P(B \cap \bar{C})$
$P(B \cap \bar{C}) = P(B) - P(B \cap C)$
Substituting the values, we get
$= 0.07 + 0.10 + 0.15 - 0.15$
$= 0.07 + 0.10$
$= 0.17$
$P(B \cap \bar{C}) = 0.17$

(c) P (A ⋃ B)
By General Addition Rule,
P (A ⋃ B) = P (A) + P (B) – P (A ⋂ B)
Now substituting the values, we get
⇒ P (A ⋃ B) = 0.20 + (0.07 + 0.10 + 0.15) – 0.07
⇒ P (A ⋃ B) = 0.20 + 0.25
⇒ P (A ⋃ B) = 0.45

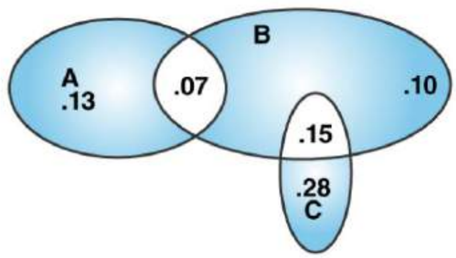
(d) $P(A \cap \bar{B})$
We know that,
$P(A \cap \bar{B}) = P(A) - P(A \cap B)$
From part (a) we can write as
$= 0.20 - 0.07$
$= 0.13$

(e) P (B ⋂ C)



From the Venn diagram we have
P (B ∩ C) = 0.15

(f) Probability of exactly one of the three occurs



P (exactly one of the three occurs) = 0.13 + 0.10 + 0.28
= 0.51

LONG ANSWER TYPE:

12. One urn contains two black balls (labelled B1 and B2) and one white ball. A second urn contains one black ball and two white balls (labelled W1 and W2). Suppose the following experiment is performed. One of the two urns is chosen at random. Next a ball is randomly chosen from the urn. Then a second ball is chosen at random from the same urn without replacing the first ball.
(a) Write the sample space showing all possible outcomes
(b) What is the probability that two black balls are chosen?
(c) What is the probability that two balls of opposite colour are chosen?

Solution:
Given that one urn contains two black balls and one white ball
and second urn contains one black ball and two white balls.
It is also given that one of the two urns is chosen, then a ball is randomly chosen from the urn, then second ball is chosen at random from the same urn without replacing the first ball

(a) Sample Space S = {B_1B_2, B_1W, B_2W, B_2B_1, WB_1, WB_2, W_1W_2, W_1B, W_2B, W_2W_1, BW_1, BW_2}
Total number of sample space = 12

(b) If two black balls are chosen
Total outcomes = 12
Favourable outcomes are B_1B_2, B_2B_1
∴ Total favourable outcomes = 2
We know that,

$$Probability = \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$$

$$\therefore \text{Required Probability} = \frac{2}{12} = \frac{1}{6}$$

(c) If two balls of opposite colours are chosen
Favourable outcomes are B_1W, B_2W, WB_1, WB_2, W_1B, W_2B, BW_1, BW_2
∴ Total favourable outcomes = 8 and total outcomes = 12
We know that,

$$Probability = \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$$

$$\therefore \text{Required Probability} = \frac{8}{12} = \frac{2}{3}$$

13. A bag contains 8 red and 5 white balls. Three balls are drawn at random. Find the Probability that
(a) All the three balls are white
(b) All the three balls are red
(c) One ball is red and two balls are white

Solution:
Given that, number of red balls = 8
Number of white balls = 5
∴ Total balls, n = 13
It is given that 3 balls are drawn at random
⇒ r = 3
∴ $n(S) = {}^nC_r = {}^{13}C_3$
(a) All the three balls are white

We know that,

$$P(A) = \frac{n(A)}{n(S)} = \frac{Number\ of\ favourable\ outcomes}{Sample\ Space}$$

Total white balls are = 5

$$\text{P (all the three balls are white)} = \frac{{}^5C_3}{{}^{13}C_3}$$

$$= \frac{\frac{5!}{3!\,(5-3)!}}{\frac{13!}{3!\,(13-3)!}} \left[\because\ {}^nC_r = \frac{n!}{(n-r)!\,r!}\right]$$

$$= \frac{\frac{5 \times 4 \times 3!}{3!\,2!}}{\frac{13 \times 12 \times 11 \times 10!}{3 \times 2 \times 1 \times 10!}}$$

$$= \frac{\frac{5 \times 4}{2 \times 1}}{13 \times 2 \times 11}$$

$$= \frac{5}{143}$$

(b) All the three balls are red

We know that,

$$P(A) = \frac{n(A)}{n(S)} = \frac{Number\ of\ favourable\ outcomes}{Sample\ Space}$$

Total red balls are = 8

$$\text{P (all the three balls are red)} = \frac{{}^8C_3}{{}^{13}C_3}$$

$$= \frac{\frac{8!}{3!\,(8-3)!}}{\frac{13!}{3!\,(13-3)!}} \left[\because\ {}^nC_r = \frac{n!}{(n-r)!\,r!}\right]$$

$$= \frac{\frac{8 \times 7 \times 6 \times 5!}{3!\,5!}}{\frac{13 \times 12 \times 11 \times 10!}{3 \times 2 \times 1 \times 10!}}$$

$$= \frac{\frac{8 \times 7 \times 6}{3 \times 2}}{13 \times 2 \times 11}$$
$$= \frac{28}{143}$$

(c) One ball is red and two balls are white

We know that,

$$P(A) = \frac{n(A)}{n(S)} = \frac{Number\ of\ favourable\ outcomes}{Sample\ Space}$$

$$\text{P(one ball is red and two balls are white)} = \frac{{}^{8}C_{1} \times {}^{5}C_{2}}{{}^{13}C_{3}}$$

$$= \frac{\frac{8!}{1!\,(8-1)!} \times \frac{5!}{2!\,(5-2)!}}{\frac{13!}{3!\,(13-3)!}}$$

$$\left[\because {}^{n}C_{r} = \frac{n!}{(n-r)!\,r!}\right]$$

$$= \frac{\frac{8 \times 7!}{7!} \times \frac{5 \times 4 \times 3!}{2 \times 1 \times 3!}}{\frac{13 \times 12 \times 11 \times 10!}{3 \times 2 \times 1 \times 10!}}$$

$$= \frac{8 \times 10}{13 \times 2 \times 11}$$

$$= \frac{40}{143}$$

14. If the letters of the word ASSASSINATION are arranged at random. Find the Probability that

(a) Four S's come consecutively in the word

(b) Two I's and two N's come together

(c) All A's are not coming together

(d) No two A's are coming together.

Solution:

Given word is ASSASSINATION

Total number of letters in ASSASSINATION is 13

In word ASSASSINATION, there are 3A's, 4S's, 2I's, 1T's, 2N's and 1O's.

Total number of ways these letters can be arranged =

$n(S) = \frac{13!}{3!4!2!2!}$

(a) Four S's come consecutively in the word

If 4S's come consecutively then word ASSASSINATION become.

So, now numbers of letters is 1 + 9 = 10

$$\therefore n(E) = \frac{10!}{3!\,2!\,2!}$$

$$\therefore \text{Required Probability} = \frac{\frac{10!}{3!\,2!\,2!}}{\frac{13!}{3!\,4!\,2!\,2!}}$$

$$= \frac{10!}{3!\,2!\,2!} \times \frac{3!\,4!\,2!\,2!}{13!}$$

The above equation can be written as

$$= \frac{10! \times 4!}{13 \times 12 \times 11 \times 10!}$$

$$= \frac{4 \times 3 \times 2 \times 1}{13 \times 12 \times 11}$$

On simplifying we get

$$= \frac{2}{143}$$

(b) Two I's and two N's come together

So, now numbers of letters is 1 + 9 = 10

$$\therefore n(E) = \frac{4!}{2!\,2!} \times \frac{10!}{3!\,4!}$$

$$\therefore \text{Required Probability} = \frac{\frac{4!\,10!}{3!\,4!\,2!\,2!}}{\frac{13!}{3!\,4!\,2!\,2!}}$$

The above equation can be written as

$$= \frac{10!\,4!}{3!\,4!\,2!\,2!} \times \frac{3!\,4!\,2!\,2!}{13!}$$

$$= \frac{10! \times 4!}{13 \times 12 \times 11 \times 10!}$$

$$= \frac{4 \times 3 \times 2 \times 1}{13 \times 12 \times 11}$$

On simplifying we get

$$= \frac{2}{143}$$

(c) All A's are not coming together

Firstly, we find the probability that all A's are coming together

If all A's are coming together then

So, now numbers of letters is 1 + 10 = 11

$$\therefore \text{Number of words when all A'scome together} = \frac{11!}{4!\,2!\,2!}$$

$$\therefore \text{Probability when all A's come together} = \frac{\frac{11!}{4!\,2!\,2!}}{\frac{13!}{3!\,4!\,2!\,2!}}$$

$$= \frac{11!}{4!\,2!\,2!} \times \frac{3!\,4!\,2!\,2!}{13!}$$

The above equation can be written as

$$= \frac{11! \times 3!}{13 \times 12 \times 11!}$$

$$= \frac{3 \times 2 \times 1}{13 \times 12}$$

On simplifying we get

$$= \frac{1}{26}$$

Now, P (all A's does not come together) = 1 – P (all A's come together)

$$= 1 - \frac{1}{26}$$

$$= \frac{25}{26}$$

(d) No two A's are coming together

First we arrange the alphabets except A's

$$\therefore \text{Number ofways of arranging all alphabets except A's} = \frac{10!}{4!\,2!\,2!}$$

There are 11 vacant places between these alphabets.

Total A's in the word ASSASSINATION are 3

∴ 3 A's can be placed in 11 place in $^{11}C_3$ ways

$$= \frac{11!}{3!\,(11-3)!}$$
$$= \frac{11!}{3!\,8!}$$
∴ Total number of words when no two A's together
$$= \frac{11!}{3!\,8!} \times \frac{10!}{4!\,2!\,2!}$$
$$\therefore \text{Required Probability} = \frac{\frac{11!}{3!\,8!} \times \frac{10!}{4!\,2!\,2!}}{\frac{13!}{3!\,4!\,2!\,2!}}$$
The above equation can be written as
$$= \frac{11!}{3!\,8!} \times \frac{10!}{4!\,2!\,2!} \times \frac{3!\,4!\,2!\,2!}{13!}$$
$$= \frac{11! \times 10 \times 9 \times 8!}{8! \times 13 \times 12 \times 11!}$$
$$= \frac{10 \times 9}{13 \times 12}$$
On simplifying we get
$$= \frac{15}{26}$$

15. A card is drawn from a deck of 52 cards. Find the probability of getting a king or a heart or a red card.

Solution:

Given total number of playing cards = 52

∴ n(S) = 52

Total number of king cards = 4

Total number of heart cards = 13

Total number of red cards = 13 + 13 = 26

∴ Favourable outcomes = 4 + 13 + 26 – 13 – 2

= 28

We know that,
$$Probability = \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$$

$\therefore$ Required Probability $= \frac{28}{52}$

$= \frac{7}{13}$

16. A sample space consists of 9 elementary outcomes $e_1, e_2, ..., e_9$ whose probabilities are
$P(e_1) = P(e_2) = .08$, $P(e_3) = P(e_4) = P(e_5) = .1$
$P(e_6) = P(e_7) = .2$, $P(e_8) = P(e_9) = .07$
Suppose $A = \{e_1, e_5, e_8\}$, $B = \{e_2, e_5, e_8, e_9\}$
(a) Calculate P (A), P (B), and P (A ∩ B)
(b) Using the addition law of probability, calculate P (A ∪ B)
(c) List the composition of the event A ∪ B, and calculate P (A ∪ B) by adding the probabilities of the elementary outcomes.
(d) Calculate P ($\overline{B}$) from P (B), also calculate P ($\overline{B}$) directly from the elementary outcomes of $\overline{B}$.

Solution:
Given
$S = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9\}$
$A = \{e_1, e_5, e_8\}$ and $B = \{e_2, e_5, e_8, e_9\}$
$P(e_1) = P(e_2) = .08$, $P(e_3) = P(e_4) = P(e_5) = .1$
$P(e_6) = P(e_7) = .2$, $P(e_8) = P(e_9) = .07$
(a) To find P (A), P (B) and P (A ⋂ B)
$A = \{e_1, e_5, e_8\}$
$P(A) = P(e_1) + P(e_5) + P(e_8)$
Substituting the values, we get
$\Rightarrow P(A) = 0.08 + 0.1 + 0.07$
$\Rightarrow P(A) = 0.25$
$B = \{e_2, e_5, e_8, e_9\}$
$P(B) = P(e_2) + P(e_5) + P(e_8) + P(e_9)$
Substituting the values, we get
$\Rightarrow P(B) = 0.08 + 0.1 + 0.07 + 0.07$ [given]
$\Rightarrow P(B) = 0.32$
Now, we have to find P (A ⋂ B)
$A = \{e_1, e_5, e_8\}$ and $B = \{e_2, e_5, e_8, e_9\}$

$\therefore$ A ∩ B = {e_5, e_8}
⇒ P (A ∩ B) = P (e_5) + P (e_8)
= 0.1 + 0.07
= 0.17

(b) To find P (A ∪ B)
By General Addition Rule:
P (A ∪ B) = P (A) + P (B) – P (A ∩ B)
from part (a), we have
P (A) = 0.25, P (B) = 0.32 and P (A ∩ B) = 0.17
Putting the values, we get
P (A ∪ B) = 0.25 + 0.32 – 0.17
= 0.40

(c) A = {e_1, e_5, e_8} and B = {e_2, e_5, e_8, e_9}
$\therefore$ A ∪ B = {e_1, e_2, e_5, e_8, e_9}
⇒ P (A ∪ B) = P (e_1) + P (e_2) + P (e_5) + P (e_8) + P (e_9)
Substituting the values, we get
= 0.08 +0.08 + 0.1 + 0.07 + 0.07
= 0.40

(d) To find $P(\overline{B})$
By Complement Rule, we have
$\mathbf{P(\overline{B}) = 1 - P(B)}$
$\Rightarrow P(\overline{B}) = 1 - 0.32$
= 0.68
Given: B = {e_2, e_5, e_8, e_9}
$\therefore \overline{B} = \{e_1, e_3, e_4, e_6, e_7\}$
$\therefore P(\overline{B}) = P(e_1) + P(e_3) + P(e_4) + P(e_6) + P(e_7)$
By substituting the given values, we get
= 0.08 + 0.1 + 0.1 + 0.2 + 0.2
= 0.68

17. Determine the probability p, for each of the following events.
(a) An odd number appears in a single toss of a fair die.
(b) At least one head appears in two tosses of a fair coin.

(c) A king, 9 of hearts, or 3 of spades appears in drawing a single card from a well shuffled ordinary deck of 52 cards.
(d) The sum of 6 appears in a single toss of a pair of fair dice.

Solution:

(a) When a fair die is thrown, the possible outcomes are
S = {1, 2, 3, 4, 5, 6}
∴ total outcomes = 6 and the odd numbers are 1, 3, 5
∴ Favourable outcomes = 3
We know that,

$$Probability = \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$$

$$\therefore \text{Required probability } = \frac{3}{6} = \frac{1}{2}$$

(b) When a fair coin is tossed two times, the sample space is
S = {HH, HT, TH, TT}
∴ Total outcomes = 4
If at least one head appears then the favourable cases are HH, HT and TH.
∴ Favourable outcomes = 3
We know that,

$$Probability = \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$$

$$\therefore \text{Required probability } = \frac{3}{4}$$

(c) When a pair of dice is rolled, total number of cases
S = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}
Total Sample Space, n(S) = 36
If sum is 6 then possible outcomes are (1,5), (2,4), (3,3), (4,2) and (5,1).
∴ Favourable outcomes = 5

We know that,

$$Probability = \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$$

$\therefore$ Required probability $= \frac{5}{36}$

OBJECTIVE TYPE QUESTIONS:

Choose the correct answer out of four given options in each of the Exercises 18 to 29 (M.C.Q.).

18. In a non-leap year, the probability of having 53 Tuesdays or 53 Wednesdays is
A. 1/7
B. 2/7
C. 3/7
D. none of these

Solution:
B. 2/7

Explanation:
We know that in a non-leap year, there are 365 days and we know that there are 7 days in a week
∴ 365 ÷ 7 = 52 weeks + 1 day
This 1 day can be Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday
∴ Total Outcomes = 7
If this day is a Tuesday or Wednesday, then the year will have 53 Tuesday or 53 Wednesday.
∴P (non-leap year has 53 Tuesdays or 53 Wednesdays) = 1/7 + 1/7 = 2/7
Hence, the correct option is (B).

19. Three numbers are chosen from 1 to 20. Find the probability that they are not consecutive

A. $\frac{186}{190}$

B. $\frac{187}{190}$

C. $\frac{188}{190}$

D. $\frac{18}{{}^{20}C_3}$

Solution:
B. 187/190

Explanation:
Since, the set of three consecutive numbers from 1 to 20 are (1, 2, 3), (2, 3, 4), (3, 4, 5), ..., (18,19,20)
Considering 3 numbers as a single digit
∴ the numbers will be 18
Now, we have to choose 3 numbers out of 20. This can be done in ${}^{20}C_3$ ways
∴ $n(S) = {}^{20}C_3$
The desired event is that the 3 numbers are choose must consecutive. So,

$$P(\text{numbers are consecutive}) = \frac{18}{{}^{20}C_3}$$

$$= \frac{18}{\frac{20!}{3!\,(20-3)!}} \left[\because\ {}^{n}C_{r} = \frac{n!}{(n-r)!\,r!}\right]$$

The above equation can be written as

$$= \frac{18}{\frac{20 \times 19 \times 18 \times 17!}{3 \times 2 \times 1 \times 17!}}$$

$$= \frac{18}{\frac{20 \times 19 \times 18}{6}}$$

$$= \frac{6}{20 \times 19}$$

On simplifying we get

$$= \frac{3}{190}$$

P(three number are not consecutive) $= 1 - \frac{3}{190}$

$= \frac{190-3}{190}$

$= \frac{187}{190}$

Hence, the correct option is (B).

20. While shuffling a pack of 52 playing cards, 2 are accidentally dropped. Find the probability that the missing cards to be of different colours

A. $\frac{29}{52}$

B. $\frac{1}{2}$

C. $\frac{26}{51}$

D. $\frac{27}{51}$

Solution:

C. 26/51

Explanation:

We know that, in a pack of 52 cards 26 are of red colour and 26 are of black colour.

It is given that 2 cards are accidentally dropped

So,

Probability of dropping a red card first $= \frac{26}{52}$

Probability of dropping a red card second $= \frac{26}{51}$

Similarly,

Probability of dropping a black card first $= \frac{26}{52}$

Probability of dropping a black card second $= \frac{26}{51}$

∴ P (both cards of different colour) $= \frac{26}{52} \times \frac{26}{51} + \frac{26}{52} \times \frac{26}{51}$

$= 2 \times \frac{26}{52} \times \frac{26}{51}$

$= \frac{26}{51}$

Hence, the correct option is (C).

21. Seven persons are to be seated in a row. The probability that two particular persons sit next to each other is

A. $\frac{1}{3}$

B. $\frac{1}{6}$

C. $\frac{2}{7}$

D. $\frac{1}{2}$

Solution:

C. 2/7

Explanation:

Given that 7 persons are to be seated in a row.

If two persons sit next to each other, then consider these two persons as 1 group.

Now we have to arrange 6 persons.

∴ Number of arrangement = 2! × 6!

Total number of arrangement of 7 persons = 7!

$$Probability = \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$$

∴ Required probability $= \frac{2! \times 6!}{7!}$

$= \frac{2 \times 1 \times 6!}{7 \times 6!}$

= 2/7

Hence, the correct option is (C)

22. Without repetition of the numbers, four digit numbers are formed with the numbers 0, 2, 3, 5. The probability of such a number divisible by 5 is

A. $\frac{1}{5}$

B. $\frac{4}{5}$

C. $\frac{1}{30}$

D. $\frac{5}{9}$

Solution:

D. 5/9

Explanation:

We have digits 0, 2, 3, 5.

We know that, if unit place digit is '0' or '5' then the number is divisible by 5

If unit place is '0'

Then first three places can be filled in 3! ways = $3 \times 2 \times 1 \times 1 = 6$

If unit place is '5'

Then first place can be filled in two ways and second and third place can be filled in 2! ways = $2 \times 2 \times 1 \times 1 = 4$

∴ Total number of ways = 6 + 4 = 10 = n(E)

Total number of ways of arranging the digits 0, 2, 3, 5 to form 4 – digit numbers without repetition is $3 \times 3 \times 2 \times 1 = 18$

$$Probability = \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$$

$$\therefore \text{Required probability } = \frac{10}{18} = \frac{5}{9}$$

Hence, the correct option is (D).