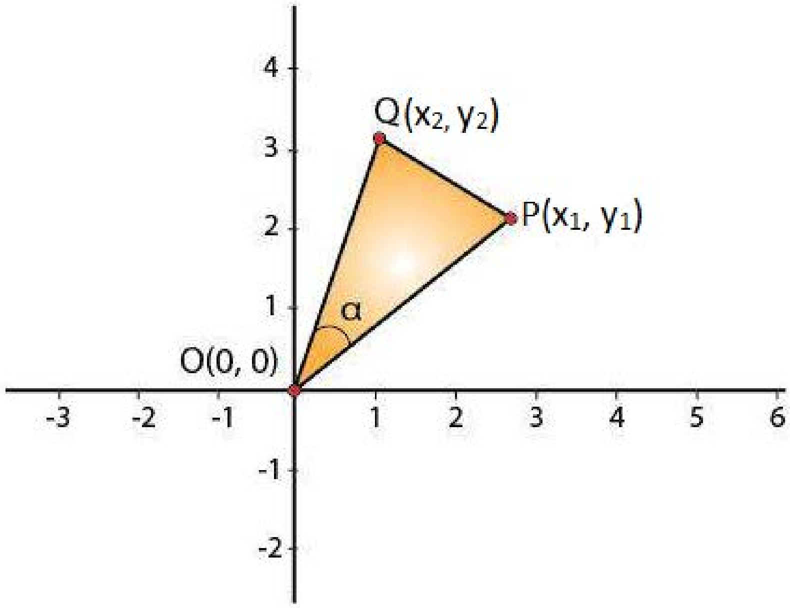
EXERCISE 22.1 PAGE NO: 22.12

1. If the line segment joining the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ subtends an angle α at the origin O, prove that : OP. OQ cos α = $x_1 x_2 + y_1 y_2$.

Solution:

Given,

Two points P and Q subtends an angle α at the origin as shown in figure:



From figure we can see that points O, P and Q forms a triangle.

Clearly in ΔOPQ we have:

$\cos\alpha = \frac{OP^2+OQ^2-PQ^2}{2OP.OQ}$ {from cosine formula}

$2\ OP.OQ\cos\alpha = OP^2 + OQ^2 - PQ^2$ … equation (1)

We know that the, coordinates of O are (0, 0) $\Rightarrow x_2 = 0$ and $y_2 = 0$

Coordinates of P are $(x_1, y_1) \Rightarrow x_1 = x_1$ and $y_1 = y_1$

By using distance formula we have:

$$OP = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(x_1-0)^2+(y_1-0)^2}$$
$$= \sqrt{x_1^2+y_1^2}$$

Similarly, $OQ = \sqrt{(x_2-0)^2+(y_2-0)^2}$
$$= \sqrt{x_2^2+y_2^2}$$

And, $PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\therefore OP^2 + OQ^2 - PQ^2 = x_1^2+y_1^2+\ x_2^2+y_2^2-\{(x_2-x_1)^2+(y_2-y_1)^2\}$

By using $(a-b)^2 = a^2 + b^2 - 2ab$

$\therefore OP^2 + OQ^2 - PQ^2 = 2x_1\,x_2 + 2y_1\,y_2$ …. Equation (2)

So now from equation (1) and (2) we have:

$2OP.\,OQ\cos\alpha = 2x_1x_2+2y_1y_2$

$OP.\,OQ\cos\alpha = x_1x_2+y_1y_2$

Hence Proved.

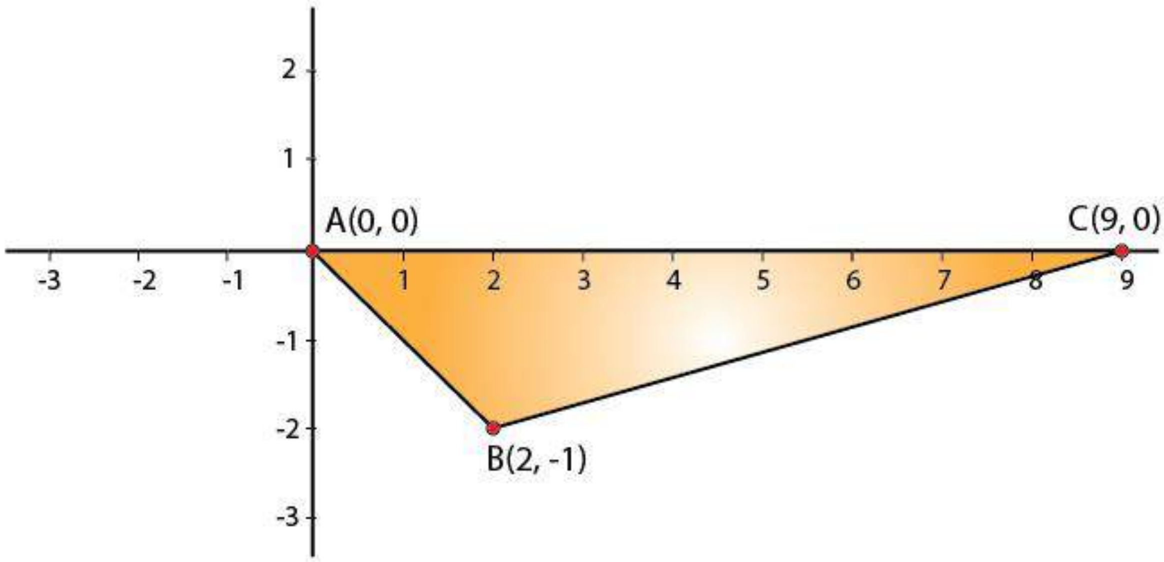
2. The vertices of a triangle ABC are A(0, 0), B (2, -1) and C (9, 0). Find cos B.

Solution:

Given:

The coordinates of triangle.

From the figure,



By using cosine formula,
In ΔABC, we have:

$\cos B = \frac{AB^2+BC^2-AC^2}{2AB.BC}$

Now by using distance formula we have:

$AB = \sqrt{(2-0)^2+(-1-0)^2} = \sqrt{5}$

$BC = \sqrt{(9-2)^2+(0-(-1))^2} = \sqrt{7^2+1^2} = \sqrt{50}$

And, $AC = \sqrt{(9-0)^2+(0-0)^2} = 9$

Now substitute the obtained values in the cosine formula, we get

$\therefore \cos B = \frac{(\sqrt{5})^2+(\sqrt{50})^2-9^2}{2\sqrt{5}\sqrt{50}} = \frac{55-81}{2\sqrt{5}\sqrt{2\times25}} = \frac{-26}{10\sqrt{10}} = \frac{-13}{5\sqrt{10}}$

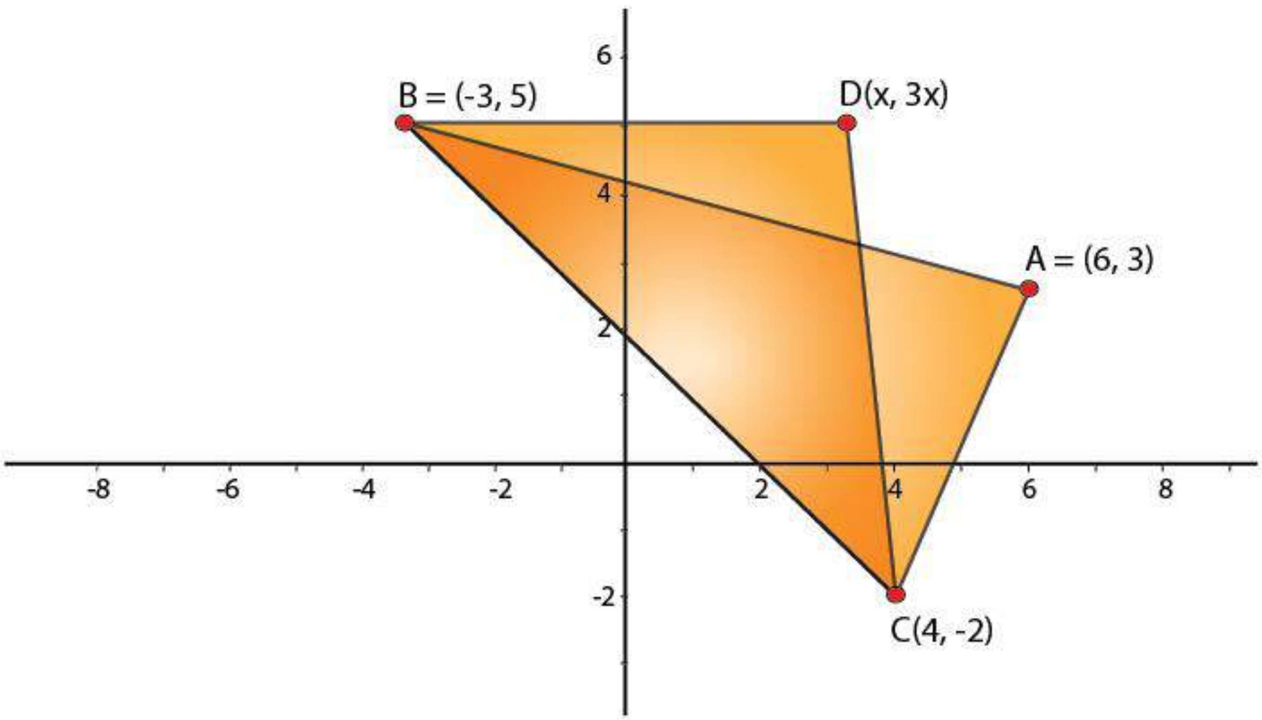
3. Four points A (6, 3), B (-3, 5), C (4, -2) and D (x, 3x) are given in such a way that $\frac{\Delta DBC}{\Delta ABC} = \frac{1}{2}$, find x.

Solution:

Given:

The coordinates of triangle are shown in the below figure.

Also, $\frac{\Delta DBC}{\Delta ABC} = \frac{1}{2}$



Now let us consider Area of a ΔPQR
Where, $P(x_1, y_1)$, $Q(x_2, y_2)$ and $R(x_3, y_3)$ be the 3 vertices of ΔPQR.
So, Area of $(\Delta PQR) = ½ [x_1(y_2 – y_3) + x_2(y_3 – y_1) + x_3(y_1 – y_2)]$

Area of $(\Delta DBC) = \frac{1}{2}[x(5 - (-2)) + (-3)(-2 - 3x) + 4(3x - 5)]$

$= \frac{1}{2}[7x + 6 + 9x + 12x - 20] = 14x - 7$

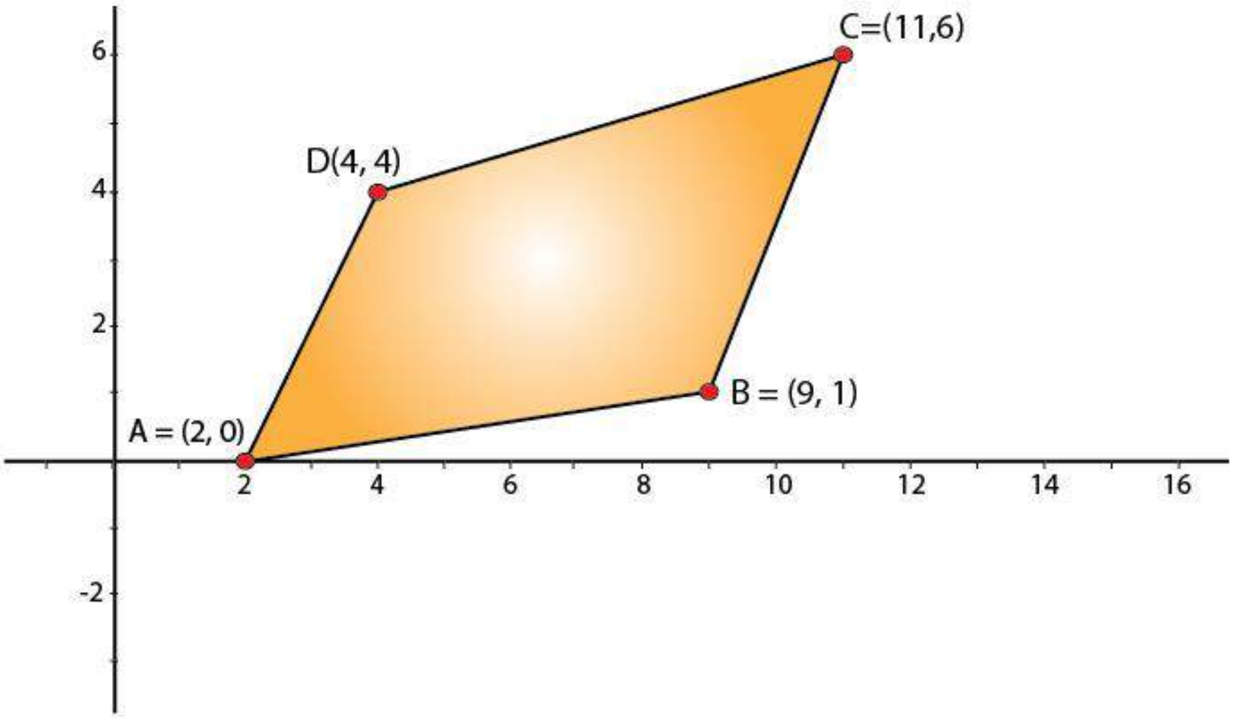
Similarly, area of $(\Delta ABC) = \frac{1}{2}[6(5 - (-2)) + (-3)(-2 - 3) + 4(3 - 5)]$

$= \frac{1}{2}[42 + 15 - 8] = \frac{49}{2} = 24.5$

$\therefore \frac{\Delta DBC}{\Delta ABC} = \frac{1}{2} = \frac{14x-7}{24.5}$

24.5 = 28x – 14
28x = 38.5
x = 38.5/28
= 1.375

4. The points A (2, 0), B (9, 1), C (11, 6) and D (4, 4) are the vertices of a quadrilateral ABCD. Determine whether ABCD is a rhombus or not.
Solution:
Given:
The coordinates of 4 points that form a quadrilateral is shown in the below figure



Now by using distance formula, we have:

AB = $\sqrt{(9-2)^2+(1-0)^2} = \sqrt{7^2+1} = \sqrt{50}$

BC = $\sqrt{(11-9)^2+(6-1)^2} = \sqrt{2^2+5^2} = \sqrt{29}$

It is clear that, AB ≠ BC [quad ABCD does not have all 4 sides equal.]

∴ ABCD is not a Rhombus