EXERCISE 22.3 PAGE NO: 22.21

1. What does the equation $(x – a)^2 + (y – b)^2 = r^2$ become when the axes are transferred to parallel axes through the point (a-c, b)?

Solution:

Given:

The equation, $(x – a)^2 + (y – b)^2 = r^2$

The given equation $(x – a)^2 + (y – b)^2 = r^2$ can be transformed into the new equation by changing x by x – a + c and y by y – b, i.e. substitution of x by x + a and y by y + b.

$((x + a - c) – a)^2 + ((y + b) - b)^2 = r^2$

$(x – c)^2 + y^2 = r^2$

$x^2 + c^2 – 2cx + y^2 = r^2$

$x^2 + y^2 -2cx = r^2 - c^2$

Hence, the transformed equation is $x^2 + y^2 -2cx = r^2 - c^2$

2. What does the equation $(a – b) (x^2 + y^2) – 2abx = 0$ become if the origin is shifted to the point (ab / (a-b), 0) without rotation?

Solution:

Given:

The equation $(a – b) (x^2 + y^2) – 2abx = 0$

The given equation $(a – b) (x^2 + y^2) – 2abx = 0$ can be transformed into new equation by changing x by [X + ab / (a-b)] and y by Y

$$(a-b)\left[\left(X+\frac{ab}{a-b}\right)^2+Y^2\right]-2ab\times\left(X+\frac{ab}{a-b}\right)=0$$

Upon expansion we get,

$$(a-b)\left(X^2+\frac{a^2b^2}{(a-b)^2}+\frac{2abX}{a-b}+Y^2\right)-2abX-\frac{2a^2b^2}{a-b}=0$$

Now let us simplify,

$$(a-b)\left(X^2+Y^2\right)+\frac{a^2b^2}{a-b}+2abX-2abX-\frac{2a^2b^2}{a-b}=0$$

$$(a-b)\left(X^2+Y^2\right)-\frac{a^2b^2}{a-b}=0$$

By taking LCM we get,

$$(a-b)^2\left(X^2+Y^2\right)=a^2b^2$$

Hence, the transformed equation is $(a – b)^2 (X^2 + Y^2) = a^2 b^2$

3. Find what the following equations become when the origin is shifted to the point

(1, 1)?
(i) $x^2 + xy - 3x - y + 2 = 0$
(ii) $x^2 - y^2 - 2x + 2y = 0$
(iii) $xy - x - y + 1 = 0$
(iv) $xy - y^2 - x + y = 0$
Solution:
(i) $x^2 + xy - 3x - y + 2 = 0$
Firstly let us substitute the value of x by x + 1 and y by y + 1
Then,
$(x + 1)^2 + (x + 1) (y + 1) - 3(x + 1) - (y + 1) + 2 = 0$
$x^2 + 1 + 2x + xy + x + y + 1 - 3x - 3 - y - 1 + 2 = 0$
Upon simplification we get,
$x^2 + xy = 0$
∴ The transformed equation is $x^2 + xy = 0$.

(ii) $x^2 - y^2 - 2x + 2y = 0$
Let us substitute the value of x by x + 1 and y by y + 1
Then,
$(x + 1)^2 - (y + 1)^2 - 2(x + 1) + 2(y + 1) = 0$
$x^2 + 1 + 2x - y^2 - 1 - 2y - 2x - 2 + 2y + 2 = 0$
Upon simplification we get,
$x^2 - y^2 = 0$
∴ The transformed equation is $x^2 - y^2 = 0$.

(iii) $xy - x - y + 1 = 0$
Let us substitute the value of x by x + 1 and y by y + 1
Then,
$(x + 1) (y + 1) - (x + 1) - (y + 1) + 1 = 0$
$xy + x + y + 1 - x - 1 - y - 1 + 1 = 0$
Upon simplification we get,
$xy = 0$
∴ The transformed equation is $xy = 0$.

(iv) $xy - y^2 - x + y = 0$
Let us substitute the value of x by x + 1 and y by y + 1
Then,
$(x + 1) (y + 1) - (y + 1)^2 - (x + 1) + (y + 1) = 0$
$xy + x + y + 1 - y^2 - 1 - 2y - x - 1 + y + 1 = 0$
Upon simplification we get,

$xy - y^2 = 0$

∴ The transformed equation is $xy - y^2 = 0$.

4. At what point the origin be shifted so that the equation $x^2 + xy – 3x + 2 = 0$ does not contain any first-degree term and constant term?

Solution:

Given:

The equation $x^2 + xy – 3x + 2 = 0$

We know that the origin has been shifted from (0, 0) to (p, q)

So any arbitrary point (x, y) will also be converted as (x + p, y + q).

The new equation is:

$(x + p)^2 + (x + p)(y + q) – 3(x + p) + 2 = 0$

Upon simplification,

$x^2 + p^2 + 2px + xy + py + qx + pq – 3x – 3p + 2 = 0$

$x^2 + xy + x(2p + q – 3) + y(q – 1) + p^2 + pq – 3p – q + 2 = 0$

For no first degree term, we have $2p + q - 3 = 0$ and $p – 1 = 0$, and

For no constant term we have $p^2 + pq – 3p - q + 2 = 0$.

By solving these simultaneous equations we have p = 1 and q = 1 from first equation.

The values p = 1 and q = 1 satisfies $p^2 + pq – 3p - q + 2 = 0$.

Hence, the point to which origin must be shifted is (p, q) = (1, 1).

5. Verify that the area of the triangle with vertices (2, 3), (5, 7) and (-3 -1) remains invariant under the translation of axes when the origin is shifted to the point (-1, 3).

Solution:

Given:

The points (2, 3), (5, 7), and (-3, -1).

The area of triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is

$= ½ [x_1(y_2 – y_3) + x_2(y_3 - y_1) + x_3(y_1 – y_2)]$

The area of given triangle $= ½ [2(7+1) + 5(-1-3) – 3(3-7)]$

$= ½ [16 – 20 + 12]$

$= ½ [8]$

$= 4$

Origin shifted to point (-1, 3), the new coordinates of the triangle are (3, 0), (6, 4), and (-2, -4) obtained from subtracting a point (-1, 3).

The new area of triangle $= ½ [3(4-(-4)) + 6(-4-0) – 2(0-4)]$

$= ½ [24-24+8]$

$= ½ [8]$

$= 4$

Since the area of the triangle before and after the translation after shifting of origin

remains same, i.e. 4.
∴ We can say that the area of a triangle is invariant to shifting of origin.