EXERCISE 23.18 PAGE NO: 23.124

1. Find the equation of the straight lines passing through the origin and making an angle of 45° with the straight line √3x + y = 11.

Solution:

Given:

Equation passes through (0, 0) and make an angle of 45° with the line √3x + y = 11. We know that, the equations of two lines passing through a point x_1,y_1 and making an angle α with the given line y = mx + c are

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Here,

$x_1 = 0, y_1 = 0, \alpha = 45°$ and $m = -\sqrt{3}$

So, the equations of the required lines are

$$y - 0 = \frac{-\sqrt{3} + \tan 45°}{1 + \sqrt{3}\tan 45°}(x - 0) \text{ and } y - 0$$

$$= \frac{-\sqrt{3} - \tan 45°}{1 - \sqrt{3}\tan 45°}(x - 0)$$

$$= \frac{-\sqrt{3} + 1}{1 + \sqrt{3}}x \text{ and } y = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}x$$

$$= -\frac{3 + 1 - 2\sqrt{3}}{3 - 1}x \text{ and } y = \frac{3 + 1 + 2\sqrt{3}}{3 - 1}x$$

$$= (\sqrt{3} - 2)x \text{ and } y = (\sqrt{3} + 2)x$$

∴ The equation of given line is $y = (\sqrt{3} - 2)x$ and $y = (\sqrt{3} + 2)x$

2. Find the equations to the straight lines which pass through the origin and are inclined at an angle of 75° to the straight line x + y + √3(y - x) = a.

Solution:

Given:

The equation passes through (0,0) and make an angle of 75° with the line x + y + √3(y - x) = a.

We know that the equations of two lines passing through a point x_1,y_1 and making an angle α with the given line y = mx + c are

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Here, equation of the given line is,

$x + y + \sqrt{3}(y - x) = a$

$(\sqrt{3} + 1)y = (\sqrt{3} - 1)x + a$

$y = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}x + \frac{a}{\sqrt{3} + 1}$

Comparing this equation with $y = mx + c$

We get,

$m = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$

$\therefore x_1 = 0, y_1 = 0, \alpha = 75°,$

$m = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = 2 - \sqrt{3}$ and $\tan 75° = 2 + \sqrt{3}$

So, the equations of the required lines are

$y - 0 = \frac{2 - \sqrt{3} + \tan 75°}{1 - (2 - \sqrt{3})\tan 75°}(x - 0)$ and $y - 0$

$= \frac{2 - \sqrt{3} - \tan 75°}{1 + (2 - \sqrt{3})\tan 75°}(x - 0)$

$y = \frac{2 - \sqrt{3} + 2 + \sqrt{3}}{1 - (2 - \sqrt{3})(2 + \sqrt{3})}x$ and $y = \frac{2 - \sqrt{3} - 2 - \sqrt{3}}{1 + (2 - \sqrt{3})(2 + \sqrt{3})}x$

$y = \frac{4}{1 - 1}x$ and $y = -\sqrt{3}x$

$x = 0$ and $\sqrt{3}x + y = 0$

$\therefore$ The equation of given line is $x = 0$ and $\sqrt{3}x + y = 0$

3. Find the equations of straight lines passing through (2, -1) and making an angle of 45° with the line 6x + 5y – 8 = 0.

Solution:

Given:

The equation passes through (2,-1) and make an angle of 45° with the line 6x + 5y – 8 = 0

We know that the equations of two lines passing through a point x_1, y_1 and making an angle α with the given line y = mx + c are

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Here, equation of the given line is,
6x + 5y – 8 = 0
5y = - 6x + 8
y = -6x/5 + 8/5
Comparing this equation with y = mx + c
We get, m = -6/5
Where, $x_1 = 2$, $y_1 = -1$, $\alpha = 45°$, m = -6/5
So, the equations of the required lines are

$$y + 1 = \frac{\left(-\frac{6}{5} + \tan 45°\right)}{\left(1 + \frac{6}{5}\tan 45°\right)}(x - 2) \text{ and } y + 1 = \frac{\left(-\frac{6}{5} - \tan 45°\right)}{\left(1 - \frac{6}{5}\tan 45°\right)}(x - 2)$$

$$y + 1 = \frac{\left(-\frac{6}{5} + 1\right)}{\left(1 + \frac{6}{5}\right)}(x - 2) \text{ and } y + 1 = \frac{\left(-\frac{6}{5} - 1\right)}{\left(1 - \frac{6}{5}\right)}(x - 2)$$

$$y + 1 = -\frac{1}{11}(x - 2) \text{ and } y + 1 = -\frac{11}{-1}(x - 2)$$

x + 11y + 9 = 0 and 11x – y – 23 = 0
∴ The equation of given line is x + 11y + 9 = 0 and 11x – y – 23 = 0

4. Find the equations to the straight lines which pass through the point (h, k) and are inclined at angle $\tan^{-1} m$ to the straight line y = mx + c.
Solution:
Given:
The equation passes through (h, k) and make an angle of $\tan^{-1} m$ with the line y = mx + c
We know that the equations of two lines passing through a point x_1, y_1 and making an angle α with the given line y = mx + c are
m′ = m
So,

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Here,
$x_1 = h$, $y_1 = k$, $\alpha = \tan^{-1} m$, m′ = m.
So, the equations of the required lines are

$$y - k = \frac{m + m}{1 - m^2}(x - h) \text{ and } y - k = \frac{m - m}{1 + m^2}(x - h)$$

$$y - k = \frac{2m}{1 - m^2}(x - h) \text{ and } y - k = 0$$

$$(y - k)(1 - m^2) = 2m(x - h) \text{ and } y = k$$

∴ The equation of given line is $(y - k)(1 - m^2) = 2m(x - h)$ and $y = k$.

5. Find the equations to the straight lines passing through the point (2, 3) and inclined at an angle of 45^0 to the lines 3x + y – 5 = 0.

Solution:

Given:

The equation passes through (2, 3) and make an angle of 45^0 with the line 3x + y – 5 = 0.

We know that the equations of two lines passing through a point x_1, y_1 and making an angle α with the given line y = mx + c are

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Here,

Equation of the given line is,

3x + y – 5 = 0

y = - 3x + 5

Comparing this equation with y = mx + c we get, m = - 3

$x_1 = 2$, $y_1 = 3$, α = 45∘, m = - 3.

So, the equations of the required lines are

$$y - 3 = \frac{-3 + \tan 45°}{1 + 3\tan 45°}(x - 2) \text{ and } y - 3 = \frac{-3 - \tan 45°}{1 - 3\tan 45°}(x - 2)$$

$$y - 3 = \frac{-3 + 1}{1 + 3}(x - 2) \text{ and } y - 3 = \frac{-3 - 1}{1 - 3}(x - 2)$$

$$y - 3 = \frac{-1}{2}(x - 2) \text{ and } y - 3 = 2(x - 2)$$

x + 2y – 8 = 0 and 2x – y – 1 = 0

∴ The equation of given line is x + 2y – 8 = 0 and 2x – y – 1 = 0