EXERCISE 24.1 PAGE NO: 24.21

1. Find the equation of the circle with:
(i) Centre (-2, 3) and radius 4.
(ii) Centre (a, b) and radius $\sqrt{a^2+b^2}$.
(iii) Centre (0, - 1) and radius 1.
(iv) Centre (a cos α, a sin α) and radius a.
(v) Centre (a, a) and radius √2 a.
Solution:
(i) Centre (-2, 3) and radius 4.
Given:
The radius is 4 and the centre (-2, 3)
By using the formula,
The equation of the circle with centre (p, q) and radius 'r' is $(x - p)^2 + (y - q)^2 = r^2$
Where, p = -2, q = 3, r = 4
Now by substituting the values in the above equation, we get
$(x - p)^2 + (y - q)^2 = r^2$
$(x - (-2))^2 + (y - 3)^2 = 4^2$
$(x + 2)^2 + (y - 3)^2 = 16$
$x^2 + 4x + 4 + y^2 - 6y + 9 = 16$
$x^2 + y^2 + 4x - 6y - 3 = 0$
∴ The equation of the circle is $x^2 + y^2 + 4x - 6y - 3 = 0$

(ii) Centre (a, b) and radius $\sqrt{a^2+b^2}$.
Given:
The radius is $\sqrt{a^2+b^2}$ and the centre (a, b)
By using the formula,
The equation of the circle with centre (p, q) and radius 'r' is $(x - p)^2 + (y - q)^2 = r^2$
Where, p = a, q = b, r = $\sqrt{a^2+b^2}$
Now by substituting the values in the above equation, we get
$(x - p)^2 + (y - q)^2 = r^2$
$(x - a)^2 + (y - b)^2 = \left(\sqrt{a^2 + b^2}\right)^2$
$x^2 - 2ax + a^2 + y^2 - 2by + b^2 = a^2 + b^2$
$x^2 + y^2 - 2ax - 2by = 0$
∴ The equation of the circle is $x^2 + y^2 - 2ax - 2by = 0$

(iii) Centre (0, -1) and radius 1.
Given:
The radius is 1 and the centre (0, -1)
By using the formula,
The equation of the circle with centre (p, q) and radius ‘r’ is $(x - p)^2 + (y - q)^2 = r^2$
Where, p = 0, q = -1, r = 1
Now by substituting the values in the above equation, we get
$(x - p)^2 + (y - q)^2 = r^2$
$(x - 0)^2 + (y - (-1))^2 = 1^2$
$(x - 0)^2 + (y + 1)^2 = 1$
$x^2 + y^2 + 2y + 1 = 1$
$x^2 + y^2 + 2y = 0$
∴ The equation of the circle is $x^2 + y^2 + 2y = 0$.

(iv) Centre (a cos α, a sin α) and radius a.
Given:
The radius is ‘a’ and the centre (a cos α, a sin α)
By using the formula,
The equation of the circle with centre (p, q) and radius ‘r’ is $(x - p)^2 + (y - q)^2 = r^2$
Where, p = a cos α, q = a sin α, r = a
Now by substituting the values in the above equation, we get
$(x - p)^2 + (y - q)^2 = r^2$
$(x – a \cos\alpha)^2 + (y – a \sin\alpha)^2 = a^2$
$x^2 - (2a\cos\alpha)x + a^2\cos^2\alpha + y^2 - (2a\sin\alpha)y + a^2\sin^2\alpha = a^2$
We know that $\sin^2\theta + \cos^2\theta = 1$
So,
$x^2 - (2a\cos\alpha)x + y^2 - 2a\sin\alpha y + a^2 = a^2$
$x^2 + y^2 - (2a\cos\alpha)x - (2a\sin\alpha)y = 0$
∴ The equation of the circle is $x^2 + y^2 - (2a\cos\alpha)\, x - (2a\sin\alpha)\, y = 0$.

(v) Centre (a, a) and radius √2 a.
Given:
The radius is √2 a and the centre (a, a)
By using the formula,
The equation of the circle with centre (p, q) and radius ‘r’ is $(x - p)^2 + (y - q)^2 = r^2$
Where, p = a, q = a, r = √2 a
Now by substituting the values in the above equation, we get
$(x - p)^2 + (y - q)^2 = r^2$

$(x - a)^2 + (y - a)^2 = (\sqrt{2}\, a)^2$
$x^2 - 2ax + a^2 + y^2 - 2ay + a^2 = 2a^2$
$x^2 + y^2 - 2ax - 2ay = 0$
∴ The equation of the circle is $x^2 + y^2 - 2ax - 2ay = 0$.

2. Find the centre and radius of each of the following circles:
(i) $(x - 1)^2 + y^2 = 4$
(ii) $(x + 5)^2 + (y + 1)^2 = 9$
(iii) $x^2 + y^2 - 4x + 6y = 5$
(iv) $x^2 + y^2 - x + 2y - 3 = 0$
Solution:
(i) $(x - 1)^2 + y^2 = 4$
Given:
The equation $(x - 1)^2 + y^2 = 4$
We need to find the centre and the radius.
By using the standard equation formula,
$(x - a)^2 + (y - b)^2 = r^2$ …. (1)
Now let us convert given circle's equation into the standard form.
$(x - 1)^2 + y^2 = 4$
$(x - 1)^2 + (y - 0)^2 = 2^2$ ..... (2)
By comparing equation (2) with (1), we get
Centre = (1, 0) and radius = 2
∴ The centre of the circle is (1, 0) and the radius is 2.

(ii) $(x + 5)^2 + (y + 1)^2 = 9$
Given:
The equation $(x + 5)^2 + (y + 1)^2 = 9$
We need to find the centre and the radius.
By using the standard equation formula,
$(x - a)^2 + (y - b)^2 = r^2$ …. (1)
Now let us convert given circle's equation into the standard form.
$(x + 5)^2 + (y + 1)^2 = 9$
$(x - (-5))^2 + (y - (- 1))^2 = 3^2$ …. (2)
By comparing equation (2) with (1), we get
Centre = (-5, -1) and radius = 3
∴ The centre of the circle is (-5, -1) and the radius is 3.

(iii) $x^2 + y^2 - 4x + 6y = 5$
Given:

The equation $x^2 + y^2 - 4x + 6y = 5$
We need to find the centre and the radius.
By using the standard equation formula,
$(x - a)^2 + (y - b)^2 = r^2$ …. (1)
Now let us convert given circle's equation into the standard form.
$x^2 + y^2 - 4x + 6y = 5$
$(x^2 - 4x + 4) + (y^2 + 6y + 9) = 5 + 4 + 9$
$(x - 2)^2 + (y + 3)^2 = 18$
$(x - 2)^2 + (y - (-3))^2 = (3\sqrt{2})^2$ … (2)
By comparing equation (2) with (1), we get
Centre = (2, -3) and radius = $3\sqrt{2}$
∴ The centre of the circle is (2, -3) and the radius is $3\sqrt{2}$.

(iv) $x^2 + y^2 - x + 2y - 3 = 0$
Given:
The equation $x^2 + y^2 - x + 2y - 3 = 0$
We need to find the centre and the radius.
By using the standard equation formula,
$(x - a)^2 + (y - b)^2 = r^2$ …. (1)
Now let us convert given circle's equation into the standard form.
$x^2 + y^2 - x + 2y - 3 = 0$
$(x^2 – x + ¼) + (y^2 + 2y + 1) – 3 – ¼ - 1 = 0$
$(x – ½)^2 + (y + 1)^2 = 17/4$ …. (2)
By comparing equation (2) with (1), we get
Centre = (½, - 1) and radius = $\sqrt{17}/2$
∴ The centre of the circle is (½, -1) and the radius is $\sqrt{17}/2$.

3. Find the equation of the circle whose centre is (1, 2) and which passes through the point (4, 6).
Solution:
Given:
Centre is (1, 2) and which passes through the point (4, 6).
Where, p = 1, q = 2
We need to find the equation of the circle.
By using the formula,
$(x - p)^2 + (y - q)^2 = r^2$
$(x - 1)^2 + (y - 2)^2 = r^2$
It passes through the point (4, 6)
$(4 - 1)^2 + (6 - 2)^2 = r^2$

$3^2 + 4^2 = r^2$
$9 + 16 = r^2$
$25 = r^2$
$r = \sqrt{25}$
$= 5$
So r = 5 units
We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by: $(x - p)^2 + (y - q)^2 = r^2$
By substitute the values in the above equation, we get
$(x - 1)^2 + (y - 2)^2 = 5^2$
$x^2 - 2x + 1 + y^2 - 4y + 4 = 25$
$x^2 + y^2 - 2x - 4y - 20 = 0.$
∴ The equation of the circle is $x^2 + y^2 - 2x - 4y - 20 = 0$.

4. Find the equation of the circle passing through the point of intersection of the lines $x + 3y = 0$ and $2x – 7y = 0$ and whose centre is the point of intersection of the lines $x + y + 1 = 0$ and $x – 2y + 4 = 0$.
Solution:
Let us find the points of intersection of the lines.
On solving the lines $x + 3y = 0$ and $2x - 7y = 0$, we get the point of intersection to be (0, 0)
On solving the lines $x + y + 1$ and $x - 2y + 4 = 0$, we get the point of intersection to be (-2, 1)
We have circle with centre (-2, 1) and passing through the point (0, 0).
We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.
So, the equation is $(x - p)^2 + (y - q)^2 = r^2$
Where, p = -2, q = 1
$(x + 2)^2 + (y - 1)^2 = r^2$ …. (1)
Equation (1) passes through (0, 0)
So, $(0 + 2)^2 + (0 - 1)^2 = r^2$
$4 + 1 = a^2$
$5 = r^2$
$r = \sqrt{5}$
We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by: $(x - p)^2 + (y - q)^2 = r^2$
By substitute the values in the above equation, we get
$(x - (-2))^2 + (y - 1)^2 = (\sqrt{5})^2$
$(x + 2)^2 + (y - 1)^2 = 5$

$x^2 + 4x + 4 + y^2 - 2y + 1 = 5$
$x^2 + y^2 + 4x - 2y = 0$
∴ The equation of the circle is $x^2 + y^2 + 4x - 2y = 0$.

5. Find the equation of the circle whose centre lies on the positive direction of y - axis at a distance 6 from the origin and whose radius is 4.
Solution:
It is given that the centre lies on the positive y - axis at a distance of 6 from the origin, we get the centre (0, 6).
We have a circle with centre (0, 6) and having radius 4.
We know that the equation of the circle with centre (p, q) and having radius 'r' is given by: $(x - p)^2 + (y - q)^2 = r^2$
Where, p = 0, q = 6, r = 4
Now by substituting the values in the equation, we get
$(x - 0)^2 + (y - 6)^2 = 4^2$
$x^2 + y^2 - 12y + 36 = 16$
$x^2 + y^2 - 12y + 20 = 0$.
∴ The equation of the circle is $x^2 + y^2 - 12y + 20 = 0$.

6. If the equations of two diameters of a circle are 2x + y = 6 and 3x + 2y = 4 and the radius is 10, find the equation of the circle.
Solution:
It is given that the circle has the radius 10 and has diameters 2x + y = 6 and 3x + 2y = 4.
We know that the centre is the intersection point of the diameters.
On solving the diameters, we get the centre to be (8, -10).
We have a circle with centre (8, -10) and having radius 10.
By using the formula,
We know that the equation of the circle with centre (p, q) and having radius 'r' is given by: $(x - p)^2 + (y - q)^2 = r^2$
Where, p = 8, q = -10, r = 10
Now by substituting the values in the equation, we get
$(x - 8)^2 + (y - (-10))^2 = 10^2$
$(x - 8)^2 + (y + 10)^2 = 100$
$x^2 - 16x + 64 + y^2 + 20y + 100 = 100$
$x^2 + y^2 - 16x + 20y + 64 = 0$.
∴ The equation of the circle is $x^2 + y^2 - 16x + 20y + 64 = 0$.

7. Find the equation of the circle
(i) which touches both the axes at a distance of 6 units from the origin.

(ii) Which touches x - axis at a distance of 5 from the origin and radius 6 units.
(iii) Which touches both the axes and passes through the point (2, 1).
(iv) Passing through the origin, radius 17 and ordinate of the centre is - 15.
Solution:
(i) which touches both the axes at a distance of 6 units from the origin.
A circle touches the axes at the points (±6, 0) and (0, ±6).
So, a circle has a centre (±6, ±6) and passes through the point (0, 6).
We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.
So, the equation is $(x - p)^2 + (y - q)^2 = r^2$
Where, p = 6, q = 6
$(x - 6)^2 + (y - 6)^2 = r^2$ …. (1)
Equation (1) passes through (0, 6)
So, $(0 - 6)^2 + (6 - 6)^2 = r^2$
$36 + 0 = r^2$
$r = \sqrt{36}$
$= 6$
We know that the equation of the circle with centre (p, q) and having radius 'r' is given by: $(x - p)^2 + (y - q)^2 = r^2$
Now by substituting the values in the equation, we get
$(x \pm 6)^2 + (y \pm 6)^2 = (6)^2$
$x^2 \pm 12x + 36 + y^2 \pm 12y + 36 = 36$
$x^2 + y^2 \pm 12x \pm 12y + 36 = 0$
∴ The equation of the circle is $x^2 + y^2 \pm 12x \pm 12y + 36 = 0$.

(ii) Which touches x - axis at a distance of 5 from the origin and radius 6 units.
A circle touches the x - axis at the points (±5, 0).
Let us assume the centre of the circle is (±5, a).
We have a circle with centre (5, a) and passing through the point (5, 0) and having radius 6.
We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.
So, the equation is $(x - p)^2 + (y - q)^2 = r^2$
Where, p = 5, q = a
$(x - 5)^2 + (y - a)^2 = r^2$ …. (1)
Equation (1) passes through (5, 0)
So, $(5 - 5)^2 + (0 - 6)^2 = r^2$
$0 + 36 = r^2$
$r = \sqrt{36}$

$= 6$

We have got the centre at (±5, ±6) and having radius 6 units.

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by: $(x - p)^2 + (y - q)^2 = r^2$

Now by substituting the values in the equation, we get

$(x \pm 5)^2 + (y \pm 6)^2 = (6)^2$

$x^2 \pm 10x + 25 + y^2 \pm 12y + 36 = 36$

$x^2 + y^2 \pm 10x \pm 12y + 25 = 0.$

∴ The equation of the circle is $x^2 + y^2 \pm 10x \pm 12y + 25 = 0$.

(iii) Which touches both the axes and passes through the point (2, 1).

Let us assume the circle touches the x-axis at the point (a, 0) and y-axis at the point (0, a).

Then the centre of the circle is (a, a) and radius is a.

Its equation will be $(x - p)^2 + (y - q)^2 = r^2$

By substituting the values we get

$(x - a)^2 + (y - a)^2 = a^2$ … (1)

So now, equation (1) passes through P (2, 1)

By substituting the values we get

$(2 - a)^2 + (1 - a)^2 = a^2$

$4 – 4a + a^2 + 1 – 2a + a^2 = a^2$

$5 – 6a + a^2 = 0$

$(a - 5)(a - 1) = 0$

So, a = 5 or 1

Case (i)

We have got the centre at (5, 5) and having radius 5 units.

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by: $(x - p)^2 + (y - q)^2 = r^2$

Now by substituting the values in the equation we get

$(x - 5)^2 + (y - 5)^2 = 5^2$

$x^2 - 10x + 25 + y^2 - 10y + 25 = 25$

$x^2 + y^2 - 10x - 10y + 25 = 0.$

∴ The equation of the circle is $x^2 + y^2 - 10x - 10y + 25 = 0$.

Case (ii)

We have got the centre at (1, 1) and having a radius 1 unit.

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by: $(x - p)^2 + (y - q)^2 = r^2$

Now by substituting the values in the equation we get

$(x - 1)^2 + (y - 1)^2 = 1^2$
$x^2 - 2x + 1 + y^2 - 2y + 1 = 1$
$x^2 + y^2 - 2x - 2y + 1 = 0$
∴ The equation of the circle is $x^2 + y^2 - 2x - 2y + 1 = 0$.

(iv) Passing through the origin, radius 17 and ordinate of the centre is - 15.
Let us assume the abscissa as 'a'
We have a circle with centre (a, - 15) and passing through the point (0, 0) and having radius 17.
We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.
By using the distance formula,
$$r = \sqrt{(a-0)^2 + (-15-0)^2}$$
$$17 = \sqrt{(a-0)^2 + (-15-0)^2}$$
$17^2 = a^2 + (-15)^2$
$289 = a^2 + 225$
$a^2 = 64$
$|a| = \sqrt{64}$
$|a| = 8$
$a = \pm 8$ .... (1)
We have got the centre at (±8, - 15) and having radius 17 units.
We know that the equation of the circle with centre (p, q) and having radius 'r' is given by: $(x - p)^2 + (y - q)^2 = r^2$
Now by substituting the values in the equation, we get
$(x \pm 8)^2 + (y - 15)^2 = 17^2$
$x^2 \pm 16x + 64 + y^2 - 30y + 225 = 289$
$x^2 + y^2 \pm 16x - 30y = 0$.
∴ The equation of the circle is $x^2 + y^2 \pm 16x - 30y = 0$.

8. Find the equation of the circle which has its centre at the point (3, 4) and touches the straight line 5x + 12y – 1 = 0.
Solution:
It is given that we need to find the equation of the circle with centre (3, 4) and touches the straight line 5x + 12y - 1 = 0.
We know that the perpendicular distance from the point (x_1, y_1) on to the line ax + by + c = 0 is given by
$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$
Let us assume 'r' be the radius of the circle.

$$r = \frac{|5(3) + 12(4) - 1|}{\sqrt{5^2 + 12^2}}$$
$$= \frac{|15 + 48 - 1|}{\sqrt{25 + 144}}$$
$$= \frac{|62|}{\sqrt{169}}$$
$$= \frac{62}{13}$$

We have a circle with centre (3, 4) and having a radius 62/13.
We know that the equation of the circle with centre (p, q) and having radius 'r' is given by: $(x - p)^2 + (y - q)^2 = r^2$
Now by substituting the values in the equation, we get
$(x - 3)^2 + (y - 4)^2 = (62/13)^2$
$x^2 – 6x + 9 + y^2 – 8y + 16 = 3844/169$
$169x^2 + 169y^2 - 1014x - 1352y + 4225 = 3844$
$169x^2 + 169y^2 - 1014x - 1352y + 381 = 0$
∴ The equation of the circle is $169x^2 + 169y^2 - 1014x - 1352y + 381 = 0$.

9. Find the equation of the circle which touches the axes and whose centre lies on x – 2y = 3.
Solution:
Let us assume the circle touches the axes at (a, 0) and (0, a) and we get the radius to be |a|.
We get the centre of the circle as (a, a). This point lies on the line x – 2y = 3
a - 2(a) = 3
-a = 3
a = - 3
Centre = (a, a) = (-3, -3) and radius of the circle(r) = |-3| = 3
We have circle with centre (-3, -3) and having radius 3.
We know that the equation of the circle with centre (p, q) and having radius 'r' is given by: $(x - p)^2 + (y - q)^2 = r^2$
Now by substituting the values in the equation, we get
$(x - (-3))^2 + (y - (-3))^2 = 3^2$
$(x + 3)^2 + (y + 3)^2 = 9$
$x^2 + 6x + 9 + y^2 + 6y + 9 = 9$
$x^2 + y^2 + 6x + 6y + 9 = 0$
∴ The equation of the circle is $x^2 + y^2 + 6x + 6y + 9 = 0$.

10. A circle whose centre is the point of intersection of the lines 2x – 3y + 4 = 0 and 3x + 4y – 5 = 0 passes through the origin. Find its equation.

Solution:

It is given that the circle has the centre at the intersection point of the lines 2x - 3y + 4 = 0 and 3x + 4y - 5 = 0 and passes through the origin

On solving the lines 2x - 3y + 4 = 0 and 3x + 4y - 5 = 0, we get the point of intersection to be $\left(\frac{-1}{17}, \frac{22}{17}\right)$

We have circle with centre $\left(\frac{-1}{17}, \frac{22}{17}\right)$ and passing through the point (0, 0).

We know that radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

By using the distance formula,

We know that the distance between the two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Let us assume r be the radius of the circle.

$$r = \sqrt{\left(\frac{-1}{17} - 0\right)^2 + \left(\frac{22}{17} - 0\right)^2}$$

$$r = \sqrt{\left(-\frac{1}{17}\right)^2 + \left(\frac{22}{17}\right)^2}$$

$$r = \sqrt{\frac{1}{289} + \frac{484}{289}}$$

$$r = \sqrt{\frac{485}{289}}$$

$$r = \frac{\sqrt{485}}{17}$$

We know that the equation of the circle with centre (p, q) and having radius 'r' is given by: $(x - p)^2 + (y - q)^2 = r^2$

Now by substituting the values in the equation, we get

$$\left(x - \left(\frac{-1}{17}\right)\right)^2 + \left(y - \frac{22}{17}\right)^2 = \left(\frac{\sqrt{485}}{17}\right)^2$$

$$\left(x + \frac{1}{17}\right)^2 + \left(y - \frac{22}{17}\right)^2 = \frac{485}{289}$$

$$x^2 + \frac{2x}{17} + \frac{1}{289} + y^2 - \frac{44y}{17} + \frac{484}{289} = \frac{485}{289}$$

$17x^2 + 17y^2 + 2x - 44y = 0$

∴ The equation of the circle is $17x^2 + 17y^2 + 2x - 44y = 0$.

EXERCISE 24.2 PAGE NO: 24.31

1. Find the coordinates of the centre radius of each of the following circle:
(i) $x^2 + y^2 + 6x - 8y - 24 = 0$
(ii) $2x^2 + 2y^2 – 3x + 5y = 7$
$(iii)\frac{1}{2}(x^2 + y^2) + x\ cos\theta + y\ sin\theta - 4 = 0$
(iv) $x^2 + y^2 – ax – by = 0$
Solution:
(i) $x^2 + y^2 + 6x - 8y - 24 = 0$
Given:
The equation of the circle is $x^2 + y^2 + 6x - 8y - 24 = 0$ …… (1)
We know that for a circle $x^2 + y^2 + 2ax + 2by + c = 0$ …… (2)
Centre = (-a, -b)
So by comparing equation (1) and (2)
Centre = (-6/2, -(-8)/2)
= (-3, 4)
Radius = $\sqrt{(a^2 + b^2 - c)}$
= $\sqrt{(3^2 + 4^2 – (-24))}$
= $\sqrt{(9 + 16 + 24)}$
= $\sqrt{(49)}$
= 7
∴ The centre of the circle is (-3, 4) and the radius is 7.

(ii) $2x^2 + 2y^2 – 3x + 5y = 7$
Given:
The equation of the circle is $2x^2 + 2y^2 – 3x + 5y = 7$ (divide by 2 we get)
$x^2 + y^2 – 3x/2 + 5y/2 = 7/2$
Now, by comparing with the equation $x^2 + y^2 + 2ax + 2by + c = 0$
Centre = (-a, -b)

Centre = $\left(\frac{-\left(\frac{-3}{2}\right)}{2}, \frac{-\left(\frac{5}{2}\right)}{2}\right)$

$= \left(\frac{3}{4}, \frac{-5}{4}\right)$

Radius = $\sqrt{(a^2 + b^2 - c)}$

Radius = $\sqrt{\left(\frac{3}{4}\right)^2 + \left(-\frac{5}{4}\right)^2 - \left(\frac{-7}{2}\right)}$

$= \sqrt{\frac{9}{16} + \frac{25}{16} + \frac{7}{2}}$

$= \sqrt{\frac{90}{16}}$

$= \frac{3\sqrt{10}}{4}$

∴ The centre and radius of the circle is $\left(\frac{3}{4}, \frac{-5}{4}\right)$ and $\frac{3\sqrt{10}}{4}$.

(iii) $\boldsymbol{\frac{1}{2}(x^2 + y^2) + x\cos\theta + y\sin\theta - 4 = 0}$

Given:

The equation of the circle is

$\frac{1}{2}(x^2 + y^2) + x\cos\theta + y\sin\theta - 4 = 0$

(Multiply by 2 we get)

$x^2 + y^2 + 2x\cos\theta + 2y\sin\theta – 8 = 0$

By comparing with the equation $x^2 + y^2 + 2ax + 2by + c = 0$

Centre = (-a, -b)

= [(-2cos θ)/2 , (-2sin θ)/2]

= (-cos θ, -sin θ)

Radius = $\sqrt{(a^2 + b^2 - c)}$

$= \sqrt{[(-\cos\theta)^2 + (\sin\theta)^2 – (-8)]}$

$= \sqrt{[\cos^2\theta + \sin^2\theta + 8]}$

$= \sqrt{[1 + 8]}$

$= \sqrt{[9]}$

= 3

∴ The centre and radius of the circle is (-cos θ, -sin θ) and 3.

(iv) $x^2 + y^2 – ax – by = 0$

Given:
Equation of the circle is $x^2 + y^2 - ax - by = 0$
By comparing with the equation $x^2 + y^2 + 2ax + 2by + c = 0$
Centre = (-a, -b)
= (-(-a)/2, -(-b)/2)
= (a/2, b/2)

Radius = $\sqrt{(a^2 + b^2 - c)}$
= $\sqrt{[(a/2)^2 + (b/2)^2]}$
= $\sqrt{[(a^2/4 + b^2/4)]}$
= $\sqrt{[(a^2 + b^2)/4]}$
= $[\sqrt{(a^2 + b^2)}]/2$
∴ The centre and radius of the circle is (a/2, b/2) and $[\sqrt{(a^2 + b^2)}]/2$

2. Find the equation of the circle passing through the points :
(i) (5, 7), (8, 1) and (1, 3)
(ii) (1, 2), (3, - 4) and (5, - 6)
(iii) (5, -8), (-2, 9) and (2, 1)
(iv) (0, 0), (-2, 1) and (-3, 2)
Solution:
(i) (5, 7), (8, 1) and (1, 3)
By using the standard form of the equation of the circle:
$x^2 + y^2 + 2ax + 2by + c = 0$ ..... (1)
Firstly let us find the values of a, b and c
Substitute the given point (5, 7) in equation (1), we get
$5^2 + 7^2 + 2a (5) + 2b (7) + c = 0$
25 + 49 + 10a + 14b + c = 0
10a + 14b + c + 74 = 0..... (2)

By substituting the given point (8, 1) in equation (1), we get
$8^2 + 1^2 + 2a (8) + 2b (1) + c = 0$
64 + 1 + 16a + 2b + c = 0
16a + 2b + c + 65 = 0..... (3)

Substituting the point (1, 3) in equation (1), we get
$1^2 + 3^2 + 2a (1) + 2b (3) + c = 0$
1 + 9 + 2a + 6b + c = 0
2a + 6b + c + 10 = 0..... (4)

Now by simplifying the equations (2), (3), (4) we get the values
a = -29/6, b = -19/6, c = 56/3

Substituting the values of a, b, c in equation (1), we get
$x^2 + y^2 + 2(-29/6)x + 2(-19/6) + 56/3 = 0$
$x^2 + y^2 – 29x/3 – 19y/3 + 56/3 = 0$
$3x^2 + 3y^2 - 29x - 19y + 56 = 0$
∴ The equation of the circle is $3x^2 + 3y^2 - 29x - 19y + 56 = 0$

(ii) (1, 2), (3, - 4) and (5, - 6)
By using the standard form of the equation of the circle:
$x^2 + y^2 + 2ax + 2by + c = 0$ ..... (1)
Substitute the points (1, 2) in equation (1), we get
$1^2 + 2^2 + 2a(1) + 2b(2) + c = 0$
$1 + 4 + 2a + 4b + c = 0$
$2a + 4b + c + 5 = 0$..... (2)

Substitute the points (3, -4) in equation (1), we get
$3^2 + (-4)^2 + 2a(3) + 2b(-4) + c = 0$
$9 + 16 + 6a - 8b + c = 0$
$6a - 8b + c + 25 = 0$..... (3)

Substitute the points (5, -6) in equation (1), we get
$5^2 + (-6)^2 + 2a(5) + 2b(-6) + c = 0$
$25 + 36 + 10a - 12b + c = 0$
$10a - 12b + c + 61 = 0$..... (4)

Now by simplifying the equations (2), (3), (4) we get
a = - 11, b = - 2, c = 25

Substitute the values of a, b and c in equation (1), we get
$x^2 + y^2 + 2(-11)x + 2(-2) + 25 = 0$
$x^2 + y^2 - 22x - 4y + 25 = 0$
∴ The equation of the circle is $x^2 + y^2 - 22x - 4y + 25 = 0$

(iii) (5, -8), (-2, 9) and (2, 1)
By using the standard form of the equation of the circle:
$x^2 + y^2 + 2ax + 2by + c = 0$ ..... (1)
Substitute the point (5, -8) in equation (1), we get

$5^2 + (-8)^2 + 2a(5) + 2b(-8) + c = 0$
$25 + 64 + 10a - 16b + c = 0$
$10a - 16b + c + 89 = 0$..... (2)

Substitute the points (-2, 9) in equation (1), we get
$(-2)^2 + 9^2 + 2a(-2) + 2b(9) + c = 0$
$4 + 81 - 4a + 18b + c = 0$
$-4a + 18b + c + 85 = 0$..... (3)

Substitute the points (2, 1) in equation (1), we get
$2^2 + 1^2 + 2a(2) + 2b(1) + c = 0$
$4 + 1 + 4a + 2b + c = 0$
$4a + 2b + c + 5 = 0$..... (4)

By simplifying equations (2), (3), (4) we get
a = 58, b = 24, c = - 285.

Now, by substituting the values of a, b, c in equation (1), we get
$x^2 + y^2 + 2(58)x + 2(24) - 285 = 0$
$x^2 + y^2 + 116x + 48y - 285 = 0$
∴ The equation of the circle is $x^2 + y^2 + 116x + 48y - 285 = 0$

(iv) (0, 0), (-2, 1) and (-3, 2)
By using the standard form of the equation of the circle:
$x^2 + y^2 + 2ax + 2by + c = 0$ ..... (1)
Substitute the points (0, 0) in equation (1), we get
$0^2 + 0^2 + 2a(0) + 2b(0) + c = 0$
$0 + 0 + 0a + 0b + c = 0$
$c = 0$..... (2)

Substitute the points (-2, 1) in equation (1), we get
$(-2)^2 + 1^2 + 2a(-2) + 2b(1) + c = 0$
$4 + 1 - 4a + 2b + c = 0$
$-4a + 2b + c + 5 = 0$..... (3)

Substitute the points (-3, 2) in equation (1), we get
$(-3)^2 + 2^2 + 2a(-3) + 2b(2) + c = 0$
$9 + 4 - 6a + 4b + c = 0$
$-6a + 4b + c + 13 = 0$..... (4)

By simplifying the equations (2), (3), (4) we get
$a = -3/2$, $b = -11/2$, $c = 0$

Now, by substituting the values of a, b, c in equation (1), we get
$x^2 + y^2 + 2(-3/2)x + 2(-11/2)y + 0 = 0$
$x^2 + y^2 - 3x - 11y = 0$
∴ The equation of the circle is $x^2 + y^2 - 3x - 11y = 0$

3. Find the equation of the circle which passes through (3, - 2), (- 2, 0) and has its centre on the line 2x – y = 3.
Solution:
Given:
The line $2x – y = 3$ … (1)
The points (3, -2), (-2, 0)
By using the standard form of the equation of the circle:
$x^2 + y^2 + 2ax + 2by + c = 0$ ..... (2)
Let us substitute the centre (-a, -b) in equation (1) we get,
$2(- a) - (- b) = 3$
$-2a + b = 3$
$2a - b + 3 = 0$...... (3)

Now Substitute the given points (3, -2) in equation (2), we get
$3^2 + (- 2)^2 + 2a(3) + 2b(- 2) + c = 0$
$9 + 4 + 6a - 4b + c = 0$
$6a - 4b + c + 13 = 0$..... (4)

Substitute the points (-2, 0) in equation (2), we get
$(- 2)^2 + 0^2 + 2a(- 2) + 2b(0) + c = 0$
$4 + 0 - 4a + c = 0$
$4a - c - 4 = 0$..... (5)

By simplifying the equations (3), (4) and (5) we get,
$a = 3/2$, $b = 6$, $c = 2$

Again by substituting the values of a, b, c in (2), we get
$x^2 + y^2 + 2 (3/2)x + 2 (6)y + 2 = 0$
$x^2 + y^2 + 3x + 12y + 2 = 0$
∴ The equation of the circle is $x^2 + y^2 + 3x + 12y + 2 = 0$.

4. Find the equation of the circle which passes through the points (3, 7), (5, 5) and has its centre on line x – 4y = 1.

Solution:

Given:

The points (3, 7), (5, 5)

The line $x - 4y = 1$…. (1)

By using the standard form of the equation of the circle:

$x^2 + y^2 + 2ax + 2by + c = 0$ ..... (2)

Let us substitute the centre (-a, -b) in equation (1) we get,

$(- a) - 4(- b) = 1$

$-a + 4b = 1$

$a - 4b + 1 = 0$...... (3)

Substitute the points (3, 7) in equation (2), we get

$3^2 + 7^2 + 2a(3) + 2b(7) + c = 0$

$9 + 49 + 6a + 14b + c = 0$

$6a + 14b + c + 58 = 0$..... (4)

Substitute the points (5, 5) in equation (2), we get

$5^2 + 5^2 + 2a(5) + 2b(5) + c = 0$

$25 + 25 + 10a + 10b + c = 0$

$10a + 10b + c + 50 = 0$..... (5)

By simplifying equations (3), (4) and (5) we get,

$a = 3, b = 1, c = - 90$

Now, by substituting the values of a, b, c in equation (2), we get

$x^2 + y^2 + 2(3)x + 2(1)y - 90 = 0$

$x^2 + y^2 + 6x + 2y - 90 = 0$

∴ The equation of the circle is $x^2 + y^2 + 6x + 2y - 90 = 0$.

5. Show that the points (3, -2), (1, 0), (-1, -2) and (1, -4) are con - cyclic.

Solution:

Given:

The points (3, -2), (1, 0), (-1, -2) and (1, -4)

Let us assume the circle passes through the points A, B, C.

So by using the standard form of the equation of the circle:

$x^2 + y^2 + 2ax + 2by + c = 0$..... (1)

Substitute the points A (3, - 2) in equation (1), we get,

$3^2 + (-2)^2 + 2a(3) + 2b(-2) + c = 0$
$9 + 4 + 6a - 4b + c = 0$
$6a - 4b + c + 13 = 0$..... (2)

Substitute the points B (1, 0) in equation (1), we get,
$1^2 + 0^2 + 2a(1) + 2b(0) + c = 0$
$1 + 2a + c = 0$ ......- (3)

Substitute the points C (-1, -2) in equation (1), we get,
$(-1)^2 + (-2)^2 + 2a(-1) + 2b(-2) + c = 0$
$1 + 4 - 2a - 4b + c = 0$
$5 - 2a - 4b + c = 0$
$2a + 4b - c - 5 = 0$..... (4)

Upon simplifying the equations (2), (3) and (4) we get,
$a = -1$, $b = 2$ and $c = 1$

Substituting the values of a, b, c in equation (1), we get
$x^2 + y^2 + 2(-1)x + 2(2)y + 1 = 0$
$x^2 + y^2 - 2x + 4y + 1 = 0$ ..... (5)

Now by substituting the point D (1, -4) in equation (5) we get,
$1^2 + (-4)^2 - 2(1) + 4(-4) + 1$
$1 + 16 - 2 - 16 + 1$
0
∴ The points (3, -2), (1, 0), (-1, -2), (1, -4) are con - cyclic.

6. Show that the points (5, 5), (6, 4), (- 2, 4) and (7, 1) all lie on a circle, and find its equation, centre, and radius.
Solution:
Given:
The points (5, 5), (6, 4), (- 2, 4) and (7, 1) all lie on a circle.
Let us assume the circle passes through the points A, B, C.
So by using the standard form of the equation of the circle:
$x^2 + y^2 + 2ax + 2by + c = 0$..... (1)

Substituting A (5, 5) in (1), we get,
$5^2 + 5^2 + 2a(5) + 2b(5) + c = 0$
$25 + 25 + 10a + 10b + c = 0$

$10a + 10b + c + 50 = 0$..... (2)

Substitute the points B (6, 4) in equation (1), we get,
$6^2 + 4^2 + 2a(6) + 2b(4) + c = 0$
$36 + 16 + 12a + 8b + c = 0$
$12a + 8b + c + 52 = 0$..... (3)

Substitute the point C (-2, 4) in equation (1), we get,
$(-2)^2 + 4^2 + 2a(-2) + 2b(4) + c = 0$
$4 + 16 - 4a + 8b + c = 0$
$20 - 4a + 8b + c = 0$
$4a - 8b - c - 20 = 0$..... (4)

Upon simplifying equations (2), (3) and (4) we get,
$a = -2$, $b = -1$ and $c = -20$

Now by substituting the values of a, b, c in equation (1), we get
$x^2 + y^2 + 2(-2)x + 2(-1)y - 20 = 0$
$x^2 + y^2 - 4x - 2y - 20 = 0$ ..... (5)

Substituting D (7, 1) in equation (5) we get,
$7^2 + 1^2 - 4(7) - 2(1) - 20$
$49 + 1 - 28 - 2 - 20$
0
∴ The points (3, -2), (1, 0), (-1, -2), (1, -4) lie on a circle.

Now let us find the centre and the radius.
We know that for a circle $x^2 + y^2 + 2ax + 2by + c = 0$,
Centre = (-a, -b)
Radius = $\sqrt{(a^2 + b^2 - c)}$

Comparing equation (5) with equation (1), we get
Centre = [-(-4)/2, -(-2)/2)]
= (2, 1)
Radius = $\sqrt{(2^2 + 1^2 - (-20))}$
= $\sqrt{(25)}$
= 5
∴ The centre and radius of the circle is (2, 1) and 5.

7. Find the equation of the circle which circumscribes the triangle formed by the lines:
(i) $x + y + 3 = 0$, $x - y + 1 = 0$ and $x = 3$
(ii) $2x + y - 3 = 0$, $x + y - 1 = 0$ and $3x + 2y - 5 = 0$
(iii) $x + y = 2$, $3x - 4y = 6$ and $x - y = 0$
(iv) $y = x + 2$, $3y = 4x$ and $2y = 3x$
Solution:
(i) $x + y + 3 = 0$, $x - y + 1 = 0$ and $x = 3$
Given:
The lines $x + y + 3 = 0$
$x - y + 1 = 0$
$x = 3$

On solving these lines we get the intersection points A (-2, -1), B (3, 4), C (3, -6)
So by using the standard form of the equation of the circle:
$x^2 + y^2 + 2ax + 2by + c = 0$..... (1)

Substitute the points (-2, -1) in equation (1), we get
$(- 2)^2 + (- 1)^2 + 2a(-2) + 2b(-1) + c = 0$
$4 + 1 - 4a - 2b + c = 0$
$5 - 4a - 2b + c = 0$
$4a + 2b - c - 5 = 0$..... (2)

Substitute the points (3, 4) in equation (1), we get
$3^2 + 4^2 + 2a(3) + 2b(4) + c = 0$
$9 + 16 + 6a + 8b + c = 0$
$6a + 8b + c + 25 = 0$..... (3)

Substitute the points (3, -6) in equation (1), we get
$3^2 + (- 6)^2 + 2a(3) + 2b(- 6) + c = 0$
$9 + 36 + 6a - 12b + c = 0$
$6a - 12b + c + 45 = 0$..... (4)

Upon simplifying equations (2), (3), (4) we get
$a = - 3$, $b = 1$, $c = -15$.

Now by substituting the values of a, b, c in equation (1), we get
$x^2 + y^2 + 2(- 3)x + 2(1)y - 15 = 0$
$x^2 + y^2 - 6x + 2y - 15 = 0$

∴ The equation of the circle is $x^2 + y^2 - 6x + 2y - 15 = 0$.

(ii) $2x + y - 3 = 0$, $x + y - 1 = 0$ and $3x + 2y - 5 = 0$
Given:
The lines $2x + y - 3 = 0$
$x + y - 1 = 0$
$3x + 2y - 5 = 0$

On solving these lines we get the intersection points A(2, - 1), B(3, - 2), C(1,1)
So by using the standard form of the equation of the circle:
$x^2 + y^2 + 2ax + 2by + c = 0$..... (1)

Substitute the points (2, -1) in equation (1), we get
$2^2 + (-1)^2 + 2a(2) + 2b(-1) + c = 0$
$4 + 1 + 4a - 2b + c = 0$
$4a - 2b + c + 5 = 0$..... (2)

Substitute the points (3, -2) in equation (1), we get
$3^2 + (-2)^2 + 2a(3) + 2b(-2) + c = 0$
$9 + 4 + 6a - 4b + c = 0$
$6a - 4b + c + 13 = 0$..... (3)

Substitute the points (1, 1) in equation (1), we get
$1^2 + 1^2 + 2a(1) + 2b(1) + c = 0$
$1 + 1 + 2a + 2b + c = 0$
$2a + 2b + c + 2 = 0$..... (4)

Upon simplifying equations (2), (3), (4) we get
$a = -13/2$, $b = -5/2$, $c = 16$

Now by substituting the values of a, b, c in equation (1), we get
$x^2 + y^2 + 2(-13/2)x + 2(-5/2)y + 16 = 0$
$x^2 + y^2 - 13x - 5y + 16 = 0$
∴ The equation of the circle is $x^2 + y^2 - 13x - 5y + 16 = 0$

(iii) $x + y = 2$, $3x - 4y = 6$ and $x - y = 0$
Given:
The lines $x + y = 2$
$3x - 4y = 6$

$x - y = 0$

On solving these lines we get the intersection points A(2,0), B(- 6, - 6), C(1,1)
So by using the standard form of the equation of the circle:
$x^2 + y^2 + 2ax + 2by + c = 0$..... (1)

Substitute the points (2, 0) in equation (1), we get
$2^2 + 0^2 + 2a(2) + 2b(0) + c = 0$
$4 + 4a + c = 0$
$4a + c + 4 = 0$..... (2)

Substitute the point (-6, -6) in equation (1), we get
$(- 6)^2 + (- 6)^2 + 2a(- 6) + 2b(- 6) + c = 0$
$36 + 36 - 12a - 12b + c = 0$
$12a + 12b - c - 72 = 0$..... (3)

Substitute the points (1, 1) in equation (1), we get
$1^2 + 1^2 + 2a(1) + 2b(1) + c = 0$
$1 + 1 + 2a + 2b + c = 0$
$2a + 2b + c + 2 = 0$..... (4)

Upon simplifying equations (2), (3), (4) we get
$a = 2, b = 3, c = - 12.$

Substituting the values of a, b, c in equation (1), we get
$x^2 + y^2 + 2(2)x + 2(3)y - 12 = 0$
$x^2 + y^2 + 4x + 6y - 12 = 0$
∴ The equation of the circle is $x^2 + y^2 + 4x + 6y - 12 = 0$

(iv) y = x + 2, 3y = 4x and 2y = 3x
Given:
The lines $y = x + 2$
$3y = 4x$
$2y = 3x$

On solving these lines we get the intersection points A(6,8), B(0,0), C(4,6)
So by using the standard form of the equation of the circle:
$x^2 + y^2 + 2ax + 2by + c = 0$..... (1)

Substitute the points (6, 8) in equation (1), we get
$6^2 + 8^2 + 2a(6) + 2b(8) + c = 0$
$36 + 64 + 12a + 16b + c = 0$
$12a + 16b + c + 100 = 0$...... (2)

Substitute the points (0, 0) in equation (1), we get
$0^2 + 0^2 + 2a(0) + 2b(0) + c = 0$
$0 + 0 + 0a + 0b + c = 0$
$c = 0$..... (3)

Substitute the points (4, 6) in equation (1), we get
$4^2 + 6^2 + 2a(4) + 2b(6) + c = 0$
$16 + 36 + 8a + 12b + c = 0$
$8a + 12b + c + 52 = 0$..... (4)

Upon simplifying equations (2), (3), (4) we get
$a = - 23$, $b = 11$, $c = 0$

Now by substituting the values of a, b, c in equation (1), we get
$x^2 + y^2 + 2(- 23)x + 2(11)y + 0 = 0$
$x^2 + y^2 - 46x + 22y = 0$
∴ The equation of the circle is $x^2 + y^2 - 46x + 22y = 0$

EXERCISE 24.3 PAGE NO: 24.37

1. Find the equation of the circle, the end points of whose diameter are (2, -3) and (-2, 4). Find its centre and radius.

Solution:

Given:

The diameters (2, -3) and (-2, 4).

By using the formula,

Centre = (-a, -b)

= [-(2-2)/2, -(-3+4)/2]

= (0, -½)

By using the distance formula,

$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

So, r = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

= $\sqrt{[(2-0)^2 + (-3-½)^2]}$

= $\sqrt{[(2)^2 + (-7/2)^2]}$

= √[4 + 49/4]

= √[65/4]

= [√65]/2

We know that the equation of the circle with centre (p, q) and having radius 'r' is given by: $(x - p)^2 + (y - q)^2 = r^2$

Now by substituting the values in the above equation, we get

$(x - 0)^2 + (y – ½)^2 = [[\sqrt{65}]/2]^2$

$x^2 + y^2 – y + ¼ = 65/4$

$4x^2 + 4y^2 - 4y + 1 = 65$

∴The equation of the circle is $4x^2 + 4y^2 - 4y - 64 = 0$ or $x^2 + y^2 - y - 16 = 0$

2. Find the equation of the circle the end points of whose diameter are the centres of the circles $x^2 + y^2 + 6x – 14y – 1 = 0$ and $x^2 + y^2 – 4x + 10y – 2 = 0$.

Solution:

Given:

$x^2 + y^2 + 6x – 14y – 1 = 0$…. (1)

So the centre = [(-6/2), -(-14/2)]

= [-3, 7]

$x^2 + y^2 – 4x + 10y – 2 = 0$… (2)
So the centre = [-(-4/2), (-10/2)]
= [2, -5]

We know that the equation of the circle is given by,
$(x – x_1) (x – x_2) + (y – y_1) (y – y_2) = 0$
$(x + 3) (x - 2) + (y - 7) (y + 5) = 0$
Upon simplification we get
$x^2 + 3x – 2x – 6 + y^2 – 7y + 5y – 35 = 0$
$x^2 + y^2 + x – 2y – 41 = 0$
∴The equation of the circle is $x^2 + y^2 + x – 2y – 41 = 0$

3. The sides of a squares are x = 6, x = 9, y = 3 and y = 6. Find the equation of a circle drawn on the diagonal of the square as its diameter.
Solution:
Given:
The sides of a squares are x = 6, x = 9, y = 3 and y = 6.
Let us assume A, B, C, D be the vertices of the square. On solving the lines, we get the coordinates as: A = (6, 3)
B = (9, 3)
C = (9, 6)
D = (6, 6)

We know that the equation of the circle with diagonal AC is given by
$(x – x_1) (x – x_2) + (y – y_1) (y – y_2) = 0$
$(x - 6) (x - 9) + (y - 3) (y - 6) = 0$
Upon simplifying, we get
$x^2 – 6x – 9x + 54 + y^2 – 3y – 6y + 18 = 0$
$x^2 + y^2 – 15x – 9y + 72 = 0$

We know that the equation of the circle with diagonal BD as diameter is given by
$(x – x_1) (x – x_2) + (y – y_1) (y – y_2) = 0$
$(x - 9) (x - 6) + (y - 3) (y - 6) = 0$
Upon simplifying, we get
$x^2 – 9x – 6x + 54 + y^2 – 3y – 6y + 18 = 0$
$x^2 + y^2 – 15x – 9y + 72 = 0$
∴The equation of the circle is $x^2 + y^2 – 15x – 9y + 72 = 0$

4. Find the equation of the circle circumscribing the rectangle whose sides are x – 3y

= 4, 3x + y = 22, x – 3y = 14 and 3x + y = 62.
Solution:
Given:
The sides $x - 3y = 4$ …. (1)
$3x + y = 22$ … (2)
$x - 3y = 14$ …. (3)
$3x + y = 62$ … (4)
Let us assume A, B, C, D be the vertices of the square. On solving the lines, we get the coordinates as: A = (7, 1)
B = (8, - 2)
C = (20, 2)
D = (19, 5)
We know that the equation of the circle with diagonal AC as diameter is given by
$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$
$(x - 7)(x - 20) + (y - 1)(y - 2) = 0$
Upon simplification we get
$x^2 + y^2 - 27x - 3y + 142 = 0$
∴The equation of the circle is $x^2 + y^2 - 27x - 3y + 142 = 0$

5. Find the equation of the circle passing through the origin and the points where the line 3x + 4y = 12 meets the axes of coordinates.
Solution:
Given:
The line $3x + 4y = 12$
The value of x is 0 on meeting the y - axis. So,
$3(0) + 4y = 12$
$4y = 12$
$y = 3$
The point is A(0, 3)

The value of y is 0 on meeting the x - axis. So,
$3x + 4(0) = 12$
$3x = 12$
$x = 4$
The point is B(4, 0)

Since the circle passes through origin and A and B
So, AB is the diameter
We know that the equation of the circle with AB as diameter is given by

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$
$(x - 0)(x - 4) + (y - 3)(y - 0) = 0$
$x^2 + y^2 - 4x - 3y = 0$
∴The equation of the circle is $x^2 + y^2 - 4x - 3y = 0$

6. Find the equation of the circle which passes through the origin and cuts off intercepts a and b respectively from x and y - axes.
Solution:
Since the circle has intercept ‘a’ from x – axis, the circle must pass through (a, 0) and (-a, 0) as it already passes through the origin.

Since the circle has intercept ‘b’ from x - axis, the circle must pass through (0, b) and (0, -b) as it already passes through the origin.
Let us assume the circle passing through the points A(a,0) and B(0,b).
We know that the equation of the circle with AB as diameter is given by
$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$
$(x - a)(x - 0) + (y - 0)(y - b) = 0$
$x^2 + y^2 + ax + by = 0$ or $x^2 + y^2 - ax - by = 0$
∴The equation of the circle is $x^2 + y^2 + ax + by = 0$ or $x^2 + y^2 - ax - by = 0$