EXERCISE 26.1 PAGE NO: 26.22

1. Find the equation of the ellipse whose focus is (1, -2), the directrix 3x – 2y + 5 = 0 and eccentricity equal to 1/2.

Solution:

Given:

Focus = (1, -2)

Directrix = 3x – 2y + 5 = 0

Eccentricity = ½

Let P(x, y) be any point on the ellipse.

We know that distance between the points (x_1, y_1) and (x_2, y_2) is given as

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

We also know that the perpendicular distance from the point (x_1, y_1) to the line ax + by + c = 0 is given as $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

So,

SP = ePM

$SP^2 = e^2PM^2$

$$(x - 1)^2 + \left(y - (-2)\right)^2 = \left(\frac{1}{2}\right)^2 \left(\frac{|3x - 2y + 5|}{\sqrt{3^2 + (-2)^2}}\right)^2$$

$$x^2 - 2x + 1 + y^2 + 4y + 4 = \frac{1}{4} \times \frac{(|3x - 2y + 5|)^2}{9 + 4}$$

$$x^2 + y^2 - 2x + 4y + 5 = \frac{1}{52} \times (9x^2 + 4y^2 + 25 - 12xy - 20y + 30x)$$

Upon cross multiplying, we get

$52x^2 + 52y^2 - 104x + 208y + 260 = 9x^2 + 4y^2 - 12xy - 20y + 30x + 25$

$43x^2 + 48y^2 + 12xy - 134x + 228y + 235 = 0$

∴ The equation of the ellipse is $43x^2 + 48y^2 + 12xy - 134x + 228y + 235 = 0$

2. Find the equation of the ellipse in the following cases:

(i) focus is (0, 1), directrix is x + y = 0 and e = ½.

(ii) focus is (- 1, 1), directrix is x - y + 3 = 0 and e = ½.

(iii) focus is (- 2, 3), directrix is 2x + 3y + 4 = 0 and e = 4/5.

(iv) focus is (1, 2), directrix is 3x + 4y - 7 = 0 and e = ½.

Solution:

(i) focus is (0, 1), directrix is x + y = 0 and e = ½

Given:

Focus is (0, 1)

Directrix is x + y = 0
e = ½
Let P(x, y) be any point on the ellipse.
We know that distance between the points (x_1, y_1) and (x_2, y_2) is given as

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

We also know that the perpendicular distance from the point (x_1, y_1) to the line ax + by + c = 0 is given as $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

So,
SP = ePM
$SP^2 = e^2PM^2$

$$(x - 0)^2 + (y - 1)^2 = \left(\frac{1}{2}\right)^2 \left(\frac{|x + y|}{\sqrt{1^2 + 1^2}}\right)^2$$

$$x^2 + y^2 - 2y + 1 = \frac{1}{4} \times \frac{(|x + y|)^2}{1 + 1}$$

$$x^2 + y^2 - 2y + 1 = \frac{1}{8} \times (x^2 + y^2 + 2xy)$$

Upon cross multiplying, we get
$8x^2 + 8y^2 - 16y + 8 = x^2 + y^2 + 2xy$
$7x^2 + 7y^2 - 2xy - 16y + 8 = 0$
∴ The equation of the ellipse is $7x^2 + 7y^2 - 2xy - 16y + 8 = 0$

(ii) focus is (- 1, 1), directrix is x - y + 3 = 0 and e = ½
Given:
Focus is (- 1, 1)
Directrix is x - y + 3 = 0
e = ½
Let P(x, y) be any point on the ellipse.
We know that distance between the points (x_1, y_1) and (x_2, y_2) is given as

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

We also know that the perpendicular distance from the point (x_1, y_1) to the line ax + by + c = 0 is given as $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

So,
SP = ePM
$SP^2 = e^2PM^2$

$$(x-(-1))^2 + (y-1)^2 = \left(\frac{1}{2}\right)^2\left(\frac{|x-y+3|}{\sqrt{1^2+1^2}}\right)^2$$

$$x^2 + 2x + 1 + y^2 - 2y + 1 = \frac{1}{4}\times\frac{(|x-y+3|)^2}{1+1}$$

$$x^2 + y^2 + 2x - 2y + 2 = \frac{1}{8}\times(x^2 + y^2 + 9 - 2xy - 6y + 6x)$$

Upon cross multiplying, we get

$8x^2 + 8y^2 + 16x - 16y + 16 = x^2 + y^2 - 2xy + 6x - 6y + 9$

$7x^2 + 7y^2 + 2xy + 10x - 10y + 7 = 0$

∴ The equation of the ellipse is $7x^2 + 7y^2 + 2xy + 10x - 10y + 7 = 0$

(iii) focus is (- 2, 3), directrix is $2x + 3y + 4 = 0$ and e = 4/5

Focus is (- 2, 3)

Directrix is $2x + 3y + 4 = 0$

e = 4/5

Let P(x, y) be any point on the ellipse.

We know that distance between the points (x_1, y_1) and (x_2, y_2) is given as

$$\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$

We also know that the perpendicular distance from the point (x_1, y_1) to the line ax + by + c = 0 is given as $\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

So,

SP = ePM

$SP^2 = e^2PM^2$

$$(x-(-2))^2 + (y-3)^2 = \left(\frac{4}{5}\right)^2\left(\frac{|2x+3y+4|}{\sqrt{2^2+3^2}}\right)^2$$

$$x^2 + 4x + 4 + y^2 - 6y + 9 = \frac{16}{25}\times\frac{(|2x+3y+4|)^2}{4+9}$$

$x^2 + y^2 + 4x - 6y + 13 = (16/325) \times (4x^2 + 9y^2 + 16 + 12xy + 16x + 24y)$

Upon cross multiplying, we get

$325x^2 + 325y^2 + 1300x - 1950y + 4225 = 64x^2 + 144y^2 + 192xy + 256x + 384y + 256$

$261x^2 + 181y^2 - 192xy + 1044x - 2334y + 3969 = 0$

∴ The equation of the ellipse is $261x^2 + 181y^2 - 192xy + 1044x - 2334y + 3969 = 0$

(iv) focus is (1, 2), directrix is $3x + 4y - 7 = 0$ and e = ½.

Given:

focus is (1, 2)

directrix is $3x + 4y - 7 = 0$

e = ½.

Let P(x, y) be any point on the ellipse.
We know that distance between the points (x_1, y_1) and (x_2, y_2) is given as
$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

We also know that the perpendicular distance from the point (x_1, y_1) to the line ax + by + c = 0 is given as $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$
So,
SP = ePM
$SP^2 = e^2PM^2$

$$(x-1)^2 + (y-2)^2 = \left(\frac{1}{2}\right)^2 \left(\frac{|3x+4y-5|}{\sqrt{3^2+4^2}}\right)^2$$
$$x^2 - 2x + 1 + y^2 - 4y + 4 = \frac{1}{4} \times \frac{(|3x+4y-5|)^2}{9+16}$$
$$x^2 + y^2 - 2x - 4y + 5 = \frac{1}{100} \times (9x^2 + 16y^2 + 25 + 24xy - 30x - 40y)$$
Upon cross multiplying, we get
$100x^2 + 100y^2 - 200x - 400y + 500 = 9x^2 + 16y^2 + 24xy - 30x - 40y + 25$
$91x^2 + 84y^2 - 24xy - 170x - 360y + 475 = 0$
∴ The equation of the ellipse is $91x^2 + 84y^2 - 24xy - 170x - 360y + 475 = 0$

3. Find the eccentricity, coordinates of foci, length of the latus - rectum of the following ellipse:
(i) $4x^2 + 9y^2 = 1$
(ii) $5x^2 + 4y^2 = 1$
(iii) $4x^2 + 3y^2 = 1$
(iv) $25x^2 + 16y^2 = 1600$
(v) $9x^2 + 25y^2 = 225$
Solution:
(i) $4x^2 + 9y^2 = 1$
Given:
The equation of ellipse => $4x^2 + 9y^2 = 1$
This equation can be expressed as
$$\frac{x^2}{\frac{1}{4}} + \frac{y^2}{\frac{1}{9}} = 1$$
By using the formula,
Eccentricity:

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

Here, $a^2 = ¼$, $b^2 = 1/9$

$$= \sqrt{\frac{\frac{1}{4} - \frac{1}{9}}{\frac{1}{4}}}$$

$$= \sqrt{\frac{\frac{5}{36}}{\frac{1}{4}}}$$

$$= \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$$

Length of latus rectum = $2b^2/a$
= [2 (1/9)] / (1/2)
= 4/9

Coordinates of foci (±ae, 0)

$$\text{foci} = \left(\pm \frac{1}{2} \times \frac{\sqrt{5}}{3}, 0\right)$$

$$= \left(\pm \frac{\sqrt{5}}{6}, 0\right)$$

∴ The eccentricity is $\frac{\sqrt{5}}{3}$, foci are $\left(\pm \frac{\sqrt{5}}{6}, 0\right)$ and length of the latus rectum is $\frac{4}{9}$.

(ii) $5x^2 + 4y^2 = 1$

Given:

The equation of ellipse => $5x^2 + 4y^2 = 1$

This equation can be expressed as

$$\frac{x^2}{\frac{1}{5}} + \frac{y^2}{\frac{1}{4}} = 1$$

Here, $a^2 = 1/5$ and $b^2 = ¼$

i.e. $a = 1/\sqrt{5}$, $b = 1/2$

Clearly $b > a$ or $b^2 > a^2$

By using the formula,

$$e = \sqrt{\frac{b^2 - a^2}{a^2}}$$

$$e = \sqrt{\frac{\frac{1}{4}-\frac{1}{5}}{\frac{1}{4}}}$$

$$= \sqrt{\frac{\frac{1}{20}}{\frac{1}{4}}}$$

$$= \sqrt{\frac{1}{5}}$$

Length of latus rectum = $2b^2/a$
= [2(1/5)] / (1/2)
= 4/5

Coordinates of foci (±ae, 0)

$$\text{foci} = \left(0, \pm\frac{1}{2} \times \sqrt{\frac{1}{5}}\right)$$

$$= \left(0, \pm\frac{1}{2\sqrt{5}}\right)$$

∴ The eccentricity is $\sqrt{\frac{1}{5}}$, foci are $\left(0, \pm\frac{1}{2\sqrt{5}}\right)$ and length of the latus rectum is $\frac{4}{5}$.

(iii) $4x^2 + 3y^2 = 1$

Given:

The equation of ellipse => $4x^2 + 3y^2 = 1$

This equation can be expressed as

$$\frac{x^2}{\frac{1}{4}} + \frac{y^2}{\frac{1}{3}} = 1$$

By using the formula,

Eccentricity:

$$e = \sqrt{\frac{a^2-b^2}{a^2}}$$

Here, $a^2 = 1/4$ and $b^2 = 1/3$

Clearly $b^2 > a^2$

$e = \sqrt{(1- (a^2/b^2))}$

$$e = \sqrt{1-\frac{\frac{1}{4}}{\frac{1}{3}}}$$

$$= \sqrt{\frac{\frac{1}{3}-\frac{1}{4}}{\frac{1}{3}}}$$

$$= \sqrt{\frac{\frac{1}{12}}{\frac{1}{3}}}$$

$$= \sqrt{\frac{1}{4}}$$

$$= \frac{1}{2}$$

Length of latus rectum = $2b^2/a$

= $[2(1/4)] / (1/\sqrt{3})$

= $\sqrt{3}/2$

Coordinates of foci ($\pm ae$, 0)

$$\text{foci} = \left(0, \pm\frac{1}{\sqrt{3}} \times \frac{1}{2}\right)$$

$$= \left(0, \pm\frac{1}{2\sqrt{3}}\right)$$

∴ The eccentricity is $\frac{\sqrt{3}}{2}$, foci are $\left(0, \pm\frac{1}{2\sqrt{3}}\right)$ and length of the latus rectum is $\frac{\sqrt{3}}{2}$.

(iv) $25x^2 + 16y^2 = 1600$

Given:

The equation of ellipse => $25x^2 + 16y^2 = 1600$

This equation can be expressed as

$$\frac{25x^2}{1600} + \frac{16y^2}{1600} = 1$$

$$\frac{x^2}{64} + \frac{y^2}{100} = 1$$

By using the formula,

Eccentricity:

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

Here, $a^2 = 64$ and $b^2 = 100$

Clearly $b^2 > a^2$

$e = \sqrt{(1 - (a^2/b^2))}$

$$e = \sqrt{1 - \frac{64}{100}}$$

$$= \sqrt{\frac{100-64}{100}}$$

$$= \sqrt{\frac{36}{100}}$$
$$= \frac{6}{10}$$
$$= \frac{3}{5}$$

Length of latus rectum = $2b^2/a$
= [2(64)] / (100)
= 32/25

Coordinates of foci ($\pm$ae, 0)

$$\text{foci} = \left(0, \pm 10 \times \frac{3}{5}\right)$$
$$= (0, \pm 6)$$

∴ The eccentricity is $\frac{3}{5}$, foci are (0, ±6) and length of the latus rectum is $\frac{32}{25}$.

(v) $9x^2 + 25y^2 = 225$
Given:
The equation of ellipse => $9x^2 + 25y^2 = 225$
This equation can be expressed as

$$\frac{9x^2}{225} + \frac{25y^2}{225} = 1$$
$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

By using the formula,
Eccentricity:

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

Here, $a^2 = 25$ and $b^2 = 9$

$$e = \sqrt{\frac{25-9}{25}}$$
$$= \sqrt{\frac{16}{25}}$$
$$= \frac{4}{5}$$

Length of latus rectum = $2b^2/a$
= [2(9)] / (5)
= 18/5

Coordinates of foci (±ae, 0)

$$\text{foci} = \left(\pm 5 \times \frac{4}{5}, 0\right)$$
$$= (\pm 4, 0)$$

∴ The eccentricity is $\frac{4}{5}$, foci are $(\pm 4, 0)$ and length of the latus rectum is $\frac{18}{5}$.

4. Find the equation to the ellipse (referred to its axes as the axes of x and y respectively) which passes through the point (-3, 1) and has eccentricity √(2/5).

Solution:

Given:

The point (-3, 1)

Eccentricity = √(2/5)

Now let us find the equation to the ellipse.

We know that the equation of the ellipse whose axes are x and y - axis is given as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \ldots. (1)$$

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$\sqrt{\frac{2}{5}} = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$\frac{2}{5} = 1 - \frac{b^2}{a^2}$$

$$\frac{b^2}{a^2} = \frac{3}{5}$$

$$b^2 = \frac{3a^2}{5} \quad \ldots. (2)$$

Now let us substitute equation (2) in equation (1), we get

$$\frac{x^2}{a^2} + \frac{y^2}{\frac{3a^2}{5}} = 1$$

$$\frac{x^2}{a^2} + \frac{5y^2}{3a^2} = 1$$

$$3x^2 + 5y^2 = 3a^2$$

It is given that the curve passes through the point (-3, 1).

So by substituting the point in the curve we get,

$3(-3)^2 + 5(1)^2 = 3a^2$

$3(9) + 5 = 3a^2$

$32 = 3a^2$

$a^2 = 32/3$
From equation (2)
$b^2 = 3a^2/5$
$= 3(32/3) / 5$
$= 32/5$
So now, the equation of the ellipse is given as:
$$\frac{x^2}{\frac{32}{3}} + \frac{y^2}{\frac{32}{5}} = 1$$
$$\frac{3x^2}{32} + \frac{5y^2}{32} = 1$$
$3x^2 + 5y^2 = 32$
∴ The equation of the ellipse is $3x^2 + 5y^2 = 32$.

5. Find the equation of the ellipse in the following cases:
(i) eccentricity e = ½ and foci (± 2, 0)
(ii) eccentricity e = 2/3 and length of latus - rectum = 5
(iii) eccentricity e = ½ and semi - major axis = 4
(iv) eccentricity e = ½ and major axis = 12
(v) The ellipse passes through (1, 4) and (- 6, 1)
Solution:
(i) Eccentricity e = ½ and foci (± 2, 0)
Given:
Eccentricity e = ½
Foci (± 2, 0)
Now let us find the equation to the ellipse.
We know that the equation of the ellipse whose axes are x and y - axis is given as
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
By using the formula,
Eccentricity:
$$e = \sqrt{\frac{a^2-b^2}{a^2}}$$
$$\frac{1}{2} = \sqrt{\frac{a^2-b^2}{a^2}}$$
$$\frac{1}{4} = 1 - \frac{b^2}{a^2}$$
$$\frac{b^2}{a^2} = \frac{3}{4}$$

$b^2 = 3a^2/4$
It is given that foci (± 2, 0) =>foci = (±ae, 0)
Where, ae = 2
a(1/2) = 2
a = 4
$a^2 = 16$
We know $b^2 = 3a^2/4$
$b^2 = 3(16)/4$
= 12
So the equation of the ellipse can be given as
$$\frac{x^2}{16} + \frac{y^2}{12} = 1$$
$$\frac{3x^2 + 4y^2}{48} = 1$$
$3x^2 + 4y^2 = 48$
∴ The equation of the ellipse is $3x^2 + 4y^2 = 48$

(ii) eccentricity e = 2/3 and length of latus rectum = 5
Given:
Eccentricity e = 2/3
Length of latus - rectum = 5
Now let us find the equation to the ellipse.
We know that the equation of the ellipse whose axes are x and y - axis is given as
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
By using the formula,
Eccentricity:
$$e = \sqrt{\frac{a^2-b^2}{a^2}}$$
$$\frac{2}{3} = \sqrt{\frac{a^2-b^2}{a^2}}$$
$$\frac{4}{9} = 1 - \frac{b^2}{a^2}$$
$$\frac{b^2}{a^2} = \frac{5}{9}$$
$$b^2 = \frac{5a^2}{9}$$
By using the formula, length of the latus rectum is $2b^2/a$

$\frac{2b^2}{a} = 5$

$b^2 = \frac{5a}{2}$

Since, $b^2 = 5a^2/9$

$\frac{5a^2}{9} = \frac{5a}{2}$

$\frac{a}{9} = \frac{1}{2}$

$a = \frac{9}{2}$

$a^2 = \frac{81}{4}$

Now, substituting the value of a^2, we get

$b^2 = \frac{5\left(\frac{81}{4}\right)}{9}$

$b^2 = \frac{45}{4}$

So the equation of the ellipse can be given as

$\frac{x^2}{\frac{81}{4}} + \frac{y^2}{\frac{45}{4}} = 1$

$\frac{4x^2}{81} + \frac{4y^2}{45} = 1$

$\frac{(20x^2 + 36y^2)}{405} = 1$

$20x^2 + 36y^2 = 405$

∴ The equation of the ellipse is $20x^2 + 36y^2 = 405$.

(iii) eccentricity e = ½ and semi - major axis = 4

Given:

Eccentricity e = ½

Semi - major axis = 4

Now let us find the equation to the ellipse.

We know that the equation of the ellipse whose axes are x and y - axis is given as

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

By using the formula,

Eccentricity:

$e = \sqrt{\frac{a^2-b^2}{a^2}}$

$\frac{1}{2} = \sqrt{\frac{a^2-b^2}{a^2}}$

$\frac{1}{4} = 1 - \frac{b^2}{a^2}$

$\frac{b^2}{a^2} = \frac{3}{4}$

$b^2 = \frac{3a^2}{4}$

It is given that the length of the semi - major axis is a

a = 4

$a^2 = 16$

We know, $b^2 = 3a^2/4$

$b^2 = 3(16)/4$

= 4

So the equation of the ellipse can be given as

$\frac{x^2}{16} + \frac{y^2}{12} = 1$

$\frac{3x^2 + 4y^2}{48} = 1$

$3x^2 + 4y^2 = 48$

∴ The equation of the ellipse is $3x^2 + 4y^2 = 48$.

(iv) eccentricity e = ½ and major axis = 12

Given:

Eccentricity e = ½

Major axis = 12

Now let us find the equation to the ellipse.

We know that the equation of the ellipse whose axes are x and y - axis is given as

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

By using the formula,

Eccentricity:

$e = \sqrt{\frac{a^2-b^2}{a^2}}$

$\frac{1}{2} = \sqrt{\frac{a^2-b^2}{a^2}}$

$\frac{1}{4} = 1 - \frac{b^2}{a^2}$

$\frac{b^2}{a^2} = \frac{3}{4}$

$b^2 = 3a^2/4$
It is given that length of major axis is 2a.
$2a = 12$
$a = 6$
$a^2 = 36$
So, by substituting the value of a^2, we get
$b^2 = 3(36)/4$
$= 27$
So the equation of the ellipse can be given as
$$\frac{x^2}{36} + \frac{y^2}{27} = 1$$
$$\frac{3x^2 + 4y^2}{108} = 1$$
$3x^2 + 4y^2 = 108$
∴ The equation of the ellipse is $3x^2 + 4y^2 = 108$.

(v) The ellipse passes through (1, 4) and (- 6, 1)
Given:
The points (1, 4) and (- 6, 1)
Now let us find the equation to the ellipse.
We know that the equation of the ellipse whose axes are x and y - axis is given as
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad …. (1)$$

Let us substitute the point (1, 4) in equation (1), we get
$$\frac{1^2}{a^2} + \frac{4^2}{b^2} = 1$$
$$\frac{1}{a^2} + \frac{16}{b^2} = 1$$
$$\frac{b^2 + 16a^2}{a^2b^2} = 1$$
$b^2 + 16a^2 = a^2 b^2$ …. (2)

Let us substitute the point (-6, 1) in equation (1), we get
$$\frac{(-6)^2}{a^2} + \frac{1^2}{b^2} = 1$$
$$\frac{36}{a^2} + \frac{1}{b^2} = 1$$
$$\frac{36b^2 + a^2}{a^2b^2} = 1$$
$a^2 + 36b^2 = a^2b^2$ …. (3)

Let us multiply equation (3) by 16 and subtract with equation (2), we get
$(16a^2 + 576b^2) - (b^2 + 16a^2) = (16a^2b^2 - a^2b^2)$
$575b^2 = 15a^2b^2$
$15a^2 = 575$
$a^2 = 575/15$
$= 115/3$

So from equation (2),
$$b^2 + 16\left(\frac{115}{3}\right) = b^2\left(\frac{115}{3}\right)$$
$$b^2\left(\frac{112}{3}\right) = \frac{1840}{3}$$
$$b^2 = \frac{115}{7}$$
So the equation of the ellipse can be given as
$$\frac{x^2}{\frac{115}{3}} + \frac{y^2}{\frac{115}{7}} = 1$$
$$\frac{3x^2}{115} + \frac{7y^2}{115} = 1$$
$3x^2 + 7y^2 = 115$
∴ The equation of the ellipse is $3x^2 + 7y^2 = 115$.

6. Find the equation of the ellipse whose foci are (4, 0) and (- 4, 0), eccentricity = 1/3.
Solution:
Given:
Foci are (4, 0) (- 4, 0)
Eccentricity = 1/3.
Now let us find the equation to the ellipse.
We know that the equation of the ellipse whose axes are x and y - axis is given as
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
By using the formula,
Eccentricity:
$$e = \sqrt{\frac{a^2-b^2}{a^2}}$$
$$\frac{1}{3} = \sqrt{\frac{a^2-b^2}{a^2}}$$
$$\frac{1}{9} = 1 - \frac{b^2}{a^2}$$

$\frac{b^2}{a^2} = \frac{8}{9}$

$b^2 = \frac{8a^2}{9}$

It is given that foci = (4, 0) (- 4, 0) => foci = (±ae,0)
Where, ae = 4
a(1/3) = 4
a = 12
$a^2 = 144$
By substituting the value of a^2, we get
$b^2 = 8a^2/9$
$b^2 = 8(144)/9$
= 128
So the equation of the ellipse can be given as

$\frac{x^2}{144} + \frac{y^2}{128} = 1$

∴ The equation of the ellipse is $\frac{x^2}{144} + \frac{y^2}{128} = 1$

7. Find the equation of the ellipse in the standard form whose minor axis is equal to the distance between foci and whose latus - rectum is 10.
Solution:
Given:
Minor axis is equal to the distance between foci and whose latus - rectum is 10.
Now let us find the equation to the ellipse.
We know that the equation of the ellipse whose axes are x and y - axis is given as

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

We know that length of the minor axis is 2b and distance between the foci is 2ae.
By using the formula,
Eccentricity:

$e = \sqrt{\frac{a^2-b^2}{a^2}}$

2b = 2ae
b = ae

$b = a\sqrt{\frac{a^2-b^2}{a^2}}$

$b^2 = a^2 - b^2$
$a^2 = 2b^2$ …. (1)

We know that the length of the latus rectum is $2b^2/a$
It is given that length of the latus rectum = 10
So by equating, we get
$2b^2/a = 10$
$a^2/ a = 10$ [Since, $a^2 = 2b^2$]
$a = 10$
$a^2 = 100$
Now, by substituting the value of a^2 we get
$2b^2/a = 10$
$2b^2/10 = 10$
$2b^2 = 10(10)$
$b^2 = 100/2$
$= 50$
So the equation of the ellipse can be given as
$$\frac{x^2}{100} + \frac{y^2}{50} = 1$$
$$\frac{x^2 + 2y^2}{100} = 1$$
$x^2 + 2y^2 = 100$
∴ The equation of the ellipse is $x^2 + 2y^2 = 100$.

8. Find the equation of the ellipse whose centre is (-2, 3) and whose semi - axis are 3 and 2 when the major axis is (i) parallel to x - axis (ii) parallel to the y - axis.
Solution:
Given:
Centre = (-2, 3)
Semi - axis are 3 and 2
(i) When major axis is parallel to x-axis
Now let us find the equation to the ellipse.
We know that the equation of the ellipse with centre (p, q) is given by $\frac{(x-p)^2}{a^2} + \frac{(y-q)^2}{b^2} = 1$
Since major axis is parallel to x - axis
So, a = 3 and b = 2.
$a^2 = 9$
$b^2 = 4$
So the equation of the ellipse can be given as

$\frac{(x+2)^2}{9} + \frac{(y-3)^2}{4} = 1$

$\frac{4(x+2)^2 + 9(y-3)^2}{36} = 1$

$4(x^2 + 4x + 4) + 9(y^2 - 6y + 9) = 36$

$4x^2 + 16x + 16 + 9y^2 - 54y + 81 = 36$

$4x^2 + 9y^2 + 16x - 54y + 61 = 0$

∴ The equation of the ellipse is $4x^2 + 9y^2 + 16x - 54y + 61 = 0$.

(ii) When major axis is parallel to y-axis

Now let us find the equation to the ellipse.

We know that the equation of the ellipse with centre (p, q) is given by $\frac{(x-p)^2}{a^2} + \frac{(y-q)^2}{b^2} = 1$

Since major axis is parallel to y - axis

So, a = 2 and b = 3.

$a^2 = 4$

$b^2 = 9$

So the equation of the ellipse can be given as

$\frac{(x+2)^2}{4} + \frac{(y-3)^2}{9} = 1$

$\frac{9(x+2)^2 + 4(y-3)^2}{36} = 1$

$9(x^2 + 4x + 4) + 4(y^2 - 6y + 9) = 36$

$9x^2 + 36x + 36 + 4y^2 - 24y + 36 = 36$

$9x^2 + 4y^2 + 36x - 24y + 36 = 0$

∴ The equation of the ellipse is $9x^2 + 4y^2 + 36x - 24y + 36 = 0$.

9. Find the eccentricity of an ellipse whose latus - rectum is
(i) Half of its minor axis
(ii) Half of its major axis

Solution:

Given:

We need to find the eccentricity of an ellipse.

(i) If latus - rectum is half of its minor axis

We know that the length of the semi - minor axis is b and the length of the latus - rectum is $2b^2/a$.

$2b^2/a = b$

a = 2b …. (1)

By using the formula,
We know that eccentricity of an ellipse is given as

$$e = \sqrt{\frac{a^2-b^2}{a^2}}$$

From equation (1)

$$e = \sqrt{\frac{a^2-b^2}{a^2}}$$

$$= \sqrt{\frac{(2b)^2-b^2}{(2b)^2}}$$

$$= \sqrt{\frac{4b^2-b^2}{4b^2}}$$

$$= \sqrt{\frac{3}{4}}$$

$$= \frac{\sqrt{3}}{2}$$

(ii) If latus - rectum is half of its major axis
We know that the length of the semi - major axis is a and the length of the latus - rectum is $2b^2/a$.
$2b^2/a$
$a^2 = 2b^2$ …. (1)
By using the formula,
We know that eccentricity of an ellipse is given as

$$e = \sqrt{\frac{a^2-b^2}{a^2}}$$

From equation (1)

$$e = \sqrt{\frac{a^2-b^2}{a^2}}$$

$$= \sqrt{\frac{2b^2-b^2}{2b^2}}$$

$$= \sqrt{\frac{b^2}{2b^2}}$$

$$= \sqrt{\frac{1}{2}}$$

$$= \frac{1}{\sqrt{2}}$$