

**EXERCISE 29.11**
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**Evaluate the following limits:**

1.  $\lim_{x \rightarrow \pi} \left(1 - \frac{x}{\pi}\right)^\pi$

**Solution:**

Given:  $\lim_{x \rightarrow \pi} \left(1 - \frac{x}{\pi}\right)^\pi$

The limit Let us substitute the value of  $x = \pi$  directly, we get

$$Z = \lim_{x \rightarrow \pi} \left(1 - \frac{x}{\pi}\right)^\pi = \left(1 - \frac{\pi}{\pi}\right)^\pi = (1 - 1)^\pi = 0^\pi = 0$$

Since, it is not of indeterminate form.

$Z = 0$   
 $\therefore$  The value of  $\lim_{x \rightarrow \pi} \left(1 - \frac{x}{\pi}\right)^\pi = 0$

2.  $\lim_{x \rightarrow 0^+} \left\{1 + \tan^2 \sqrt{x}\right\}^{1/2x}$

**Solution:**

Given:  $\lim_{x \rightarrow 0^+} \left\{1 + \tan^2 \sqrt{x}\right\}^{1/2x}$

The limit Let us use the theorem given below

If  $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$  such that  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$  exists, then  $\lim_{x \rightarrow a} [1 + f(x)]^{\frac{1}{g(x)}} = e^{\lim_{x \rightarrow a} \frac{f(x)}{g(x)}}$ .

So here,

$$f(x) = \tan^2 \sqrt{x}$$

$$g(x) = 2x$$

Then,

$$\begin{aligned} \lim_{x \rightarrow 0^+} \left\{1 + \tan^2 \sqrt{x}\right\}^{1/2x} &= e^{\lim_{x \rightarrow 0^+} \left(\frac{\tan^2 \sqrt{x}}{2x}\right)} \\ &= e^{\lim_{x \rightarrow 0^+} \left(\frac{\tan \sqrt{x}}{\sqrt{x}}\right) \times \left(\frac{\tan \sqrt{x}}{\sqrt{x}}\right) \times \frac{1}{2}} \\ &= e^{1 \times 1 \times \frac{1}{2}} \\ &= \sqrt{e} \end{aligned}$$

$\therefore$  The value of  $\lim_{x \rightarrow 0^+} \left\{1 + \tan^2 \sqrt{x}\right\}^{1/2x} = \sqrt{e}$

3.  $\lim_{x \rightarrow 0} (\cos x)^{1/\sin x}$

**Solution:**

Given:  $\lim_{x \rightarrow 0} (\cos x)^{1/\sin x}$

The limit  $x \rightarrow 0$

Let us use the theorem given below

If  $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$  such that  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$  exists, then  $\lim_{x \rightarrow a} [1 + f(x)]^{\frac{1}{g(x)}} = e^{\lim_{x \rightarrow a} \frac{f(x)}{g(x)}}$ .

So here,

$f(x) = \cos x - 1$

$g(x) = \sin x$

Then,

$$\begin{aligned} \lim_{x \rightarrow 0} (\cos x)^{1/\sin x} &= e^{\lim_{x \rightarrow 0} \left( \frac{\cos x - 1}{\sin x} \right)} \\ &= e^{\lim_{x \rightarrow 0} \left( \frac{-2 \sin^2 \frac{x}{2}}{2 \sin \frac{x}{2} \cos \frac{x}{2}} \right)} \\ &= e^{\lim_{x \rightarrow 0} \left( -\tan \frac{x}{2} \right)} \\ &= e^0 \\ &= 1 \end{aligned}$$

$\therefore$  The value of  $\lim_{x \rightarrow 0} (\cos x)^{1/\sin x} = 1$

4.  $\lim_{x \rightarrow 0} (\cos x + \sin x)^{1/x}$

**Solution:**

Given:  $\lim_{x \rightarrow 0} (\cos x + \sin x)^{1/x}$

The limit  $x \rightarrow 0$

Let us add and subtract '1' to the given expression, we get

$\lim_{x \rightarrow 0} [1 + \cos x + \sin x - 1]^{\frac{1}{x}}$

Let us use the theorem given below

If  $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$  such that  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$  exists, then  $\lim_{x \rightarrow a} [1 + f(x)]^{\frac{1}{g(x)}} = e^{\lim_{x \rightarrow a} \frac{f(x)}{g(x)}}$ .

So here,

$f(x) = \cos x + \sin x - 1$

$g(x) = x$

Then,

$$\lim_{x \rightarrow 0} (\cos x + \sin x)^{1/x} = e^{\lim_{x \rightarrow 0} \left( \frac{\cos x + \sin x - 1}{x} \right)}$$

Upon computing, we get

$$\begin{aligned} &= e^{\lim_{x \rightarrow 0} \left[ \frac{\sin x}{x} - \frac{(1 - \cos x)}{x} \right]} \\ &= e^{\lim_{x \rightarrow 0} \left( \frac{\sin x}{x} - \frac{2 \sin^2 \frac{x}{2}}{x} \right)} \\ &= e^{\lim_{x \rightarrow 0} \left( \frac{\sin x}{x} - \frac{2 \sin(\frac{x}{2}) \times \sin(\frac{x}{2})}{2 \times \frac{x}{2}} \right)} \end{aligned}$$

Now, substitute the value of x, we get

$$\begin{aligned} &= e^{1-0} \\ &= e^1 \\ &= e \end{aligned}$$

$$\therefore \text{The value of } \lim_{x \rightarrow 0} (\cos x + \sin x)^{1/x} = e$$

5.  $\lim_{x \rightarrow 0} (\cos x + a \sin bx)^{1/x}$

**Solution:**

Given:  $\lim_{x \rightarrow 0} (\cos x + a \sin bx)^{1/x}$

The limit  $\lim_{x \rightarrow 0}$

Let us add and subtract '1' to the given expression, we get

$$\lim_{x \rightarrow 0} [1 + \cos x + a \sin bx - 1]^{\frac{1}{x}}$$

Let us use the theorem given below

If  $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$  such that  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$  exists, then  $\lim_{x \rightarrow a} [1 + f(x)]^{\frac{1}{g(x)}} = e^{\lim_{x \rightarrow a} \frac{f(x)}{g(x)}}$ .

So here,

$$f(x) = \cos x + a \sin bx - 1$$

$$g(x) = x$$

Then,

$$\lim_{x \rightarrow 0} (\cos x + a \sin bx)^{1/x} = e^{\lim_{x \rightarrow 0} \left[ \frac{\cos x + a \sin bx - 1}{x} \right]}$$

Let us compute now, we get

$$\begin{aligned} &= e^{\lim_{x \rightarrow 0} \left[ \frac{b \times a \sin bx}{bx} - \frac{(1 - \cos x)}{x} \right]} \\ &= e^{\lim_{x \rightarrow 0} \left( \frac{ab \sin bx}{bx} - \frac{2 \sin^2 \frac{x}{2}}{x} \right)} \end{aligned}$$

Now, substitute the value of  $x$ , we get  
 $= e^{ab}$

$\therefore$  The value of  $\lim_{x \rightarrow 0} (\cos x + a \sin bx)^{1/x} = e^{ab}$

