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1. Find the domain of each of the following real valued functions of real variable:
(i) f (x) = 1/x
(ii) f (x) = 1/(x-7)
(iii) f (x) = (3x-2)/(x+1)
(iv) f (x) = (2x+1)/(x^2-9)
(v) f (x) = (x^2+2x+1)/(x^2-8x+12)
Solution:
(i) f (x) = 1/x
We know, f (x) is defined for all real values of x, except for the case when x = 0.
∴ Domain of f = R – {0}

(ii) f (x) = 1/(x-7)
We know, f (x) is defined for all real values of x, except for the case when x – 7 = 0 or x = 7.
∴ Domain of f = R – {7}

(iii) f (x) = (3x-2)/(x+1)
We know, f(x) is defined for all real values of x, except for the case when x + 1 = 0 or x = –1.
∴ Domain of f = R – {–1}

(iv) f (x) = (2x+1)/(x^2-9)
We know, f (x) is defined for all real values of x, except for the case when $x^2 – 9 = 0$.
$x^2 – 9 = 0$
$x^2 – 3^2 = 0$
(x + 3)(x – 3) = 0
x + 3 = 0 or x – 3 = 0
x = ± 3
∴ Domain of f = R – {–3, 3}

(v) f (x) = (x^2+2x+1)/(x^2-8x+12)
We know, f(x) is defined for all real values of x, except for the case when $x^2 – 8x + 12 = 0$.
$x^2 – 8x + 12 = 0$
$x^2 – 2x – 6x + 12 = 0$
x(x – 2) – 6(x – 2) = 0
(x – 2)(x – 6) = 0

x – 2 = 0 or x – 6 = 0
x = 2 or 6
∴ Domain of f = R – {2, 6}

2. Find the domain of each of the following real valued functions of real variable:
(i) f (x) = √(x-2)
(ii) f (x) = 1/(√(x^2-1))
(iii) f (x) = √(9-x^2)
(iv) f (x) = √(x-2)/(3-x)
Solution:
(i) f (x) = √(x-2)
We know the square of a real number is never negative.
f (x) takes real values only when x – 2 ≥ 0
x ≥ 2
∴ x ∈ [2, ∞)
∴ Domain (f) = [2, ∞)

(ii) f (x) = 1/(√(x^2-1))
We know the square of a real number is never negative.
f (x) takes real values only when $x^2 – 1 \geq 0$
$x^2 – 1^2 \geq 0$
(x + 1) (x – 1) ≥ 0
x ≤ –1 or x ≥ 1
∴ x ∈ (–∞, –1] ∪ [1, ∞)
In addition, f (x) is also undefined when $x^2 – 1 = 0$ because denominator will be zero and the result will be indeterminate.
$x^2 – 1 = 0 \Rightarrow x = \pm 1$
So, x ∈ (–∞, –1] ∪ [1, ∞) – {–1, 1}
x ∈ (–∞, –1) ∪ (1, ∞)
∴ Domain (f) = (–∞, –1) ∪ (1, ∞)

(iii) f (x) = √(9-x^2)
We know the square of a real number is never negative.
f (x) takes real values only when $9 – x^2 \geq 0$
$9 \geq x^2$
$x^2 \leq 9$
$x^2 – 9 \leq 0$
$x^2 – 3^2 \leq 0$
(x + 3)(x – 3) ≤ 0

$x \geq -3$ and $x \leq 3$
$x \in [-3, 3]$
∴ Domain (f) = [–3, 3]

(iv) f (x) = √(x-2)/(3-x)
We know the square root of a real number is never negative.
f (x) takes real values only when x – 2 and 3 – x are both positive and negative.
(a) Both x – 2 and 3 – x are positive
$x - 2 \geq 0$
$x \geq 2$

$3 - x \geq 0$
$x \leq 3$
Hence, $x \geq 2$ and $x \leq 3$
∴ $x \in [2, 3]$

(b) Both x – 2 and 3 – x are negative
$x - 2 \leq 0$
$x \leq 2$

$3 - x \leq 0$
$x \geq 3$
Hence, $x \leq 2$ and $x \geq 3$
However, the intersection of these sets is null set. Thus, this case is not possible.
Hence, $x \in [2, 3] - [3]$
$x \in [2, 3]$
∴ Domain (f) = [2, 3]

3. Find the domain and range of each of the following real valued functions:
(i) f (x) = (ax+b)/(bx-a)
(ii) f (x) = (ax-b)/(cx-d)
(iii) f (x) = √(x-1)
(iv) f (x) = √(x-3)
(v) f (x) = (x-2)/(2-x)
(vi) f (x) = |x-1|
(vii) f (x) = -|x|
(viii) f (x) = √(9-x^2)
Solution:
(i) f (x) = (ax+b)/(bx-a)

f(x) is defined for all real values of x, except for the case when bx – a = 0 or x = a/b.
Domain (f) = R – (a/b)
Let f (x) = y
(ax+b)/(bx-a) = y
ax + b = y(bx – a)
ax + b = bxy – ay
ax – bxy = –ay – b
x(a – by) = –(ay + b)
∴ x = - (ay+b)/(a-by)
When a – by = 0 or y = a/b
Hence, f(x) cannot take the value a/b.
∴ Range (f) = R – (a/b)

(ii) f (x) = (ax-b)/(cx-d)
f(x) is defined for all real values of x, except for the case when cx – d = 0 or x = d/c.
Domain (f) = R – (d/c)
Let f (x) = y
(ax-b)/(cx-d) = y
ax – b = y(cx – d)
ax – b = cxy – dy
ax – cxy = b – dy
x(a – cy) = b – dy
∴ x = (b-dy)/(a-cy)
When a – cy = 0 or y = a/c,
Hence, f(x) cannot take the value a/c.
∴ Range (f) = R – (a/c)

(iii) f (x) = √(x-1)
We know the square of a real number is never negative.
f(x) takes real values only when x – 1 ≥ 0
x ≥ 1
∴ x ∈ [1, ∞)
Thus, domain (f) = [1, ∞)
When x ≥ 1, we have x – 1 ≥ 0
Hence, √(x-1) ≥ 0 ⇒ f (x) ≥ 0
f(x) ∈ [0, ∞)
∴ Range (f) = [0, ∞)

(iv) f (x) = √(x-3)

We know the square of a real number is never negative.
f (x) takes real values only when $x - 3 \geq 0$
$x \geq 3$
$\therefore x \in [3, \infty)$
Domain (f) = $[3, \infty)$
When $x \geq 3$, we have $x - 3 \geq 0$
Hence, $\sqrt{(x-3)} \geq 0 \Rightarrow f(x) \geq 0$
$f(x) \in [0, \infty)$
$\therefore$ Range (f) = $[0, \infty)$

(v) f (x) = (x-2)/(2-x)
f(x) is defined for all real values of x, except for the case when 2 – x = 0 or x = 2.
Domain (f) = R – {2}
We have, f (x) = (x-2)/(2-x)
f (x) = -(2-x)/(2-x)
= –1
When $x \neq 2$, f(x) = –1
$\therefore$ Range (f) = {–1}

(vi) f (x) = |x-1|

$$we\ know\ |x| = \begin{cases} -x, x < 0 \\ x, x \geq 0 \end{cases}$$

Now we have,

$$|x-1| = \begin{cases} -(x-1), x-1 < 0 \\ x-1, x-1 \geq 0 \end{cases}$$

$$\therefore f(x) = |x-1| = \begin{cases} 1-x, x < 1 \\ x-1, x \geq 1 \end{cases}$$

Hence, f(x) is defined for all real numbers x.
Domain (f) = R
When, x < 1, we have x – 1 < 0 or 1 – x > 0.
$|x - 1| > 0 \Rightarrow f(x) > 0$

When, $x \geq 1$, we have $x - 1 \geq 0$.
$|x - 1| \geq 0 \Rightarrow f(x) \geq 0$

$\therefore f(x) \geq 0$ or $f(x) \in [0, \infty)$
Range (f) = $[0, \infty)$

(vii) f (x) = -|x|

$we\ know\ |x| = \begin{cases} -x, x < 0 \\ x, x \geq 0 \end{cases}$

Now we have,

$-|x| = \begin{cases} -(-x), x < 0 \\ -x, x \geq 0 \end{cases}$

$\therefore f(x) = -|x| = \begin{cases} x, x < 0 \\ -x, x \geq 0 \end{cases}$

Hence, f(x) is defined for all real numbers x.

Domain (f) = R

When, $x < 0$, we have $-|x| < 0$

$f(x) < 0$

When, $x \geq 0$, we have $-x \leq 0$.

$-|x| \leq 0 \Rightarrow f(x) \leq 0$

$\therefore f(x) \leq 0$ or $f(x) \in (-\infty, 0]$

Range (f) = $(-\infty, 0]$

(viii) $f(x) = \sqrt{(9-x^2)}$

We know the square of a real number is never negative.

f(x) takes real values only when $9 - x^2 \geq 0$

$9 \geq x^2$

$x^2 \leq 9$

$x^2 - 9 \leq 0$

$x^2 - 3^2 \leq 0$

$(x + 3)(x - 3) \leq 0$

$x \geq -3$ and $x \leq 3$

$\therefore x \in [-3, 3]$

Domain (f) = $[-3, 3]$

When, $x \in [-3, 3]$, we have $0 \leq 9 - x^2 \leq 9$

$0 \leq \sqrt{(9-x^2)} \leq 3 \Rightarrow 0 \leq f(x) \leq 3$

$\therefore f(x) \in [0, 3]$

Range (f) = $[0, 3]$