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Differentiate the following functions with respect to x:

1. $x^3 \sin x$

Solution:

Let us consider $y = x^3 \sin x$

We need to find dy/dx

We know that y is a product of two functions say u and v where,

$u = x^3$ and $v = \sin x$

$\therefore y = uv$

Now let us apply product rule of differentiation.

By using product rule, we get

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ … Equation (1)

As, $u = x^3$

$\frac{du}{dx} = 3x^{3-1} = 3x^2$ … Equation (2) {Since, $\frac{d}{dx}(x^n) = nx^{n-1}$}

As, $v = \sin x$

$\frac{dv}{dx} = \frac{d}{dx}(\sin x) = \cos x$ … Equation (3) {Since, $\frac{d}{dx}(\sin x) = \cos x$}

From equation (1), we can find dy/dx

$\frac{dy}{dx} = x^3\frac{dv}{dx} + \sin x\frac{du}{dx}$

$\frac{dy}{dx} = x^3\cos x + 3x^2\sin x$ {Using equation 2 & 3}

$\therefore \frac{dy}{dx} = x^3\cos x + 3x^2\sin x$

2. $x^3 e^x$

Solution:

Let us consider $y = x^3 e^x$

We need to find dy/dx

We know that y is a product of two functions say u and v where,

$u = x^3$ and $v = e^x$

$\therefore y = uv$

Now let us apply product rule of differentiation.

By using product rule, we get

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ … Equation (1)

As, $u = x^3$

$\frac{du}{dx} = 3x^{3-1} = 3x^2$ … Equation (2) $\{\frac{d}{dx}(x^n) = nx^{n-1}\}$

As, $v = e^x$

$\frac{dv}{dx} = \frac{d}{dx}(e^x) = e^x$ … Equation (3) {Since, $\frac{d}{dx}(e^x) = e^x$}

Now from equation (1), we can find dy/dx

$\frac{dy}{dx} = x^3\frac{dv}{dx} + e^x\frac{du}{dx}$

$\frac{dy}{dx} = x^3e^x + 3x^2e^x$ {Using equation 2 & 3}

$\therefore \frac{dy}{dx} = x^2e^x(3+x)$

3. x^2 e^x log x

Solution:

Let us consider y = x^2 e^x log x

We need to find dy/dx

We know that y is a product of two functions say u and v where,

u = x^2 and v = e^x, w = 1/x

∴ y = uv

Now let us apply product rule of differentiation.

By using product rule, we get

$\frac{dy}{dx} = uw\frac{dv}{dx} + vw\frac{du}{dx} + uv\frac{dw}{dx}$ …equation 1

As, $u = x^2$

$\frac{du}{dx} = 2x^{2-1} = 2x$ … Equation (2) {Since, $\frac{d}{dx}(x^n) = nx^{n-1}$}

As, $v = e^x$

$\frac{dv}{dx} = \frac{d}{dx}(e^x) = e^x$ … Equation (3) {Since, $\frac{d}{dx}(e^x) = e^x$}

As, w = log x

$\frac{dw}{dx} = \frac{d}{dx}(\log x) = \frac{1}{x}$ … Equation (4) {Since, $\frac{d}{dx}(\log_e x) = \frac{1}{x}$}

Now, from equation 1, we can find dy/dx

$\frac{dy}{dx} = x^2\log x\frac{dv}{dx} + e^x\log x\frac{du}{dx} + x^2e^x\frac{dw}{dx}$

$$\frac{dy}{dx} = x^2 e^x \log x + 2xe^x \log x + x^2 e^x \frac{1}{x} \text{ \{Using equation 2, 3 \& 4\}}$$

$$\therefore \frac{dy}{dx} = xe^x(1 + x \log x + 2 \log x)$$

4. x^n tan x

Solution:

Let us consider y = x^n tan x

We need to find dy/dx

We know that y is a product of two functions say u and v where,

u = x^n and v = tan x

∴ y = uv

Now let us apply product rule of differentiation.

By using product rule, we get

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx} \ldots \text{ Equation 1}$$

As, $u = x^n$

$$\frac{du}{dx} = nx^{n-1} \ldots \text{ Equation 2 } \left\{\text{Since, } \frac{d}{dx}(x^n) = nx^{n-1}\right\}$$

As, v = tan x

$$\frac{dv}{dx} = \frac{d}{dx}(\tan x) = \sec^2 x \ldots \text{ Equation 3 } \left\{\text{Since, } \frac{d}{dx}(\tan x) = \sec^2 x\right\}$$

Now, from equation 1, we can find dy/dx

$$\frac{dy}{dx} = x^n\frac{dv}{dx} + \tan x\frac{du}{dx}$$

$$\frac{dy}{dx} = x^n \sec^2 x + nx^{n-1}\tan x \text{ \{Using equation 2 \& 3\}}$$

$$\therefore \frac{dy}{dx} = x^{n-1}(n \tan x + x \sec^2 x)$$

5. x^n $\log_a$ x

Solution:

Let us consider y = x^n $\log_a$ x

We need to find dy/dx

We know that y is a product of two functions say u and v where,

u = x^n and v = $\log_a$ x

∴ y = uv

Now let us apply product rule of differentiation.

By using product rule, we get

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ … Equation (1)

As, $u = x^n$

$\frac{du}{dx} = nx^{n-1}$ … Equation (2) {Since, $\frac{d}{dx}(x^n) = nx^{n-1}$}

As, $v = \log_a x$

$\frac{dv}{dx} = \frac{d}{dx}(\log_a x) = \frac{1}{x\log_e a}$ … Equation (3) {Since, $\frac{d}{dx}(\log_a x) = \frac{1}{x\log_e a}$}

Now, from equation 1, we can find dy/dx

$\frac{dy}{dx} = x^n\frac{dv}{dx} + \log_a x\frac{du}{dx}$

$\frac{dy}{dx} = x^n\frac{1}{x\log_e a} + nx^{n-1}\log_a x$ {Using equation 2 & 3}

$\therefore \frac{dy}{dx} = x^{n-1}\left(n\log_a x + \frac{1}{\log a}\right)$