EXERCISE 5.3 PAGE NO: 5.39

1. Find the values of the following trigonometric ratios:
(i) sin 5π/3
(ii) sin 17π
(iii) tan 11π/6
(iv) cos (-25π/4)
(v) tan 7π/4
(vi) sin 17π/6
(vii) cos 19π/6
(viii) sin (-11π/6)
(ix) cosec (-20π/3)
(x) tan (-13π/4)
(xi) cos 19π/4
(xii) sin 41π/4
(xiii) cos 39π/4
(xiv) sin 151π/6

Solution:

(i) sin 5π/3

$5\pi/3 = (5/3 \times 180)^o$

$= 300^o$

$= (90\times3 + 30)^o$

Since, 300^o lies in IV quadrant in which sine function is negative.

$\sin 5\pi/3 = \sin (300)^o$

$= \sin (90\times3 + 30)^o$

$= -\cos 30^o$

$= -\sqrt{3}/2$

(ii) sin 17π

$\text{Sin } 17\pi = \sin 3060^o$

$= \sin (90\times34 + 0)^o$

Since, 3060^o lies in the negative direction of x-axis i.e., on boundary line of II and III quadrants.

$\text{Sin } 17\pi = \sin (90\times34 + 0)^o$

$= -\sin 0^o$

$= 0$

(iii) tan 11π/6

$\tan 11\pi/6 = (11/6 \times 180)^o$

$= 330^o$

Since, 330^o lies in the IV quadrant in which tangent function is negative.

$\tan 11\pi/6 = \tan (300)^o$
$= \tan (90\times3 + 60)^o$
$= - \cot 60^o$
$= - 1/\sqrt{3}$

(iv) $\cos (-25\pi/4)$

$\cos (-25\pi/4) = \cos (-1125)^o$
$= \cos (1125)^o$

Since, 1125^o lies in the I quadrant in which cosine function is positive.

$\cos (1125)^o = \cos (90\times12 + 45)^o$
$= \cos 45^o$
$= 1/\sqrt{2}$

(v) $\tan 7\pi/4$

$\tan 7\pi/4 = \tan 315^o$
$= \tan (90\times3 + 45)^o$

Since, 315^o lies in the IV quadrant in which tangent function is negative.

$\tan 315^o = \tan (90\times3 + 45)^o$
$= - \cot 45^o$
$= -1$

(vi) $\sin 17\pi/6$

$\sin 17\pi/6 = \sin 510^o$
$= \sin (90\times5 + 60)^o$

Since, 510^o lies in the II quadrant in which sine function is positive.

$\sin 510^o = \sin (90\times5 + 60)^o$
$= \cos 60^o$
$= 1/2$

(vii) $\cos 19\pi/6$

$\cos 19\pi/6 = \cos 570^o$
$= \cos (90\times6 + 30)^o$

Since, 570^o lies in III quadrant in which cosine function is negative.

$\cos 570^o = \cos (90\times6 + 30)^o$
$= - \cos 30^o$
$= - \sqrt{3}/2$

(viii) sin $(-11\pi/6)$

sin $(-11\pi/6)$ = sin (-330^o)

= - sin $(90\times3 + 60)^o$

Since, 330^o lies in the IV quadrant in which the sine function is negative.

sin (-330^o) = - sin $(90\times3 + 60)^o$

= - (-cos 60^o)

= - (-1/2)

= 1/2

(ix) cosec $(-20\pi/3)$

cosec $(-20\pi/3)$ = cosec $(-1200)^o$

= - cosec $(1200)^o$

= - cosec $(90\times13 + 30)^o$

Since, 1200^o lies in the II quadrant in which cosec function is positive.

cosec $(-1200)^o$ = - cosec $(90\times13 + 30)^o$

= - sec 30^o

= $-2/\sqrt{3}$

(x) tan $(-13\pi/4)$

tan $(-13\pi/4)$ = tan $(-585)^o$

= - tan $(90\times6 + 45)^o$

Since, 585^o lies in the III quadrant in which the tangent function is positive.

tan $(-585)^o$ = - tan $(90\times6 + 45)^o$

= - tan 45^o

= -1

(xi) cos $19\pi/4$

cos $19\pi/4$ = cos 855^o

= cos $(90\times9 + 45)^o$

Since, 855^o lies in the II quadrant in which the cosine function is negative.

cos 855^o = cos $(90\times9 + 45)^o$

= - sin 45^o

= $- 1/\sqrt{2}$

(xii) sin $41\pi/4$

sin $41\pi/4$ = sin 1845^o

= sin $(90\times20 + 45)^o$

Since, 1845^o lies in the I quadrant in which the sine function is positive.

sin 1845^o = sin $(90\times20 + 45)^o$

= sin 45°
= 1/√2

(xiii) cos 39π/4
cos 39π/4 = cos 1755°
= cos (90×19 + 45)°
Since, 1755° lies in the IV quadrant in which the cosine function is positive.
cos 1755° = cos (90×19 + 45)°
= sin 45°
= 1/√2

(xiv) sin 151π/6
sin 151π/6 = sin 4530°
= sin (90×50 + 30)°
Since, 4530° lies in the III quadrant in which the sine function is negative.
sin 4530° = sin (90×50 + 30)°
= - sin 30°
= -1/2

2. prove that:
(i) tan 225° cot 405° + tan 765° cot 675° = 0
(ii) sin 8π/3 cos 23π/6 + cos 13π/3 sin 35π/6 = 1/2
(iii) cos 24° + cos 55° + cos 125° + cos 204° + cos 300° = 1/2
(iv) tan (-125°) cot (-405°) – tan (-765°) cot (675°) = 0
(v) cos 570° sin 510° + sin (-330°) cos (-390°) = 0
(vi) tan 11π/3 – 2 sin 4π/6 – 3/4 cosec^2 π/4 + 4 cos^2 17π/6 = (3 - 4√3)/2
(vii) 3 sin π/6 sec π/3 – 4 sin 5π/6 cot π/4 = 1
Solution:
(i) tan 225° cot 405° + tan 765° cot 675° = 0
Let us consider LHS:
tan 225° cot 405° + tan 765° cot 675°
tan (90° × 2 + 45°) cot (90° × 4 + 45°) + tan (90° × 8 + 45°) cot (90° × 7 + 45°)
We know that when n is odd, cot → tan.
tan 45° cot 45° + tan 45° [-tan 45°]
tan 45° cot 45° - tan 45° tan 45°
1 × 1 – 1 × 1
1 – 1
0 = RHS
∴ LHS = RHS

Hence proved.

(ii) sin 8π/3 cos 23π/6 + cos 13π/3 sin 35π/6 = 1/2
Let us consider LHS:
sin 8π/3 cos 23π/6 + cos 13π/3 sin 35π/6
sin 480° cos 690° + cos 780° sin 1050°
sin (90° × 5 + 30°) cos (90° × 7 + 60°) + cos (90° × 8 + 60°) sin (90° × 11 + 60°)
We know that when n is odd, sin → cos and cos → sin.
cos 30° sin 60° + cos 60° [-cos 60°]
√3/2 × √3/2 – 1/2 × 1/2
3/4 - 1/4
2/4
1/2
= RHS
∴ LHS = RHS
Hence proved.

(iii) cos 24° + cos 55° + cos 125° + cos 204° + cos 300° = 1/2
Let us consider LHS:
cos 24° + cos 55° + cos 125° + cos 204° + cos 300°
cos 24° + cos (90° × 1 – 35°) + cos (90° × 1 + 35°) + cos (90° × 2 + 24°) + cos (90° × 3 + 30°)
We know that when n is odd, cos → sin.
cos 24° + sin 35° - sin 35° - cos 24° + sin 30°
0 + 0 + 1/2
1/2
= RHS
∴ LHS = RHS
Hence proved.

(iv) tan (-125°) cot (-405°) – tan (-765°) cot (675°) = 0
Let us consider LHS:
tan (-125°) cot (-405°) – tan (-765°) cot (675°)
We know that tan (-x) = -tan (x) and cot (-x) = -cot (x).
[-tan (225°)] [-cot (405°)] – [-tan (765°)] cot (675°)
tan (225°) cot (405°) + tan (765°) cot (675°)
tan (90° × 2 + 45°) cot (90° × 4 + 45°) + tan (90° × 8 + 45°) cot (90° × 7 + 45°)
tan 45° cot 45° + tan 45° [-tan 45°]
1 × 1 + 1 × (-1)

1 – 1
0
= RHS
∴ LHS = RHS
Hence proved.

(v) $\cos 570^\circ \sin 510^\circ + \sin(-330^\circ)\cos(-390^\circ) = 0$
Let us consider LHS:
$\cos 570^\circ \sin 510^\circ + \sin(-330^\circ)\cos(-390^\circ)$
We know that $\sin(-x) = -\sin(x)$ and $\cos(-x) = +\cos(x)$.
$\cos 570^\circ \sin 510^\circ + [-\sin(330^\circ)]\cos(390^\circ)$
$\cos 570^\circ \sin 510^\circ - \sin(330^\circ)\cos(390^\circ)$
$\cos(90^\circ \times 6 + 30^\circ)\sin(90^\circ \times 5 + 60^\circ) – \sin(90^\circ \times 3 + 60^\circ)\cos(90^\circ \times 4 + 30^\circ)$
We know that cos is negative at $90^\circ + \theta$ i.e. in Q_2 and when n is odd, sin → cos and cos → sin.
$-\cos 30^\circ \cos 60^\circ - [-\cos 60^\circ]\cos 30^\circ$
$-\cos 30^\circ \cos 60^\circ + \cos 60^\circ \cos 30^\circ$
0
= RHS
∴ LHS = RHS
Hence proved.

(vi) $\tan 11\pi/3 – 2 \sin 4\pi/6 – 3/4 \operatorname{cosec}^2 \pi/4 + 4 \cos^2 17\pi/6 = (3 - 4\sqrt{3})/2$
Let us consider LHS:
$\tan 11\pi/3 – 2 \sin 4\pi/6 – 3/4 \operatorname{cosec}^2 \pi/4 + 4 \cos^2 17\pi/6$
$\tan (11 \times 180^\circ)/3 – 2 \sin (4 \times 180^\circ)/6 – 3/4 \operatorname{cosec}^2 180^\circ/4 + 4 \cos^2 (17 \times 180^\circ)/6$
$\tan 660^\circ – 2 \sin 120^\circ - 3/4 (\operatorname{cosec} 45^\circ)^2 + 4 (\cos 510^\circ)^2$
$\tan(90^\circ \times 7 + 30^\circ) – 2 \sin(90^\circ \times 1 + 30^\circ) – 3/4 [\operatorname{cosec} 45^\circ]^2 + 4 [\cos(90^\circ \times 5 + 60^\circ)]^2$
We know that tan and cos is negative at $90^\circ + \theta$ i.e. in Q_2 and when n is odd, tan → cot, sin → cos and cos → sin.
$[-\cot 30^\circ] – 2 \cos 30^\circ - 3/4 [\operatorname{cosec} 45^\circ]^2 + [-\sin 60^\circ]^2$
$- \cot 30^\circ - 2 \cos 30^\circ - 3/4 [\operatorname{cosec} 45^\circ]^2 + [\sin 60^\circ]^2$
$-\sqrt{3} - 2\sqrt{3}/2 – 3/4 (\sqrt{2})^2 + 4 (\sqrt{3}/2)^2$
$-\sqrt{3} - \sqrt{3} – 6/4 + 12/4$
$(3 - 4\sqrt{3})/2$
= RHS
∴ LHS = RHS
Hence proved.

(vii) 3 sin π/6 sec π/3 – 4 sin 5π/6 cot π/4 = 1
Let us consider LHS:
3 sin π/6 sec π/3 – 4 sin 5π/6 cot π/4
3 sin 180°/6 sec 180°/3 – 4 sin 5(180°)/6 cot 180°/4
3 sin 30° sec 60° - 4 sin 150° cot 45°
3 sin 30° sec 60° - 4 sin (90° × 1 + 60°) cot 45°
We know that when n is odd, sin → cos.
3 sin 30° sec 60° - 4 cos 60° cot 45°
3 (1/2) (2) – 4 (1/2) (1)
3 – 2
1
= RHS
∴ LHS = RHS
Hence proved.

3. Prove that:

(i)

$$\frac{cos(2\pi + x)\, cosec(2\pi + x)\, tan(\pi/2 + x)}{sec(\pi/2 + x)\, cos\, x\, cot(\pi + x)} = 1$$

(ii)

$$\frac{cosec\,(90^o + x) + cot(450^o + x)}{cosec(90^o - x) + tan(180^o - x)} + \frac{tan(180^o + x) + sec(180^o - x)}{tan(360^o + x) - sec(-x)} = 2$$

(iii)

$$\frac{sin(\pi + x)\, cos(\frac{\pi}{2} + x)\, tan(\frac{3\pi}{2} - x)\, cot(2\pi - x)}{sin(2\pi - x)\, cos(2\pi + x)\, cosec(-x)\, sin(\frac{3\pi}{2} - x)} = 1$$

(iv)

$$\left\{1 + cot\, x - sec(\frac{\pi}{2} + x)\right\} \left\{1 + cot\, x + sec(\frac{\pi}{2} + x)\right\} = 2\, cot\, x$$

(v)

$$\frac{tan(\frac{\pi}{2} - x)\, sec(\pi - x)\, sin(-x)}{sin(\pi + x)\, cot(2\pi - x)\, cosec(\frac{\pi}{2} - x)} = 1$$

Solution:

(i)

$$\frac{cos(2\pi + x)\, cosec(2\pi + x)\, tan(\pi/2 + x)}{sec(\pi/2 + x)\, cos\, x\, cot(\pi + x)} = 1$$

Let us consider LHS:

$$\frac{cos(2\pi + x)\, cosec(2\pi + x)\, tan(\pi/2 + x)}{sec(\pi/2 + x)\, cos\, x\, cot(\pi + x)}$$

$$\frac{cos\, x\, cosec\, x[-cot\, x]}{[-cosec\, x]\, cos\, x\, cot\, x}$$

$$\frac{-cos\, x cosec\, x\, cot\, x}{-cosec\, x\, cos\, x\, cot\, x}$$

1 = RHS

∴ LHS = RHS

Hence proved.

(ii)

$$\frac{cosec\,(90^o + x) + cot(450^o + x)}{cosec(90^o - x) + tan(180^o - x)} + \frac{tan(180^o + x) + sec(180^o - x)}{tan(360^o + x) - sec(-x)} = 2$$

Let us consider LHS:

$$\frac{cosec\,(90^o + x) + cot(450^o + x)}{cosec(90^o - x) + tan(180^o - x)} + \frac{tan(180^o + x) + sec(180^o - x)}{tan(360^o + x) - sec(-x)}$$

$$\frac{cosec\,(90^o + x) + cot(90^o \times 5 + x)}{cosec(90^o - x) + tan(90^o \times 2 - x)} + \frac{tan(90^o \times 2 + x) + sec(90^o \times 2 - x)}{tan(90^o \times 4 + x) - sec(-x)}$$

We know that when n is odd, cosec → sec and also sec (-x) = sec x.

$$\frac{sec\, x + cot(90^o \times 5 + x)}{cosec(90^o - x) + tan(90^o \times 2 - x)} + \frac{tan(90^o \times 2 + x) + sec(90^o \times 2 - x)}{tan(90^o \times 4 + x) - sec(-x)}$$

$$\frac{sec\, x - tan\, x}{sec\, x - tan\, x} + \frac{tan\, x - sec\, x}{tan\, x - sec\, x}$$

1 + 1

2 = RHS

∴ LHS = RHS

Hence proved.

(iii)

$$\frac{sin(\pi + x)\, cos(\frac{\pi}{2} + x)\, tan(\frac{3\pi}{2} - x)\, cot(2\pi - x)}{sin(2\pi - x)\, cos(2\pi + x)\, cosec(-x)\, sin(\frac{3\pi}{2} - x)} = 1$$

Let us consider LHS:

$$\frac{sin(\pi + x)\, cos(\frac{\pi}{2} + x)\, tan(\frac{3\pi}{2} - x)\, cot(2\pi - x)}{sin(2\pi - x)\, cos(2\pi + x)\, cosec(-x)\, sin(\frac{3\pi}{2} - x)}$$

$$\frac{sin(180^o - x)\, cos(90^o + x)\, tan(270^o - x)\, cot(360^o - x)}{sin(360^o - x)\, cos(360^o + x)\ \ cosec(-x)\, sin(270^o - x)}$$

We know that cosec (-x) = - cosec x.

$$\frac{sin(90^o \times 2 - x)\, cos(90^o \times 1 + x)\, tan(90^o \times 3 - x)\, cot(90^o \times 4 - x)}{sin(90^o \times 4 - x)\, cos(90^o \times 4 + x)\, [-cosec(x)]\, sin(90^o \times 3 - x)}$$

We know that when n is odd, cos → sin, tan → cot and sin → cos.

$$\frac{(-sin\, x)(-sin\, x)\, cot\, x\, (-cot\, x)}{(-sin\, x)\, cos\, x\, (-cosec\, x)(-cos\, x)}$$

$$\frac{sin^2 x\, cot^2 x}{sin\, x\, cosec\, x\, cos\, x\, cos\, x}$$

$$\frac{sin^2 x\ \times \frac{cos^2 x}{sin^2 x}}{sin\, x\ \times \frac{1}{sin\, x} \times cos^2 x}$$

$$\frac{cos^2 x}{cos^2 x}$$

1 = RHS

∴ LHS = RHS

Hence proved.

(iv)

$$\left\{1 + cot\, x - sec(\frac{\pi}{2} + x)\right\} \left\{1 + cot\, x + sec(\frac{\pi}{2} + x)\right\} = 2\, cot\, x$$

Let us consider LHS:

$$\left\{1 + cot\, x - sec(\frac{\pi}{2} + x)\right\} \left\{1 + cot\, x + sec(\frac{\pi}{2} + x)\right\}$$

{1 + cot x – (-cosec x)} {1 + cot x + (-cosec x)}

{1 + cot x + cosec x} {1 + cot x – cosec x}

{(1 + cot x) + (cosec x)} {(1 + cot x) – (cosec x)}

By using the formula, $(a + b)(a – b) = a^2 – b^2$
$(1 + \cot x)^2 – (\text{cosec } x)^2$
$1 + \cot^2 x + 2 \cot x – \text{cosec}^2 x$
We know that $1 + \cot^2 x = \text{cosec}^2 x$
$\text{cosec}^2 x + 2 \cot x – \text{cosec}^2 x$
$2 \cot x$ = RHS
∴ LHS = RHS
Hence proved.

(v)

$$\frac{tan(\frac{\pi}{2}-x)\ sec(\pi-x)\ sin(-x)}{sin(\pi+x)\ cot(2\pi-x)\ cosec(\frac{\pi}{2}-x)}=1$$

Let us consider LHS:

$$\frac{tan(\frac{\pi}{2}-x)\ sec(\pi-x)\ sin(-x)}{sin(\pi+x)\ cot(2\pi-x)\ cosec(\frac{\pi}{2}-x)}$$

$$\frac{tan(90^o-x)\ sec(180^o-x)\ sin(-x)}{sin(180^o+x)\ cot(360^o-x)cosec(90^o-x)}$$

We know that sin (-x) = -sin x.

$$\frac{tan(90^o\times1-x)\ sec(90^o\times2-x)\ [-sin(x)]}{sin(90^o\times2+x)\ cot(90^o\times4-x)cosec(90^o\times1-x)}$$

We know that when n is odd, tan → cot and cosec → sec.

$$\frac{(cot\,x)(-sec\,x)(-sin\,x)}{(-sin\,x)(-cot\,x)(sec\,x)}$$

$$\frac{cot\,x\ sec\,x\ sin\,x}{sin\,x\ cot\,x\ sec\,x}$$

1 = RHS
∴ LHS = RHS
Hence proved.

4. Prove that: $\sin^2 \pi/18 + \sin^2 \pi/9 + \sin^2 7\pi/18 + \sin^2 4\pi/9 = 2$
Solution:
Let us consider LHS:
$\sin^2 \pi/18 + \sin^2 \pi/9 + \sin^2 7\pi/18 + \sin^2 4\pi/9$

$\sin^2 \pi/18 + \sin^2 2\pi/18 + \sin^2 7\pi/18 + \sin^2 8\pi/18$
$\sin^2 \pi/18 + \sin^2 2\pi/18 + \sin^2 (\pi/2 - 2\pi/18) + \sin^2 (\pi/2 – \pi/18)$
We know that when n is odd, sin → cos.
$\sin^2 \pi/18 + \sin^2 2\pi/18 + \cos^2 2\pi/18 + \cos^2 2\pi/18$
when rearranged,
$\sin^2 \pi/18 + \cos^2 2\pi/18 + \sin^2 \pi/18 + \cos^2 2\pi/18$
We know that $\sin^2 x + \cos^2 x = 1$.
So,
1 + 1
2 = RHS
∴ LHS = RHS
Hence proved.