Prove the following identities:

1. $\sec^4 x - \sec^2 x = \tan^4 x + \tan^2 x$

Solution:

Let us consider LHS: $\sec^4 x - \sec^2 x$

$(\sec^2 x)^2 - \sec^2 x$

By using the formula, $\sec^2 \theta = 1 + \tan^2 \theta$.

$(1 + \tan^2 x)^2 - (1 + \tan^2 x)$

$1 + 2\tan^2 x + \tan^4 x - 1 - \tan^2 x$

$\tan^4 x + \tan^2 x$

= RHS

∴ LHS = RHS

Hence proved.

2. $\sin^6 x + \cos^6 x = 1 - 3\sin^2 x \cos^2 x$

Solution:

Let us consider LHS: $\sin^6 x + \cos^6 x$

$(\sin^2 x)^3 + (\cos^2 x)^3$

By using the formula, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$(\sin^2 x + \cos^2 x)[(\sin^2 x)^2 + (\cos^2 x)^2 - \sin^2 x \cos^2 x]$

By using the formula, $\sin^2 x + \cos^2 x = 1$ and $a^2 + b^2 = (a + b)^2 - 2ab$

$1 \times [(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x - \sin^2 x \cos^2 x$

$1^2 - 3\sin^2 x \cos^2 x$

$1 - 3\sin^2 x \cos^2 x$

= RHS

∴ LHS = RHS

Hence proved.

3. $(\operatorname{cosec} x - \sin x)(\sec x - \cos x)(\tan x + \cot x) = 1$

Solution:

Let us consider LHS: $(\operatorname{cosec} x - \sin x)(\sec x - \cos x)(\tan x + \cot x)$

By using the formulas

$\operatorname{cosec} \theta = 1/\sin \theta$;

$\sec \theta = 1/\cos \theta$;

$\tan \theta = \sin \theta / \cos \theta$;

$\cot \theta = \cos \theta / \sin \theta$

Now,

$$\left(\frac{1}{\sin x} - \sin x\right)\left(\frac{1}{\cos x} - \cos x\right)\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)$$

$$\frac{1 - \sin^2 x}{\sin x} \times \frac{1 - \cos^2 x}{\cos x} \times \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$

By using the formula,

Sin^2 x + cos^2 x =1;

$$\frac{\cos^2 x}{\sin x} \times \frac{\sin^2 x}{\cos x} \times \frac{1}{\sin x \cos x}$$

1 = RHS

∴ LHS = RHS

Hence proved.

4. cosec x (sec x – 1) – cot x (1 – cos x) = tan x – sin x

Solution:

Let us consider LHS: cosec x (sec x – 1) – cot x (1 – cos x)

By using the formulas

cosec θ = 1/sin θ;

sec θ = 1/cos θ;

tan θ = sin θ / cos θ;

cot θ = cos θ / sin θ

Now,

$$\frac{1}{\sin x}\left(\frac{1}{\cos x} - 1\right) - \frac{\cos x}{\sin x}(1 - \cos x)$$

$$\frac{1}{\sin x}\left(\frac{1 - \cos x}{\cos x}\right) - \frac{\cos x}{\sin x}(1 - \cos x)$$

$$\left(\frac{1 - \cos x}{\sin x}\right)\left(\frac{1}{\cos x} - \cos x\right)$$

$$\left(\frac{1 - \cos x}{\sin x}\right)\left(\frac{1 - \cos^2 x}{\cos x}\right)$$

By using the formula, 1 – cos^{2}x = sin^{2}x;

$$\left(\frac{1 - \cos x}{\sin x}\right)\left(\frac{\sin^2 x}{\cos x}\right)$$

$$(1 - \cos x)\left(\frac{\sin x}{\cos x}\right)$$

$$\frac{\sin x}{\cos x} - \sin x$$

$$\tan x - \sin x$$

= RHS

∴ LHS = RHS

Hence Proved.

5.

$$\frac{1 - \sin x \cos x}{\cos x (\sec x - \operatorname{cosec} x)} \cdot \frac{\sin^2 x - \cos^2 x}{\sin^3 x + \cos^3 x} = \sin x$$

Solution:

Let us consider the LHS:

$$\frac{1 - \sin x \cos x}{\cos x (\sec x - \operatorname{cosec} x)} \cdot \frac{\sin^2 x - \cos^2 x}{\sin^3 x + \cos^3 x}$$

By using the formula,

cosec θ = 1/sin θ;

sec θ = 1/cos θ;

Now,

$$\frac{1 - \sin x \cos x}{\cos x \left(\frac{1}{\cos x} - \frac{1}{\sin x}\right)} \times \frac{(\sin x)^2 - (\cos x)^2}{(\sin x)^3 + (\cos x)^3}$$

By using the formula, a³ + b³ = (a + b) (a² + b² – ab)

$$\frac{1 - \sin x \cos x}{\cos x \left(\frac{\sin x - \cos x}{\cos x \sin x}\right)} \times \frac{(\sin x + \cos x)(\sin x - \cos x)}{(\sin x + \cos x) \left[(\sin x)^2 + (\cos x)^2 - \sin x \cos x\right]}$$

$$\frac{\sin x (1 - \sin x \cos x)}{\sin x - \cos x} \times \frac{(\sin x + \cos x)(\sin x - \cos x)}{(\sin x + \cos x) \left[(\sin x)^2 + (\cos x)^2 - \sin x \cos x\right]}$$

$$\frac{\sin x \,(1-\sin x \cos x)}{1} \times \frac{1}{[(\sin x)^2 + (\cos x)^2 - \sin x \, \cos x]}$$

By using the formula, $\sin^2 x + \cos^2 x = 1$.

$$\sin x \,(1 - \sin x \cos x) \times \frac{1}{(1 - \sin x \cos x)}$$

sin x
= RHS
∴ LHS = RHS
Hence Proved.

6.
$$\frac{\tan x}{1 - \cot x} + \frac{\cot x}{1 - \tan x} = (\sec x \; cosec \; x + 1)$$

Solution:
Let us consider the LHS:
$$\frac{\tan x}{1 - \cot x} + \frac{\cot x}{1 - \tan x}$$

By using the formula,
tan θ = sin θ / cos θ;
cot θ = cos θ / sin θ
Now,

$$\frac{\dfrac{\sin x}{\cos x}}{1 - \dfrac{\cos x}{\sin x}} + \frac{\dfrac{\cos x}{\sin x}}{1 - \dfrac{\sin x}{\cos x}}$$

$$\frac{\dfrac{\sin x}{\cos x}}{\dfrac{\sin x - \cos x}{\sin x}} + \frac{\dfrac{\cos x}{\sin x}}{\dfrac{\cos x - \sin x}{\cos x}}$$

$$\frac{\sin^2 x}{\cos x \,(\sin x - \cos x)} - \frac{\cos^2 x}{\sin x \,(\sin x - \cos x)}$$

$$\frac{\sin^3 x - \cos^3 x}{\sin x \cos x \,(\sin x - \cos x)}$$

By using the formula, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$$\frac{(\sin x - \cos x)[(\sin x)^2 + (\cos x)^2 + \sin x \cos x]}{\sin x \cos x (\sin x - \cos x)}$$

We know, $\sin^2 x + \cos^2 x = 1$.

$$\frac{[1 + \sin x \cos x]}{\sin x \cos x}$$

$$\frac{1}{\sin x \cos x} + \frac{\sin x \cos x}{\sin x \cos x}$$

$$\frac{1}{\sin x} \times \frac{1}{\cos x} + 1$$

By using the formula,
cosec θ = 1/sin θ,
sec θ = 1/cos θ;
cosec x × sec x + 1
sec x cosec x + 1
=RHS
∴ LHS = RHS
Hence Proved.

7.

$$\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} + \frac{\sin^3 x - \cos^3 x}{\sin x - \cos x} = 2$$

Solution:
Let us consider LHS:

$$\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} + \frac{\sin^3 x - \cos^3 x}{\sin x - \cos x}$$

By using the formula $a^3 \pm b^3 = (a \pm b)(a^2 + b^2 \mp ab)$

$$\frac{(\sin x + \cos x)[(\sin x)^2 + (\cos x)^2 - \sin x \cos x]}{\sin x + \cos x}$$
$$+ \frac{(\sin x - \cos x)[(\sin x)^2 + (\cos x)^2 + \sin x \cos x]}{\sin x - \cos x}$$

We know, sin²x + cos²x = 1.
1 - sinx cosx + 1 + sinx cosx
2
= RHS
∴ LHS = RHS
Hence Proved.

8. (sec x sec y + tan x tan y)² – (sec x tan y + tan x sec y)² = 1
Solution:
Let us consider LHS:
(sec x sec y + tan x tan y)² – (sec x tan y + tan x sec y)²
Expanding the above equation we get,
[(sec x sec y)² + (tan x tan y)² + 2 (sec x sec y) (tan x tan y)] – [(sec x tan y)² + (tan x sec y)² + 2 (sec x tan y) (tan x sec y)]
[sec² x sec² y + tan² x tan² y + 2 (sec x sec y) (tan x tan y)] – [sec² x tan² y + tan² x sec² y + 2 (sec² x tan² y) (tan x sec y)]
sec² x sec² y - sec² x tan² y + tan² x tan² y - tan² x sec² y
sec² x (sec² y - tan² y) + tan² x (tan² y - sec² y)
sec² x (sec² y - tan² y) - tan² x (sec² y - tan² y)
We know, sec² x – tan² x = 1.
sec² x × 1 – tan² x × 1
sec² x – tan² x
1 = RHS
∴ LHS = RHS
Hence proved.

9.

$$\frac{\cos x}{1-\sin x} = \frac{1+\cos x + \sin x}{1+\cos x - \sin x}$$

Solution:
Let us Consider RHS:

$$\frac{1+\cos x + \sin x}{1+\cos x - \sin x}$$

$$\frac{(1+\cos x) + (\sin x)}{(1+\cos x) - (\sin x)}$$

$$\frac{(1+\cos x) + (\sin x)}{(1+\cos x) - (\sin x)} \times \frac{(1+\cos x) + (\sin x)}{(1+\cos x) + (\sin x)}$$

$$\frac{[(1+\cos x) + (\sin x)]^2}{(1+\cos x)^2 - (\sin x)^2}$$

$$\frac{[(1+\cos x) + (\sin x)]^2}{(1+\cos x)^2 - (\sin x)^2}$$

$$\frac{(1+\cos x)^2 + (\sin x)^2 + 2(1+\cos x)(\sin x)}{(1+\cos^2 x + 2\cos x) - (\sin^2 x)}$$

$$\frac{(1+\cos x)^2 + (\sin x)^2 + 2(1+\cos x)(\sin x)}{(1+\cos^2 x + 2\cos x) - (\sin^2 x)}$$

$$\frac{(1+\cos x)^2 + (\sin x)^2 + 2(1+\cos x)(\sin x)}{(1+\cos^2 x + 2\cos x) - (\sin^2 x)}$$

$$\frac{1+\cos^2 x + 2\cos x + \sin^2 x + 2\sin x + 2\sin x \cos x}{1+\cos^2 x + 2\cos x - \sin^2 x}$$

We know, $\sin^2 x + \cos^2 x = 1$.

$$\frac{1+1+2\cos x + 2\sin x + 2\sin x \cos x}{(1-\sin^2 x) + \cos^2 x + 2\cos x}$$

We know, $1 - \cos^2 x = \sin^2 x$.

$$\frac{2+2\cos x + 2\sin x + 2\sin x \cos x}{\cos^2 x + \cos^2 x + 2\cos x}$$

$$\frac{2+2\cos x + 2\sin x + 2\sin x \cos x}{2\cos^2 x + 2\cos x}$$

$$\frac{2+2\cos x + 2\sin x + 2\sin x \cos x}{\cos^2 x + \cos^2 x + 2\cos x}$$

$$\frac{1+\cos x + \sin x + \sin x \cos x}{\cos x (\cos x + 1)}$$

$$\frac{1(1+\cos x) + \sin x (\cos x + 1)}{\cos x (\cos x + 1)}$$

$$\frac{(1+\sin x)(\cos x + 1)}{\cos x (\cos x + 1)}$$

$$\frac{1 + \sin x}{\cos x} \times \frac{\cos x}{\cos x}$$

$$\frac{(1 + \sin x)\cos x}{\cos^2 x}$$

We know, $1 - \sin^2 x = \cos^2 x$.

$$\frac{(1 + \sin x)\cos x}{1 - \sin^2 x}$$

$$\frac{(1 + \sin x)\cos x}{(1 - \sin x)(1 + \sin x)}$$

$$\frac{\cos x}{1 - \sin x}$$

= LHS

∴ LHS = RHS

Hence Proved.

10.

$$\frac{\tan^3 x}{1 + \tan^2 x} + \frac{\cot^3 x}{1 + \cot^2 x} = \frac{1 - 2\sin^2 x \cos^2 x}{\sin x \cos x}$$

Solution:

Let us consider LHS:

$$\frac{\tan^3 x}{1 + \tan^2 x} + \frac{\cot^3 x}{1 + \cot^2 x}$$

By using the formulas,

$1 + \tan^2 x = \sec^2 x$ and $1 + \cot^2 x = \text{cosec}^2 x$

$$\frac{\tan^3 x}{\sec^2 x} + \frac{\cot^3 x}{\text{cosec}^2 x}$$

$$\frac{\dfrac{\sin^3 x}{\cos^3 x}}{\dfrac{1}{\cos^2 x}} + \frac{\dfrac{\cos^3 x}{\sin^3 x}}{\dfrac{1}{\sin^2 x}}$$

$$\frac{\sin^3 x}{\cos x} + \frac{\cos^3 x}{\sin x}$$

$$\frac{\sin^4 x + \cos^4 x}{\cos x \sin x}$$

$$\frac{(\sin^2 x)^2 + (\cos^2 x)^2}{\cos x \sin x}$$

We know, $a^2 + b^2 = (a+b)^2 - 2ab$

$$\frac{(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x}{\sin x \cos x}$$

$$\frac{1^2 - 2\sin^2 x \cos^2 x}{\sin x \cos x}$$

$$\frac{1 - 2\sin^2 x \cos^2 x}{\sin x \cos x}$$

= RHS
∴ LHS = RHS
Hence Proved.

11.

$$1 - \frac{\sin^2 x}{1 + \cot x} - \frac{\cos^2 x}{1 + \tan x} = \sin x \cos x$$

Solution:
Let us consider LHS:

$$1 - \frac{\sin^2 x}{1 + \cot x} - \frac{\cos^2 x}{1 + \tan x}$$

By using the formula,
tan θ = sin θ / cos θ;
cot θ = cos θ / sin θ
Now,

$$1 - \frac{\sin^2 x}{1 + \frac{\cos x}{\sin x}} - \frac{\cos^2 x}{1 + \frac{\sin x}{\cos x}}$$

$$1 - \frac{\sin^3 x}{\sin x + \cos x} - \frac{\cos^3 x}{\sin x + \cos x}$$

$$\frac{\sin x + \cos x - (\sin^3 x + \cos^3 x)}{\sin x + \cos x}$$

By using the formula, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$$\frac{\sin x + \cos x - ((\sin x + \cos x)(\sin x)^2 + (\cos x)^2 - \sin x \cos x))}{\sin x + \cos x}$$

$$\frac{(\sin x + \cos x)(1 - \sin^2 x - \cos^2 x + \sin x \cos x)}{\sin x + \cos x}$$

$1 - (\sin^2 x + \cos^2 x) + \sin x \cos x$

We know, $\sin^2 x + \cos^2 x = 1$.

$1 - 1 + \sin x \cos x$

Sin x cos x

= RHS

∴ LHS = RHS

Hence proved.

12.

$$\left(\frac{1}{\sec^2 x - \cos^2 x} + \frac{1}{cosec^2 x - \sin^2 x}\right) \sin^2 x \cos^2 x = \frac{1 - \sin^2 x \cos^2 x}{2 + \sin^2 x \cos^2 x}$$

Solution:

Let us consider LHS:

$$\left(\frac{1}{\sec^2 x - \cos^2 x} + \frac{1}{cosec^2 x - \sin^2 x}\right) \sin^2 x \cos^2 x$$

By using the formula,

cosec θ = 1/sin θ,

sec θ = 1/cos θ;

$$\left(\frac{1}{\frac{1}{\cos^2 x} - \cos^2 x} + \frac{1}{\frac{1}{\sin^2 x} - \sin^2 x}\right) \sin^2 x \cos^2 x$$

$$\left(\frac{\cos^2 x}{1 - \cos^4 x} + \frac{\sin^2 x}{1 - \sin^4 x}\right) \sin^2 x \cos^2 x$$

$$\left(\frac{\cos^2 x(1 - \sin^4 x) + \sin^2 x(1 - \cos^4 x)}{(1 - \cos^4 x)(1 - \sin^4 x)}\right) \sin^2 x \cos^2 x$$

$$\left(\frac{\cos^2 x - \cos^2 x \sin^4 x + \sin^2 x - \sin^2 x \cos^4 x}{(1 + \sin^2 x)(1 - \sin^2 x)(1 + \cos^2 x)(1 - \cos^2 x)}\right) \sin^2 x \cos^2 x$$

We know, $\sin^2 x + \cos^2 x = 1$.

$$\left(\frac{1 - \cos^2 x \sin^4 x - \sin^2 x \cos^4 x}{(1 + \sin^2 x)\cos^2 x(1 + \cos^2 x)\sin^2 x}\right) \sin^2 x \cos^2 x$$

$$\left(\frac{1 - \cos^2 x \sin^2 x (\sin^2 x + \cos^2 x)}{(1 + \sin^2 x)(1 + \cos^2 x)}\right)$$

$$\frac{1 - \sin^2 x \cos^2 x}{2 + \sin^2 x \cos^2 x}$$

= RHS
∴ LHS = RHS
Hence proved.

13. $(1 + \tan\alpha \tan\beta)^2 + (\tan\alpha - \tan\beta)^2 = \sec^2\alpha \sec^2\beta$

Solution:

Let us consider LHS: $(1 + \tan\alpha \tan\beta)^2 + (\tan\alpha - \tan\beta)^2$

$1 + \tan^2\alpha \tan^2\beta + 2\tan\alpha \tan\beta + \tan^2\alpha + \tan^2\beta - 2\tan\alpha \tan\beta$

$1 + \tan^2\alpha \tan^2\beta + \tan^2\alpha + \tan^2\beta$

$\tan^2\alpha (\tan^2\beta + 1) + 1(1 + \tan^2\beta)$

$(1 + \tan^2\beta)(1 + \tan^2\alpha)$

We know, $1 + \tan^2\theta = \sec^2\theta$

So,

$\sec^2\alpha \sec^2\beta$

= RHS
∴ LHS = RHS
Hence proved.

1. Find the values of the other five trigonometric functions in each of the following:
(i) cot x = 12/5, x in quadrant III
(ii) cos x = -1/2, x in quadrant II
(iii) tan x = 3/4, x in quadrant III
(iv) sin x = 3/5, x in quadrant I
Solution:
(i) cot x = 12/5, x in quadrant III
In third quadrant, tan x and cot x are positive. sin x, cos x, sec x, cosec x are negative.
By using the formulas,
tan x = 1/cot x

         = 1/(12/5)

       = 5/12

cosec x = -√(1 + cot² x)

           = -√(1 + (12/5)²)

           = -√(25+144)/25

           = -√(169/25)

           = -13/5

sin x = 1/cosec x

       = 1/(-13/5)

       = -5/13

cos x = - √(1 - sin² x)

       = - √(1 – (-5/13)²)

       = - √(169-25)/169

       = - √(144/169)

       = -12/13

sec x = 1/cos x

       = 1/(-12/13)

       = -13/12

∴ sin x = -5/13, cos x = -12/13, tan x = 5/12, cosec x = -13/5, sec x = -13/12

(ii) cos x = -1/2, x in quadrant II
In second quadrant, sin x and cosec x are positive. tan x, cot x, cos x, sec x are negative.

By using the formulas,
sin x = √(1 – cos² x)
 = √(1 – (-1/2)²)
 = √(4-1)/4
 = √(3/4)
 = √3/2

tan x = sin x/cos x
 = (√3/2)/(-1/2)
 = -√3

cot x = 1/tan x
 = 1/-√3
 = -1/√3

cosec x = 1/sin x
 = 1/(√3/2)
 = 2/√3

sec x = 1/cos x
 = 1/(-1/2)
 = -2

∴ sin x = √3/2, tan x = -√3, cosec x = 2/√3, cot x = -1/√3 sec x = -2

(iii) tan x = 3/4, x in quadrant III
In third quadrant, tan x and cot x are positive. sin x, cos x, sec x, cosec x are negative.
By using the formulas,
sin x = √(1 – cos² x)
 = - √(1-(-4/5)²)
 = - √(25-16)/25
 = - √(9/25)
 = - 3/5

cos x = 1/sec x
 = 1/(-5/4)
 = -4/5

cot x = 1/tan x

= 1/(3/4)
= 4/3

cosec x = 1/sin x
= 1/(-3/5)
= -5/3

sec x = -√(1 + tan² x)
= - √(1+(3/4)²)
= - √(16+9)/16
= - √ (25/16)
= -5/4

∴ sin x = -3/5, cos x = -4/5, cosec x = -5/3, sec x = -5/4, cot x = 4/3

(iv) sin x = 3/5, x in quadrant I
In first quadrant, all trigonometric ratios are positive.
So, by using the formulas,
tan x = sin x/cos x
= (3/5)/(4/5)
= 3/4

cosec x = 1/sin x
= 1/(3/5)
= 5/3

cos x = √(1-sin² x)
= √(1 – (-3/5)²)
= √(25-9)/25
= √(16/25)
= 4/5

sec x = 1/cos x
= 1/(4/5)
= 5/4

cot x = 1/tan x
= 1/(3/4)
= 4/3

∴ cos x = 4/5, tan x = 3/4, cosec x = 5/3, sec x = 5/4, cot x = 4/3

2. If sin x = 12/13 and lies in the second quadrant, find the value of sec x + tan x.
Solution:
Given:
Sin x = 12/13 and x lies in the second quadrant.
We know, in second quadrant, sin x and cosec x are positive and all other ratios are negative.
By using the formulas,
Cos x = √(1-sin² x)
        = - √(1-(12/13)²)
        = - √(1- (144/169))
        = - √(169-144)/169
        = -√(25/169)
        = - 5/13

We know,
tan x = sin x/cos x
sec x = 1/cos x
Now,
tan x = (12/13)/(-5/13)
       = -12/5

sec x = 1/(-5/13)
       = -13/5

Sec x + tan x = -13/5 + (-12/5)
             = (-13-12)/5
             = -25/5
             = -5
∴ Sec x + tan x = -5

3. If sin x = 3/5, tan y = 1/2 and π/2 < x< π< y< 3π/2 find the value of 8 tan x -√5 sec y.
Solution:
Given:
sin x = 3/5, tan y = 1/2 and π/2 < x< π< y< 3π/2
We know that, x is in second quadrant and y is in third quadrant.
In second quadrant, cos x and tan x are negative.

In third quadrant, sec y is negative.
By using the formula,
cos x = - √(1-sin² x)
tan x = sin x/cos x
Now,
cos x = - √(1-sin² x)
  = - √(1 – (3/5)²)
  = - √(1 – 9/25)
  = - √((25-9)/25)
  = - √(16/25)
  = - 4/5

tan x = sin x/cos x
  = (3/5)/(-4/5)
  = 3/5 × -5/4
  = -3/4

We know that sec y = - √(1+tan² y)
  = - √(1 + (1/2)²)
  = - √(1 + 1/4)
  = - √((4+1)/4)
  = - √(5/4)
  = - √5/2

Now, 8 tan x - √5 sec y = 8(-3/4) - √5(-√5/2)
  = -6 + 5/2
  = (-12+5)/2
  = -7/2
∴ 8 tan x - √5 sec y = -7/2

4. If sin x + cos x = 0 and x lies in the fourth quadrant, find sin x and cos x.
Solution:
Given:
Sin x + cos x = 0 and x lies in fourth quadrant.
Sin x = -cos x
Sin x/cos x = -1
So, tan x = -1 (since, tan x = sin x/cos x)
We know that, in fourth quadrant, cos x and sec x are positive and all other ratios are
negative.

By using the formulas,

Sec x = √(1 + tan² x)

Cos x = 1/sec x

Sin x = - √(1- cos² x)

Now,

Sec x = √(1 + tan² x)

        = √(1 + (-1)²)

        = √2

Cos x = 1/sec x

        = 1/√2

Sin x = - √(1 – cos² x)

        = - √(1 – (1/√2)²)

        = - √(1 – (1/2))

        = - √((2-1)/2)

        = - √(1/2)

        = -1/√2

∴ sin x = -1/√2 and cos x = 1/√2

5. If cos x = -3/5 and π<x<3π/2 find the values of other five trigonometric functions

and hence evaluate $\dfrac{cosec\ x + cot\ x}{sec\ x - tan\ x}$

Solution:

Given:

cos x= -3/5 and π <x < 3π/2

We know that in the third quadrant, tan x and cot x are positive and all other rations are negative.

By using the formulas,

Sin x = - √(1-cos² x)

Tan x = sin x/cos x

Cot x = 1/tan x

Sec x = 1/cos x

Cosec x = 1/sin x

Now,

Sin x = - √(1-cos² x)

        = - √(1-(-3/5)²)

        = - √(1-9/25)

        = - √((25-9)/25)

= - √(16/25)

= - 4/5

Tan x = sin x/cos x

= (-4/5)/(-3/5)

= -4/5 × -5/3

= 4/3

Cot x = 1/tan x

= 1/(4/3)

= 3/4

Sec x = 1/cos x

= 1/(-3/5)

= -5/3

Cosec x = 1/sin x

= 1/(-4/5)

= -5/4

∴ $\dfrac{cosec\ x + cot\ x}{sec\ x - tan\ x}$ = [(-5/4) + (3/4)] / [(-5/3) – (4/3)]

= [(-5+3)/4] / [(-5-4)/3]

= [-2/4] / [-9/3]

= [-1/2] / [-3]

= 1/6

EXERCISE 5.3 **PAGE NO: 5.39**

1. Find the values of the following trigonometric ratios:
(i) sin 5π/3
(ii) sin 17π
(iii) tan 11π/6
(iv) cos (-25π/4)
(v) tan 7π/4
(vi) sin 17π/6
(vii) cos 19π/6
(viii) sin (-11π/6)
(ix) cosec (-20π/3)
(x) tan (-13π/4)
(xi) cos 19π/4
(xii) sin 41π/4
(xiii) cos 39π/4
(xiv) sin 151π/6
Solution:
(i) sin 5π/3
5π/3 = (5/3 × 180)º
        = 300º
        = (90×3 + 30)º
Since, 300º lies in IV quadrant in which sine function is negative.
sin 5π/3 = sin (300)º
            = sin (90×3 + 30)º
            = - cos 30º
            = - √3/2

(ii) sin 17π
Sin 17π = sin 3060º
        = sin (90×34 + 0)º
Since, 3060º lies in the negative direction of x-axis i.e., on boundary line of II and III quadrants.
Sin 17π = sin (90×34 + 0)º
        = - sin 0º
        = 0

(iii) tan 11π/6
tan 11π/6 = (11/6 × 180)º

= 330º
Since, 330º lies in the IV quadrant in which tangent function is negative.
tan 11π/6 = tan (300)º
= tan (90×3 + 60)º
= - cot 60º
= - 1/√3

(iv) cos (-25π/4)
cos (-25π/4) = cos (-1125)º
= cos (1125)º
Since, 1125º lies in the I quadrant in which cosine function is positive.
cos (1125)º = cos (90×12 + 45)º
= cos 45º
= 1/√2

(v) tan 7π/4
tan 7π/4 = tan 315º
= tan (90×3 + 45)º
Since, 315º lies in the IV quadrant in which tangent function is negative.
tan 315º = tan (90×3 + 45)º
= - cot 45º
= -1

(vi) sin 17π/6
sin 17π/6 = sin 510º
= sin (90×5 + 60)º
Since, 510º lies in the II quadrant in which sine function is positive.
sin 510º = sin (90×5 + 60)º
= cos 60º
= 1/2

(vii) cos 19π/6
cos 19π/6 = cos 570º
= cos (90×6 + 30)º
Since, 570º lies in III quadrant in which cosine function is negative.
cos 570º = cos (90×6 + 30)º
= - cos 30º
= - √3/2

(viii) sin (-11π/6)
sin (-11π/6) = sin (-330º)
        = - sin (90×3 + 60)º
Since, 330º lies in the IV quadrant in which the sine function is negative.
sin (-330º) = - sin (90×3 + 60)º
        = - (-cos 60º)
        = - (-1/2)
        = 1/2

(ix) cosec (-20π/3)
cosec (-20π/3) = cosec (-1200)º
        = - cosec (1200)º
        = - cosec (90×13 + 30)º
Since, 1200º lies in the II quadrant in which cosec function is positive.
cosec (-1200)º = - cosec (90×13 + 30)º
        = - sec 30º
        = -2/√3

(x) tan (-13π/4)
tan (-13π/4) = tan (-585)º
        = - tan (90×6 + 45)º
Since, 585º lies in the III quadrant in which the tangent function is positive.
tan (-585)º = - tan (90×6 + 45)º
        = - tan 45º
        = -1

(xi) cos 19π/4
cos 19π/4 = cos 855º
        = cos (90×9 + 45)º
Since, 855º lies in the II quadrant in which the cosine function is negative.
cos 855º = cos (90×9 + 45)º
        = - sin 45º
        = - 1/√2

(xii) sin 41π/4
sin 41π/4 = sin 1845º
        = sin (90×20 + 45)º
Since, 1845º lies in the I quadrant in which the sine function is positive.
sin 1845º = sin (90×20 + 45)º

= sin 45º
= 1/√2

(xiii) cos 39π/4
cos 39π/4 = cos 1755º
= cos (90×19 + 45)º
Since, 1755º lies in the IV quadrant in which the cosine function is positive.
cos 1755º = cos (90×19 + 45)º
= sin 45º
= 1/√2

(xiv) sin 151π/6
sin 151π/6 = sin 4530º
= sin (90×50 + 30)º
Since, 4530º lies in the III quadrant in which the sine function is negative.
sin 4530º = sin (90×50 + 30)º
= - sin 30º
= -1/2

2. prove that:
(i) tan 225º cot 405º + tan 765º cot 675º = 0
(ii) sin 8π/3 cos 23π/6 + cos 13π/3 sin 35π/6 = 1/2
(iii) cos 24º + cos 55º + cos 125º + cos 204º + cos 300º = 1/2
(iv) tan (-125º) cot (-405º) – tan (-765º) cot (675º) = 0
(v) cos 570º sin 510º + sin (-330º) cos (-390º) = 0
(vi) tan 11π/3 – 2 sin 4π/6 – 3/4 cosec² π/4 + 4 cos² 17π/6 = (3 - 4√3)/2
(vii) 3 sin π/6 sec π/3 – 4 sin 5π/6 cot π/4 = 1
Solution:
(i) tan 225º cot 405º + tan 765º cot 675º = 0
Let us consider LHS:
tan 225° cot 405° + tan 765° cot 675°
tan (90° × 2 + 45°) cot (90° × 4 + 45°) + tan (90° × 8 + 45°) cot (90° × 7 + 45°)
We know that when n is odd, cot → tan.
tan 45° cot 45° + tan 45° [-tan 45°]
tan 45° cot 45° - tan 45° tan 45°
1 × 1 – 1 × 1
1 – 1
0 = RHS
∴ LHS = RHS

Hence proved.

(ii) sin 8π/3 cos 23π/6 + cos 13π/3 sin 35π/6 = 1/2
Let us consider LHS:
sin 8π/3 cos 23π/6 + cos 13π/3 sin 35π/6
sin 480° cos 690° + cos 780° sin 1050°
sin (90° × 5 + 30°) cos (90° × 7 + 60°) + cos (90° × 8 + 60°) sin (90° × 11 + 60°)
We know that when n is odd, sin → cos and cos → sin.
cos 30° sin 60° + cos 60° [-cos 60°]
√3/2 × √3/2 – 1/2 × 1/2
3/4 - 1/4
2/4
1/2
= RHS
∴ LHS = RHS
Hence proved.

(iii) cos 24° + cos 55° + cos 125° + cos 204° + cos 300° = 1/2
Let us consider LHS:
cos 24° + cos 55° + cos 125° + cos 204° + cos 300°
cos 24° + cos (90° × 1 – 35°) + cos (90° × 1 + 35°) + cos (90° × 2 + 24°) + cos (90° × 3 + 30°)
We know that when n is odd, cos → sin.
cos 24° + sin 35° - sin 35° - cos 24° + sin 30°
0 + 0 + 1/2
1/2
= RHS
∴ LHS = RHS
Hence proved.

(iv) tan (-125°) cot (-405°) – tan (-765°) cot (675°) = 0
Let us consider LHS:
tan (-125°) cot (-405°) – tan (-765°) cot (675°)
We know that tan (-x) = -tan (x) and cot (-x) = -cot (x).
[-tan (225°)] [-cot (405°)] – [-tan (765°)] cot (675°)
tan (225°) cot (405°) + tan (765°) cot (675°)
tan (90° × 2 + 45°) cot (90° × 4 + 45°) + tan (90° × 8 + 45°) cot (90° × 7 + 45°)
tan 45° cot 45° + tan 45° [-tan 45°]
1 × 1 + 1 × (-1)

1 – 1
0
= RHS
∴ LHS = RHS
Hence proved.

(v) cos 570º sin 510º + sin (-330º) cos (-390º) = 0
Let us consider LHS:
cos 570º sin 510º + sin (-330º) cos (-390º)
We know that sin (-x) = -sin (x) and cos (-x) = +cos (x).
cos 570º sin 510º + [-sin (330º)] cos (390º)
cos 570º sin 510º - sin (330º) cos (390º)
cos (90° × 6 + 30°) sin (90° × 5 + 60°) – sin (90° × 3 + 60°) cos (90° × 4 + 30°)
We know that cos is negative at 90° + θ i.e. in Q_2 and when n is odd, sin → cos and cos → sin.
-cos 30° cos 60° - [-cos 60°] cos 30°
-cos 30° cos 60° + cos 60° cos 30°
0
= RHS
∴ LHS = RHS
Hence proved.

(vi) tan 11π/3 – 2 sin 4π/6 – 3/4 cosec² π/4 + 4 cos² 17π/6 = (3 - 4√3)/2
Let us consider LHS:
tan 11π/3 – 2 sin 4π/6 – 3/4 cosec² π/4 + 4 cos² 17π/6
tan (11 × 180º)/3 – 2 sin (4 × 180º)/6 – 3/4 cosec² 180º/4 + 4 cos² (17 × 180º)/6
tan 660º – 2 sin 120º - 3/4 (cosec 45º)² + 4 (cos 510º)²
tan (90° × 7 + 30°) – 2 sin (90° × 1 + 30°) – 3/4 [cosec 45°]² + 4 [cos (90° × 5 + 60°)]²
We know that tan and cos is negative at 90° + θ i.e. in Q_2 and when n is odd, tan → cot, sin → cos and cos → sin.
[-cot 30°] – 2 cos 30° - 3/4 [cosec 45°]² + [-sin 60°]²
- cot 30° - 2 cos 30° - 3/4 [cosec 45°]² + [sin 60°]²
-√3 - 2√3/2 – 3/4 (√2)² + 4 (√3/2)²
-√3 - √3 – 6/4 + 12/4
(3 - 4√3)/2
= RHS
∴ LHS = RHS
Hence proved.

(vii) 3 sin π/6 sec π/3 – 4 sin 5π/6 cot π/4 = 1

Let us consider LHS:

3 sin π/6 sec π/3 – 4 sin 5π/6 cot π/4

3 sin 180º/6 sec 180º/3 – 4 sin 5(180º)/6 cot 180º/4

3 sin 30° sec 60° - 4 sin 150° cot 45°

3 sin 30° sec 60° - 4 sin (90° × 1 + 60°) cot 45°

We know that when n is odd, sin → cos.

3 sin 30° sec 60° - 4 cos 60° cot 45°

3 (1/2) (2) – 4 (1/2) (1)

3 – 2

1

= RHS

∴ LHS = RHS

Hence proved.

3. Prove that:

(i)

$$\frac{cos(2\pi + x)\, cosec(2\pi + x)\, tan(\pi/2 + x)}{sec(\pi/2 + x)\, cos\, x\, cot(\pi + x)} = 1$$

(ii)

$$\frac{cosec\,(90^o + x) + cot(450^o + x)}{cosec(90^o - x) + tan(180^o - x)} + \frac{tan(180^o + x) + sec(180^o - x)}{tan(360^o + x) - sec(-x)} = 2$$

(iii)

$$\frac{sin(\pi + x)\, cos(\frac{\pi}{2} + x)\, tan(\frac{3\pi}{2} - x)\, cot(2\pi - x)}{sin(2\pi - x)\, cos(2\pi + x)\, cosec(-x)\, sin(\frac{3\pi}{2} - x)} = 1$$

(iv)

$$\left\{1 + cot\, x - sec(\frac{\pi}{2} + x)\right\}\left\{1 + cot\, x + sec(\frac{\pi}{2} + x)\right\} = 2\, cot\, x$$

(v)

$$\frac{tan(\frac{\pi}{2} - x)\, sec(\pi - x)\, sin(-x)}{sin(\pi + x)\, cot(2\pi - x)\, cosec(\frac{\pi}{2} - x)} = 1$$

Solution:

(i)

$$\frac{cos(2\pi + x) \, cosec(2\pi + x) \, tan(\pi/2 + x)}{sec(\pi/2 + x) \, cos \, x \, cot(\pi + x)} = 1$$

Let us consider LHS:

$$\frac{cos(2\pi + x) \, cosec(2\pi + x) \, tan(\pi/2 + x)}{sec(\pi/2 + x) \, cos \, x \, cot(\pi + x)}$$

$$\frac{cos \, x \, cosec \, x \, [-cot \, x]}{[-cosec \, x] \, cos \, x \, cot \, x}$$

$$\frac{-cos \, x \, cosec \, x \, cot \, x}{-cosec \, x \, cos \, x \, cot \, x}$$

1 = RHS
∴ LHS = RHS
Hence proved.

(ii)

$$\frac{cosec \, (90^o + x) + cot(450^o + x)}{cosec(90^o - x) + tan(180^o - x)} + \frac{tan(180^o + x) + sec(180^o - x)}{tan(360^o + x) - sec(-x)} = 2$$

Let us consider LHS:

$$\frac{cosec \, (90^o + x) + cot(450^o + x)}{cosec(90^o - x) + tan(180^o - x)} + \frac{tan(180^o + x) + sec(180^o - x)}{tan(360^o + x) - sec(-x)}$$

$$\frac{cosec \, (90^o + x) + cot(90^o \times 5 + x)}{cosec(90^o - x) + tan(90^o \times 2 - x)} + \frac{tan(90^o \times 2 + x) + sec(90^o \times 2 - x)}{tan(90^o \times 4 + x) - sec(-x)}$$

We know that when n is odd, cosec → sec and also sec (-x) = sec x.

$$\frac{sec \, x + cot(90^o \times 5 + x)}{cosec(90^o - x) + tan(90^o \times 2 - x)} + \frac{tan(90^o \times 2 + x) + sec(90^o \times 2 - x)}{tan(90^o \times 4 + x) - sec(-x)}$$

$$\frac{sec \, x - tan \, x}{sec \, x - tan \, x} + \frac{tan \, x - sec \, x}{tan \, x - sec \, x}$$

1 + 1
2 = RHS
∴ LHS = RHS
Hence proved.

(iii)

$$\frac{sin(\pi + x)\, cos(\frac{\pi}{2} + x)\, tan(\frac{3\pi}{2} - x)\, cot(2\pi - x)}{sin(2\pi - x)\, cos(2\pi + x)\, cosec(-x)\, sin(\frac{3\pi}{2} - x)} = 1$$

Let us consider LHS:

$$\frac{sin(\pi + x)\, cos(\frac{\pi}{2} + x)\, tan(\frac{3\pi}{2} - x)\, cot(2\pi - x)}{sin(2\pi - x)\, cos(2\pi + x)\, cosec(-x)\, sin(\frac{3\pi}{2} - x)}$$

$$\frac{sin(180^o - x)\, cos(90^o + x)\, tan(270^o - x)\, cot(360^o - x)}{sin(360^o - x)\, cos(360^o + x)\, cosec(-x)\, sin(270^o - x)}$$

We know that cosec (-x) = - cosec x.

$$\frac{sin(90^o \times 2 - x)\, cos(90^o \times 1 + x)\, tan(90^o \times 3 - x)\, cot(90^o \times 4 - x)}{sin(90^o \times 4 - x)\, cos(90^o \times 4 + x)\, [-cosec(x)]\, sin(90^o \times 3 - x)}$$

We know that when n is odd, cos → sin, tan → cot and sin → cos.

$$\frac{(-sin\, x)(-sin\, x)\, cot\, x\, (-cot\, x)}{(-sin\, x)\, cos\, x\, (-cosec\, x)(-cos\, x)}$$

$$\frac{sin^2 x\, cot^2 x}{sin\, x\, cosec\, x\, cos\, x\, cos\, x}$$

$$\frac{sin^2 x \times \frac{cos^2 x}{sin^2 x}}{sin\, x \times \frac{1}{sin\, x} \times cos^2 x}$$

$$\frac{cos^2 x}{cos^2 x}$$

1 = RHS
∴ LHS = RHS
Hence proved.

(iv)

$$\left\{1 + cot\, x - sec(\frac{\pi}{2} + x)\right\} \left\{1 + cot\, x + sec(\frac{\pi}{2} + x)\right\} = 2\, cot\, x$$

Let us consider LHS:

$$\left\{1 + cot\, x - sec(\frac{\pi}{2} + x)\right\} \left\{1 + cot\, x + sec(\frac{\pi}{2} + x)\right\}$$

{1 + cot x − (-cosec x)} {1 + cot x + (-cosec x)}
{1 + cot x + cosec x} {1 + cot x − cosec x}
{(1 + cot x) + (cosec x)} {(1 + cot x) − (cosec x)}

By using the formula, $(a + b)(a - b) = a^2 - b^2$

$(1 + \cot x)^2 - (\csc x)^2$

$1 + \cot^2 x + 2 \cot x - \csc^2 x$

We know that $1 + \cot^2 x = \csc^2 x$

$\csc^2 x + 2 \cot x - \csc^2 x$

$2 \cot x = \text{RHS}$

$\therefore \text{LHS} = \text{RHS}$

Hence proved.

(v)

$$\frac{\tan(\frac{\pi}{2} - x) \sec(\pi - x) \sin(-x)}{\sin(\pi + x) \cot(2\pi - x) \csc(\frac{\pi}{2} - x)} = 1$$

Let us consider LHS:

$$\frac{\tan(\frac{\pi}{2} - x) \sec(\pi - x) \sin(-x)}{\sin(\pi + x) \cot(2\pi - x) \csc(\frac{\pi}{2} - x)}$$

$$\frac{\tan(90^o - x) \sec(180^o - x) \sin(-x)}{\sin(180^o + x) \cot(360^o - x) \csc(90^o - x)}$$

We know that $\sin(-x) = -\sin x$.

$$\frac{\tan(90^o \times 1 - x) \sec(90^o \times 2 - x) [-\sin(x)]}{\sin(90^o \times 2 + x) \cot(90^o \times 4 - x) \csc(90^o \times 1 - x)}$$

We know that when n is odd, tan → cot and cosec → sec.

$$\frac{(\cot x)(-\sec x)(-\sin x)}{(-\sin x)(-\cot x)(\sec x)}$$

$$\frac{\cot x \sec x \sin x}{\sin x \cot x \sec x}$$

$1 = \text{RHS}$

$\therefore \text{LHS} = \text{RHS}$

Hence proved.

4. Prove that: $\sin^2 \pi/18 + \sin^2 \pi/9 + \sin^2 7\pi/18 + \sin^2 4\pi/9 = 2$

Solution:

Let us consider LHS:

$\sin^2 \pi/18 + \sin^2 \pi/9 + \sin^2 7\pi/18 + \sin^2 4\pi/9$

sin^2 π/18 + sin^2 2π/18 + sin^2 7π/18 + sin^2 8π/18

sin^2 π/18 + sin^2 2π/18 + sin^2 (π/2 - 2π/18) + sin^2 (π/2 – π/18)

We know that when n is odd, sin → cos.

sin^2 π/18 + sin^2 2π/18 + cos^2 2π/18 + cos^2 2π/18

when rearranged,

sin^2 π/18 + cos^2 2π/18 + sin^2 π/18 + cos^2 2π/18

We know that sin^2 x + cos^2x = 1.

So,

1 + 1

2 = RHS

∴ LHS = RHS

Hence proved.