Exercise 4.5   Page No: 4.37

1. In fig. 4.136, ΔACB ∼ ΔAPQ. If BC = 8 cm, PQ = 4 cm, BA = 6.5 cm and AP = 2.8 cm, find CA and AQ.
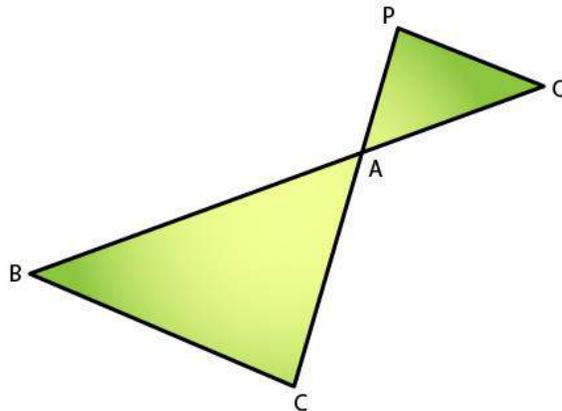**Solution:**

Given,

        ΔACB ∼ ΔAPQ
        BC = 8 cm, PQ = 4 cm, BA = 6.5 cm and AP = 2.8 cm
Required to find: CA and AQ

We know that,
ΔACB ∼ ΔAPQ   [given]
BA/ AQ = CA/ AP = BC/ PQ   [Corresponding Parts of Similar Triangles]
So,
        6.5/ AQ = 8/ 4
        AQ = (6.5 x 4)/ 8
        AQ = 3.25 cm
Similarly, as
        CA/ AP = BC/ PQ
        CA/ 2.8 = 8/ 4
        CA = 2.8 x 2
        CA = 5.6 cm

Hence, CA = 5.6 cm and AQ = 3.25 cm.

2. In fig.4.137, AB ∥ QR, find the length of PB.
Solution:

Given,

        ΔPQR, AB ∥ QR and
        AB = 3 cm, QR = 9 cm and PR = 6 cm
Required to find: PB
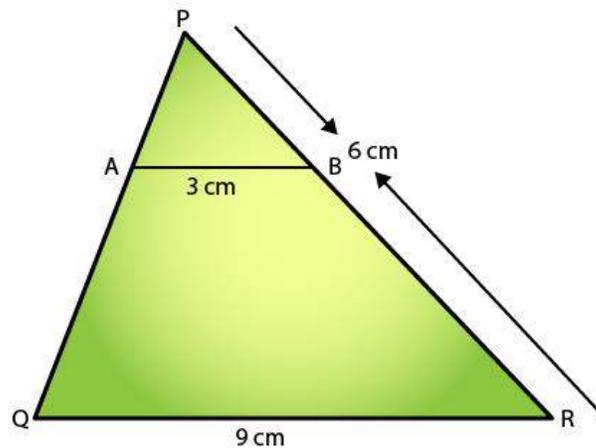
In ΔPAB and ΔPQR
We have,
∠P = ∠P   [Common]
∠PAB = ∠PQR   [Corresponding angles as AB‖QR with PQ as the transversal]
⇒ ΔPAB ∼ ΔPQR   [By AA similarity criteria]

Hence,
        AB/ QR = PB/ PR   [Corresponding Parts of Similar Triangles are propositional]
⇒   3/ 9 = PB/6
        PB = 6/3

Therefore, PB = 2 cm

3. In fig. 4.138 given, XY‖BC. Find the length of XY.
Solution:
        Given,
            XY‖BC
            AX = 1 cm, XB = 3 cm and BC = 6 cm
        Required to find: XY
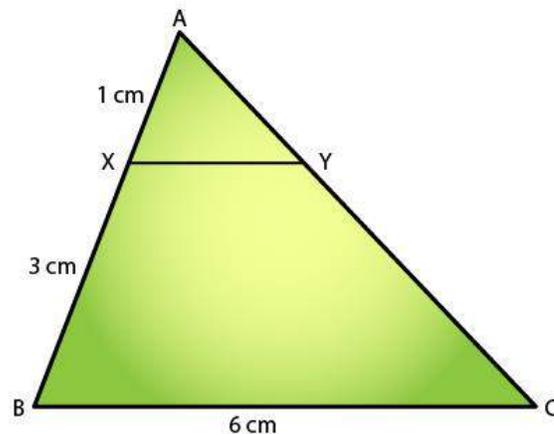
In ΔAXY and ΔABC
We have,

∠A = ∠A	[Common]
∠AXY = ∠ABC	[Corresponding angles as AB‖QR with PQ as the transversal]
⇒ ΔAXY ∼ ΔABC	[By AA similarity criteria]

Hence,

$\qquad$ XY/ BC = AX/ AB $\qquad$ [Corresponding Parts of Similar Triangles are propositional]
We know that,

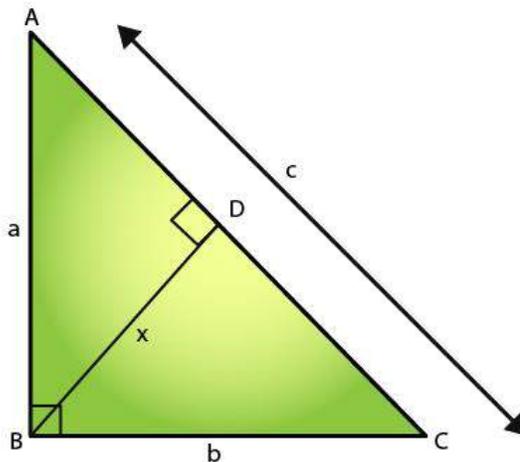$\qquad$ (AB = AX + XB = 1 + 3 = 4)
$\qquad$ XY/6 = 1/4
$\qquad$ XY/1 = 6/4

Therefore, XY = 1.5 cm

4. In a right-angled triangle with sides a and b and hypotenuse c, the altitude drawn on the hypotenuse is x. Prove that ab = cx.
Solution:

Consider ΔABC to be a right angle triangle having sides a and b and hypotenuse c. Let BD be the altitude drawn on the hypotenuse AC.
Required to prove: ab = cx

We know that,
In ΔACB and ΔCDB

∠B = ∠B	[Common]
∠ACB = ∠CDB = 90º	
⇒ ΔACB ∼ ΔCDB	[By AA similarity criteria]

Hence,

AB/ BD = AC/ BC	[Corresponding Parts of Similar Triangles are propositional]
$\qquad$ a/ x = c/ b	
⇒ $\qquad$ xc = ab	

Therefore, ab = cx

5. In fig. 4.139, ∠ABC = 90 and BD⊥AC. If BD = 8 cm, and AD = 4 cm, find CD.
Solution:

Given,
 ∠ABC = 90º and BD⊥AC
 BD = 8 cm
 AD = 4 cm
Required to find: CD.
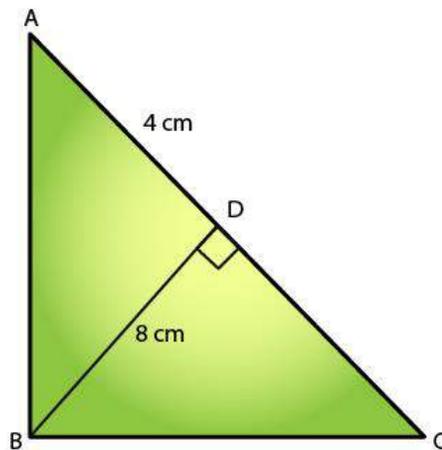
We know that,
 ABC is a right angled triangle and BD⊥AC.
 Then, ΔDBA~ΔDCB   [By AA similarity]
 BD/ CD = AD/ BD
 BD² = AD x DC
 (8)² = 4 x DC
 DC = 64/4 = 16 cm

Therefore, CD = 16 cm

6. In fig.4.140, ∠ABC = 90º and BD ⊥ AC. If AC = 5.7 cm, BD = 3.8 cm and CD = 5.4 cm, Find BC.
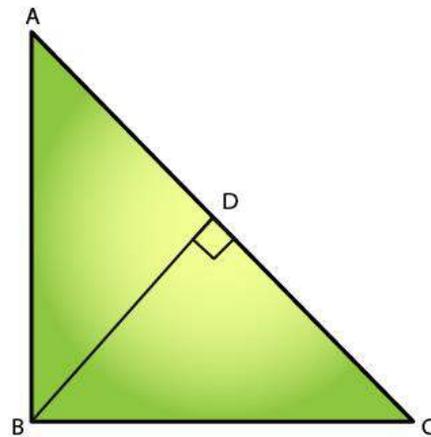**Solution:**

Given:
 BD ⊥ AC
 AC = 5.7 cm, BD = 3.8 cm and CD = 5.4 cm
 ∠ABC = 90º
Required to find: BC

We know that,

   ΔABC ~ ΔBDC   [By AA similarity]
   ∠BCA = ∠DCA = 90º
   ∠AXY = ∠ABC   [Common]

Thus,

   AB/ BD = BC/ CD   [Corresponding Parts of Similar Triangles are propositional]
   5.7/ 3.8 = BC/ 5.4
   BC = (5.7 x 5.4)/ 3.8 = 8.1

Therefore, BC = 8.1 cm

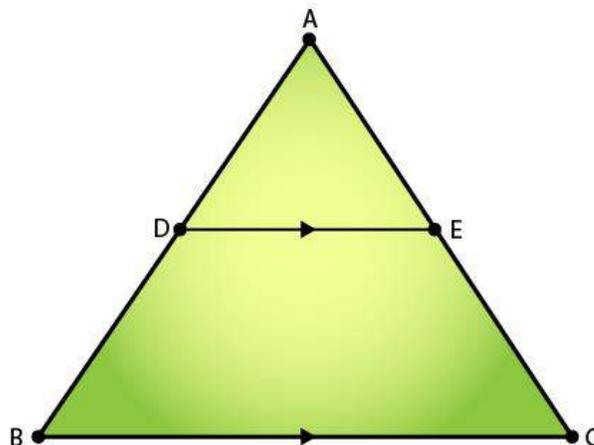**7. In the fig.4.141 given, DE ‖ BC such that AE = (1/4)AC. If AB = 6 cm, find AD.**
Solution:

Given:

   DE‖BC
   AE = (1/4)AC
   AB = 6 cm.

Required to find: AD.

In ΔADE and ΔABC
We have,
∠A = ∠A   [Common]
∠ADE = ∠ABC   [Corresponding angles as AB||QR with PQ as the transversal]
⇒ ΔADE ∼ ΔABC   [By AA similarity criteria]

Then,
　　　　AD/AB = AE/ AC   [Corresponding Parts of Similar Triangles are propositional]
　　　　AD/6 = 1/4
　　　　4 x AD = 6
　　　　AD = 6/4

Therefore, AD = 1.5 cm

8. In the fig.4.142 given, if AB ⊥ BC, DC ⊥ BC, and DE ⊥ AC, prove that ΔCED ∼ ΔABC
Solution:

Given:
　　　　AB ⊥ BC,
　　　　DC ⊥ BC,
　　　　DE ⊥ AC
Required to prove: ΔCED∼ΔABC
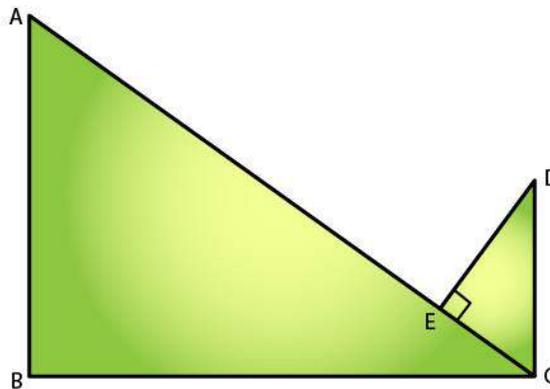
We know that,
From ΔABC and ΔCED
　　　∠B = ∠E = 90°   [given]
　　　∠BAC = ∠ECD   [alternate angles since, AB || CD with BC as transversal]
Therefore, ΔCED∼ΔABC   [AA similarity]

9. Diagonals AC and BD of a trapezium ABCD with AB ∥ DC intersect each other at the point O. Using similarity criterion for two triangles, show that OA/ OC = OB/ OD
Solution:

Given: OC is the point of intersection of AC and BD in the trapezium ABCD, with AB ∥ DC.
Required to prove: OA/ OC = OB/ OD
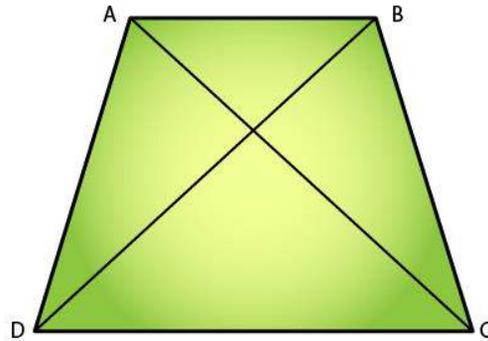
We know that,
  In ∆AOB and ∆COD
  ∠AOB = ∠COD   [Vertically Opposite Angles]
  ∠OAB = ∠OCD   [Alternate angles]
  Then, ∆AOB ~ ∆COD

Therefore, OA/ OC = OB/ OD   [Corresponding sides are proportional]

10. If ∆ ABC and ∆ AMP are two right triangles, right angled at B and M, respectively such that ∠MAP = ∠BAC. Prove that
(i) ∆ABC ~ ∆AMP
(ii) CA/ PA = BC/ MP
Solution:

 (i) Given:
   ∆ ABC and ∆ AMP are the two right triangles.
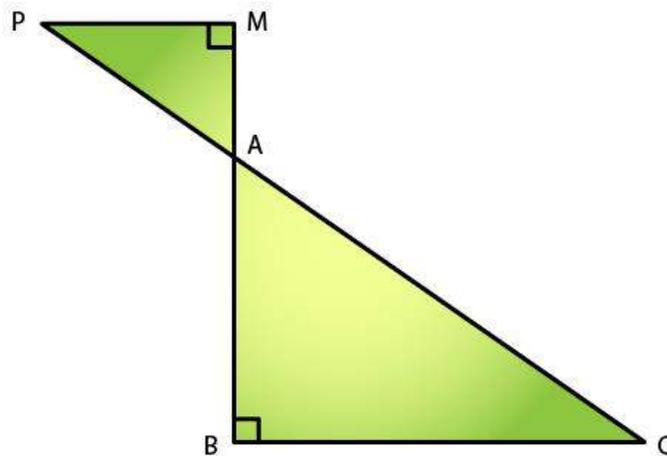
 We know that,
   ∠AMP = ∠B = 90°
   ∠MAP = ∠BAC   [Vertically Opposite Angles]

⇒   ΔABC~ΔAMP   [AA similarity]

(ii) Since, ΔABC~ΔAMP
          CA/ PA = BC/ MP   [Corresponding sides are proportional]
          Hence proved.

11. A vertical stick 10 cm long casts a shadow 8 cm long. At the same time, a tower casts a shadow 30 m long. Determine the height of the tower.
Solution:
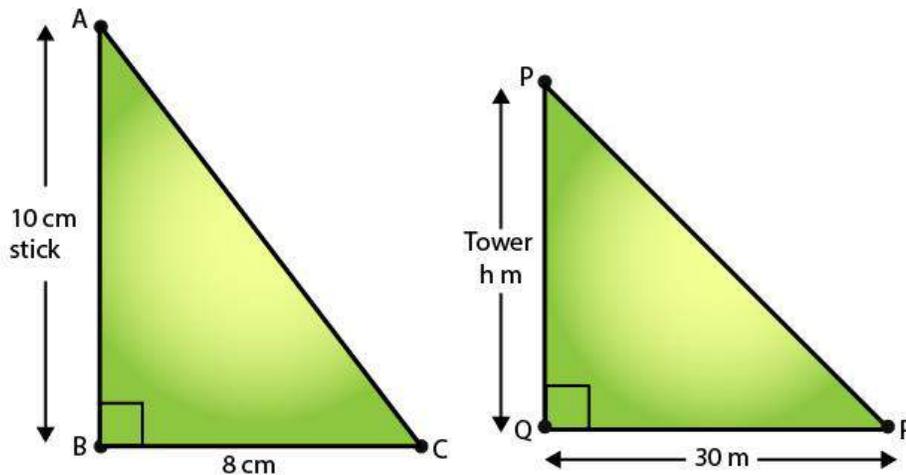
Given:
          Length of stick = 10cm
          Length of the stick's shadow = 8cm
          Length of the tower's shadow = 30m = 3000cm
Required to find: the height of the tower = PQ.

In ΔABC ~ ΔPQR
∠ABC = ∠PQR = 90º
∠ACB = ∠PRQ   [Angular Elevation of Sun is same for a particular instant of time]
⇒ ΔABC ~ ΔPQR   [By AA similarity]

So, we have
          AB/BC = PQ/QR   [Corresponding sides are proportional]
          10/8 = PQ/ 3000
          PQ = (3000x10)/ 8
          PQ = 30000/8
          PQ = 3750/100

Therefore, PQ = 37.5 m

12. In fig.4.143, ∠A = ∠CED, prove that ΔCAB ~ ΔCED. Also find the value of x.
Solution:

Given:
∠A = ∠CED
Required to prove: ΔCAB ~ ΔCED
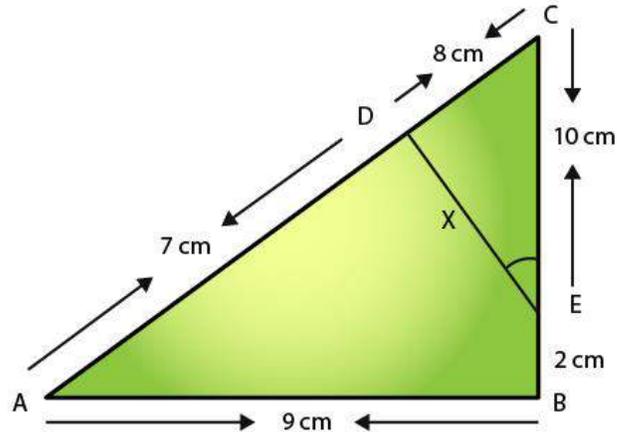
In ΔCAB ~ ΔCED
∠C = ∠C   [Common]
∠A = ∠CED   [Given]

⇒ ΔCAB ~ ΔCED   [By AA similarity]

Hence, we have
        CA/ CE = AB/ ED   [Corresponding sides are proportional]
        15/10 = 9/x
        x = (9 x 10)/ 15

Therefore, x = 6 cm