

Exercise 5.2

Evaluate each of the following:

1. $\sin 45^\circ \sin 30^\circ + \cos 45^\circ \cos 30^\circ$

Solution:

$$\sin 45^\circ \sin 30^\circ + \cos 45^\circ \cos 30^\circ$$

Value of trigonometric ratios are:

$$\sin 45^\circ = \frac{1}{\sqrt{2}} \quad \sin 30^\circ = \frac{1}{2}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} \quad \cos 30^\circ = \frac{\sqrt{3}}{2}$$

Substituting in the given equation, we get

$$\begin{aligned} & \frac{1}{\sqrt{2}} \cdot \frac{1}{2} + \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} \\ &= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}+1}{2} \\ &= \frac{\sqrt{3}+1}{2\sqrt{2}} \end{aligned}$$

2. $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

Solution:

$$\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$$

By trigonometric ratios we have ,

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \quad \sin 30^\circ = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2} \quad \cos 60^\circ = \frac{1}{2}$$

Substituting the values in given equation

$$\begin{aligned} &= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2} \\ &= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1 \end{aligned}$$

3. $\cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$

Solution:

$$\cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$$

We know that by trigonometric ratios

$$\cos 60^\circ = \frac{1}{2} \quad \cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \quad \sin 45^\circ = \frac{1}{\sqrt{2}}$$

Substituting the values in given equation

$$\begin{aligned} & \frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} \\ &= \frac{1-\sqrt{3}}{2\sqrt{2}} \end{aligned}$$

4. $\sin^2 30^\circ + \sin^2 45^\circ + \sin^2 60^\circ + \sin^2 90^\circ$

Solution:

$$\sin^2 30^\circ + \sin^2 45^\circ + \sin^2 60^\circ + \sin^2 90^\circ$$

We know that by trigonometric ratios

$$\sin 30^\circ = \frac{1}{2} \quad \sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \quad \sin 90^\circ = 1$$

Substituting the values in given equation, we get

$$\begin{aligned} &= \left[\frac{1}{2}\right]^2 + \left[\frac{1}{\sqrt{2}}\right]^2 + \left[\frac{\sqrt{3}}{2}\right]^2 + 1 \\ &= \frac{1}{4} + \frac{1}{2} + \frac{3}{4} + 1 \\ &= \frac{5}{2} \end{aligned}$$

5. $\cos^2 30^\circ + \cos^2 45^\circ + \cos^2 60^\circ + \cos^2 90^\circ$

Solution:

$$\cos^2 30^\circ + \cos^2 45^\circ + \cos^2 60^\circ + \cos^2 90^\circ$$

We know that by trigonometric ratios

$$\cos 30^\circ = \frac{\sqrt{3}}{2} \quad \cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\cos 60^\circ = \frac{1}{2} \quad \cos 90^\circ = 0$$

Substituting the values in given equation

$$\begin{aligned} & \left[\frac{\sqrt{3}}{2} \right]^2 + \left[\frac{1}{\sqrt{2}} \right]^2 + \left[\frac{1}{2} \right]^2 + 0 \\ &= \frac{3}{4} + \frac{1}{2} + \frac{1}{4} \\ &= \frac{3}{2} \end{aligned}$$

6. $\tan^2 30^\circ + \tan^2 45^\circ + \tan^2 60^\circ$

Solution:

$$\tan^2 30^\circ + \tan^2 45^\circ + \tan^2 60^\circ$$

We know that by trigonometric ratios

$$\tan 30^\circ = \frac{1}{\sqrt{3}} \quad \tan 60^\circ = \sqrt{3}$$

$$\tan 45^\circ = 1$$

Substituting the values in given equation

$$\begin{aligned} & \left[\frac{1}{\sqrt{3}} \right]^2 + [\sqrt{3}]^2 + 1 \\ &= \frac{1}{3} + 3 + 1 \\ &= \frac{13}{3} \end{aligned}$$

7. $2\sin^2 30^\circ - 3\cos^2 45^\circ + \tan^2 60^\circ$

Solution:

$$2\sin^2 30^\circ - 3\cos^2 45^\circ + \tan^2 60^\circ$$

We know that by trigonometric ratios

$$\sin 30^\circ = \frac{1}{2} \quad \cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\tan 60^\circ = \sqrt{3}$$

Substituting the values in given equation

$$= 2\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{\sqrt{2}}\right)^2 + (\sqrt{3})^2$$

$$= 2\left(\frac{1}{4}\right) - 3\left(\frac{1}{2}\right) + 3$$

$$= \frac{1-3+6}{2}$$

$$= 2$$

8. $\sin^2 30^\circ \cos^2 45^\circ + 4\tan^2 30^\circ + (1/2) \sin^2 90^\circ - 2\cos^2 90^\circ + (1/24) \cos 20^\circ$

Solution:

$$\sin^2 30^\circ \cos^2 45^\circ + 4\tan^2 30^\circ + \frac{1}{2} \sin^2 90^\circ - 2\cos^2 90^\circ + \frac{1}{24} \cos 20^\circ$$

We know that by trigonometric ratios

$$\sin 30^\circ = \frac{1}{2}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\sin 90^\circ = 1$$

$$\cos 90^\circ = 0$$

$$\cos 0^\circ = 1$$

Substituting the values in given equation

$$\begin{aligned} & \left[\frac{1}{2}\right]^2 \cdot \left[\frac{1}{\sqrt{2}}\right]^2 + 4\left[\frac{1}{\sqrt{3}}\right]^2 + \frac{1}{2}[1]^2 - 2[0]^2 + \frac{1}{24}[1]^2 \\ &= \frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24} \\ &= \frac{48}{24} \\ &= 2 \end{aligned}$$

9. $4(\sin^4 60^\circ + \cos^4 30^\circ) - 3(\tan^2 60^\circ - \tan^2 45^\circ) + 5\cos^2 45^\circ$

Solution:

$$4(\sin^4 60^\circ + \cos^4 30^\circ) - 3(\tan^2 60^\circ - \tan^2 45^\circ) + 5\cos^2 45^\circ$$

We know that by trigonometric ratios we have ,

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \quad \cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\tan 60^\circ = \sqrt{3} \quad \cos 30^\circ = \frac{\sqrt{3}}{2}$$

Substituting the values in given equation

$$\begin{aligned} &= 4 \cdot \frac{18}{16} - 6 + \frac{5}{2} \\ &= \frac{1}{4} - 6 + \frac{5}{2} \\ &= \frac{14}{2} - 6 = 7 - 6 = 1 \end{aligned}$$

10. $(\operatorname{cosec}^2 45^\circ \sec^2 30^\circ)(\sin^2 30^\circ + 4\cot^2 45^\circ - \sec^2 60^\circ)$

Solution:

$$(\operatorname{cosec}^2 45^\circ \sec^2 30^\circ) (\sin^2 30^\circ + 4\cot^2 45^\circ - \sec^2 60^\circ)$$

We know that by trigonometric ratios,

$$\operatorname{cosec} 45^\circ = \sqrt{2} \sec 30^\circ = \frac{2}{\sqrt{3}}$$

$$\sin 30^\circ = \frac{1}{2} \cot 45^\circ = 1$$

$$\sec 60^\circ = 2$$

Substituting the values in given equation

$$\left([\sqrt{2}]^2 \cdot \left[\frac{2}{\sqrt{3}} \right]^2 \right) \left(\left[\frac{1}{2} \right]^2 + 4(1) - (2)^2 \right)$$

$$= (2 \cdot (4/3)) \left[(1/4) + 4 - 4 \right] = (8/3) \cdot (1/4)$$

$$= 2/3$$

11. $\operatorname{cosec}^3 30^\circ \cos 60^\circ \tan^3 45^\circ \sin^2 90^\circ \sec^2 45^\circ \cot 30^\circ$

Solution:

$$\operatorname{cosec}^3 30^\circ \cos 60^\circ \tan^3 45^\circ \sin^2 90^\circ \sec^2 45^\circ \cot 30^\circ$$

Using trigonometric values, we have

$$= (2)^3 \times \left(\frac{1}{2}\right) \times (1^3) \times (1^2) \times (\sqrt{2}^2) \times (\sqrt{3})$$

$$= 8 \times \left(\frac{1}{2}\right) \times (1) \times (1) \times (2) \times (\sqrt{3})$$

$$= 8\sqrt{3}$$

12. $\cot^2 30^\circ - 2\cos^2 60^\circ - (3/4)\sec^2 45^\circ - 4\sec^2 30^\circ$

Solution:

Using trigonometric values, we have

$$\begin{aligned} & \cot^2 30^\circ - 2\cos^2 60^\circ - \frac{3}{4}\sec^2 45^\circ - 4\sec^2 30^\circ \\ &= (\sqrt{3}^2) - 2\left(\frac{1}{2}\right)^2 - \left(\frac{3}{4} \times \sqrt{2}^2\right) - \left(4 \times \left(\frac{2}{\sqrt{3}}\right)^2\right) \\ &= 3 - \frac{1}{2} - \frac{3}{2} - \frac{16}{3} \\ &= \frac{-13}{3} \end{aligned}$$

13. $(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ)$

Solution:

$$(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ)$$

Using trigonometric values, we have

$$\begin{aligned} &= \left(1 + \frac{1}{\sqrt{2}} + \frac{1}{2}\right) \left(1 - \frac{1}{\sqrt{2}} + \frac{1}{2}\right) \\ &= \left(\frac{3}{2} + \frac{1}{\sqrt{2}}\right) \left(\frac{3}{2} - \frac{1}{\sqrt{2}}\right) \\ &= \left(\left(\frac{3}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2\right) \\ &= \frac{9}{4} - \frac{1}{2} \\ &= \frac{7}{4} \end{aligned}$$

$$\frac{\sin 30^\circ - \sin 90^\circ + 2 \cos 0^\circ}{\tan 30^\circ \tan 60^\circ}$$

14.

Solution:

Given,

$$\frac{\sin 30^\circ - \sin 90^\circ + 2 \cos 0^\circ}{\tan 30^\circ \tan 60^\circ}$$

Using trigonometric values, we have

$$\begin{aligned} &= \frac{\frac{1}{2} - 1 + 2}{\frac{1}{\sqrt{3}} \times \sqrt{3}} \\ &= \frac{3}{2} \end{aligned}$$

15. $4/\cot^2 30^\circ + 1/\sin^2 60^\circ - \cos^2 45^\circ$

Solution:

$$\begin{aligned} &\frac{4}{\cot^2 30^\circ} + \frac{1}{\sin^2 60^\circ} - \cos^2 45^\circ \\ &= \frac{4}{(\sqrt{3})^2} + \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} - \left(\frac{1}{\sqrt{2}}\right)^2 \\ &= \frac{4}{3} + \frac{4}{3} - \frac{1}{2} \\ &= \frac{16-3}{6} \\ &= \frac{13}{6} \end{aligned}$$

16. $4(\sin^4 30^\circ + \cos^2 60^\circ) - 3(\cos^2 45^\circ - \sin^2 90^\circ) - \sin^2 60^\circ$

Solution:

Using trigonometric values, we have

$$\begin{aligned}
 & 4(\sin^4 30^\circ + \cos^2 60^\circ) - 3(\cos^2 45^\circ - \sin^2 90^\circ) - \sin^2 60^\circ \\
 &= 4\left(\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^2\right) - 3\left(\left(\frac{1}{\sqrt{2}}\right)^2 - 1\right) - \left(\frac{\sqrt{3}}{2}\right)^2 \\
 &= 4\left(\frac{1}{16} + \frac{1}{4}\right) + \frac{3}{2} - \frac{3}{4} \\
 &= \frac{8}{4} = 2
 \end{aligned}$$

17.
$$\frac{\tan^2 60^\circ + 4\cos^2 45^\circ + 3\sec^2 30^\circ + 5\cos^2 90^\circ}{\operatorname{cosec} 30^\circ + \sec 60^\circ - \cot^2 30^\circ}$$

Solution:

Using trigonometric values, we have

$$\begin{aligned}
 & \frac{\tan^2 60^\circ + 4\cos^2 45^\circ + 3\sec^2 30^\circ + 5\cos^2 90^\circ}{\operatorname{cosec} 30^\circ + \sec 60^\circ - \cot^2 30^\circ} \\
 &= \frac{(\sqrt{3})^2 + 4\left(\frac{1}{\sqrt{2}}\right)^2 + 3\left(\frac{2}{\sqrt{3}}\right)^2 + 5(0)}{2 + 2 - (\sqrt{3})^2} \\
 &= 3 + 2 + 4 \\
 &= 9
 \end{aligned}$$

18.
$$\frac{\sin 30^\circ}{\sin 45^\circ} + \frac{\tan 45^\circ}{\sec 60^\circ} - \frac{\sin 60^\circ}{\cot 45^\circ} - \frac{\cos 30^\circ}{\sin 90^\circ}$$

Solution:

Using trigonometric values, we have

$$\begin{aligned}
 & \frac{\sin 30^\circ}{\sin 45^\circ} + \frac{\tan 45^\circ}{\sec 60^\circ} - \frac{\sin 60^\circ}{\cot 45^\circ} - \frac{\cos 30^\circ}{\sin 90^\circ} \\
 &= \frac{\frac{1}{2}}{\frac{1}{\sqrt{2}}} + \frac{1}{2} - \frac{\frac{\sqrt{3}}{2}}{1} - \frac{\frac{\sqrt{3}}{2}}{1} \\
 &= \frac{\sqrt{2}}{2} + \frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \\
 &= \frac{\sqrt{2} + 1 - 2\sqrt{3}}{2}
 \end{aligned}$$

$$19. \frac{\tan 45^\circ}{\operatorname{cosec} 30^\circ} + \frac{\sec 60^\circ}{\cot 45^\circ} - \frac{5 \sin 90^\circ}{2 \cos 0^\circ}$$

Solution:

Using trigonometric values, we have

$$\begin{aligned} & \frac{\tan 45^\circ}{\operatorname{cosec} 30^\circ} + \frac{\sec 60^\circ}{\cot 45^\circ} - \frac{5 \sin 90^\circ}{2 \cos 0^\circ} \\ &= \frac{1}{2} + \frac{2}{1} - \frac{5(1)}{2(1)} \\ &= \frac{5}{2} - \frac{5}{2} \\ &= 0 \end{aligned}$$

Find the value of x in each of the following: (20-25)

20. $2 \sin 3x = \sqrt{3}$

Solution:

Given,

$$\begin{aligned} 2 \sin 3x &= \sqrt{3} \\ \sin 3x &= \frac{\sqrt{3}}{2} \\ \sin 3x &= \sin 60^\circ \\ 3x &= 60^\circ \\ x &= 20^\circ \end{aligned}$$