Exercise 5.3

1. Evalute the following:
(i) sin 20°/ cos 70°
(ii) cos 19°/ sin 71°
(iii) sin 21°/ cos 69°
(iv) tan 10°/ cot 80°
(v) sec 11°/ cosec 79°
Solution:

(i) We have,
sin 20°/ cos 70° = sin (90° – 70°)/ cos 70° = cos 70°/ cos70° = 1 [∵ sin (90 - θ) = cos θ]

(ii) We have,
cos 19°/ sin 71° = cos (90° – 71°)/ sin 71° = sin 71°/ sin 71° = 1 [∵ cos (90 - θ) = sin θ]

(iii) We have,
sin 21°/ cos 69° = sin (90° – 69°)/ cos 69° = cos 69°/ cos69° = 1 [∵ sin (90 - θ) = cos θ]

(iv) We have,
tan 10°/ cot 80° = tan (90° – 10°) / cot 80° = cot 80°/ cos80° = 1 [∵ tan (90 - θ) = cot θ]

(v) We have,
sec 11°/ cosec 79° = sec (90° – 79°)/ cosec 79° = cosec 79°/ cosec 79° = 1
[∵ sec (90 - θ) = cosec θ]

2. Evaluate the following:

(i) $\left(\frac{\sin 49^\circ}{\cos 41^\circ}\right)^2+\left(\frac{\cos 41^\circ}{\sin 49^\circ}\right)^2$

Solution:

We have, [∵ sin (90 - θ) = cos θ and cos (90 - θ) = sin θ]

$$\left(\frac{\sin 49^\circ}{\cos 41^\circ}\right)^2+\left(\frac{\cos 41^\circ}{\sin 49^\circ}\right)^2$$

$$=\left(\frac{\sin(90^\circ-41^\circ)}{\cos 41^\circ}\right)^2+\left(\frac{\cos(90^\circ-49^\circ)}{\sin 49^\circ}\right)^2$$

$$=\left(\frac{\cos 41^\circ}{\cos 41^\circ}\right)^2+\left(\frac{\sin 49^\circ}{\sin 49^\circ}\right)^2$$

$= 1^2 + 1^2 = 1 + 1$
$= 2$

(ii) cos 48°- sin 42°
Solution:

We know that, $\cos(90° - \theta) = \sin\theta$.
So,
$\cos 48° - \sin 42° = \cos(90° - 42°) - \sin 42° = \sin 42° - \sin 42° = 0$
Thus the value of $\cos 48° - \sin 42°$ is 0.

(iii) $\frac{\cot 40°}{\tan 50°} - \frac{1}{2}\left(\frac{\cos 35°}{\sin 55°}\right)$

Solution:
We have, [∵ $\cot(90 - \theta) = \tan\theta$ and $\cos(90 - \theta) = \sin\theta$]

$$\frac{\cot 40°}{\tan 50°} - \frac{1}{2}\left(\frac{\cos 35°}{\sin 55°}\right)$$

$$= \frac{\cot(90° - 50°)}{\tan 50°} - \frac{1}{2}\left(\frac{\cos(90° - 55°)}{\sin 55°}\right)$$

$$= \frac{\tan 50°}{\tan 50°} - \frac{1}{2}\left(\frac{\sin 55°}{\sin 55°}\right)$$

= 1 – 1/2(1)
= 1/2

(iv) $\left(\frac{\sin 27°}{\cos 63°}\right)^2 - \left(\frac{\cos 63°}{\sin 27°}\right)^2$

Solution:
We have, [∵ $\sin(90 - \theta) = \cos\theta$ and $\cos(90 - \theta) = \sin\theta$]

$$\left(\frac{\sin 27°}{\cos 63°}\right)^2 - \left(\frac{\cos 63°}{\sin 27°}\right)^2$$

$$= \left(\frac{\sin(90° - 63°)}{\cos 63°}\right)^2 - \left(\frac{\cos(90° - 27°)}{\sin 27°}\right)^2$$

$$= \left(\frac{\cos 63°}{\cos 63°}\right)^2 - \left(\frac{\sin 27°}{\sin 27°}\right)^2$$

= 1 – 1
= 0

(v) $\frac{\tan 35°}{\cot 55°} + \frac{\cot 78°}{\tan 12°} - 1$

Solution:

We have, [∵ cot (90 - θ) = tan θ and tan (90 - θ) = cot θ]

$\frac{\tan 35°}{\cot 55°} + \frac{\cot 78°}{\tan 12°} - 1$

= tan (90° – 35°)/ cot 55° + cot (90° – 12°)/ tan 12° – 1
= cot 55°/ cot 55° + tan 12°/ tan 12° – 1
= 1 + 1 – 1
= 1

(vi) $\frac{\sec 70°}{\operatorname{cosec} 20°} + \frac{\sin 59°}{\cos 31°}$

Solution:

We have , [∵ sin (90 - θ) = cos θ and sec (90 - θ) = cosec θ]

$\frac{\sec 70°}{\operatorname{cosec} 20°} + \frac{\sin 59°}{\cos 31°}$

= sec (90° – 20°)/ cosec 20° + sin (90° – 31°)/ cos 31°
= cosec 20°/ cosec 20° + cos 12°/ cos 12°
= 1 + 1
= 2

(vii) cosec 31° - sec 59°
Solution:

We have,
cosec 31° - sec 59°
Since, cosec (90 - θ) = cos θ
So,
cosec 31° - sec 59° = cosec (90° - 59°) – sec 59° = sec 59° - sec 59° = 0
Thus,
cosec 31° - sec 59° = 0

(viii) (sin 72° + cos 18°) (sin 72° - cos 18°)
Solution:

We know that,
sin (90 - θ) = cos θ

So, the given can be expressed as

(sin 72° + cos 18°) (sin (90 - 18)° - cos 18°)
= (sin 72° + cos 18°) (cos 18° - cos 18°)
= (sin 72° + cos 18°) x 0
= 0

(ix) sin 35° sin 55° - cos 35° cos 55°
Solution:

We know that,
sin (90 - θ) = cos θ

So, the given can be expressed as
sin (90 - 55)° sin (90 - 35)° - cos 35° cos 55°
= cos 55° cos 35° - cos 35° cos 55°
= 0

(x) tan 48° tan 23° tan 42° tan 67°
Solution:

We know that,
tan (90 - θ) = cot θ
So, the given can be expressed as
tan (90 - 42)° tan (90 - 67)° tan 42° tan 67°
= cot 42° cot 67° tan 42° tan 67°
= (cot 42° tan 42°)(cot 67° tan 67°)
= 1 x 1 [∵ tan θ x cot θ = 1]
= 1

(xi) sec 50° sin 40° + cos 40° cosec 50°
Solution:

We know that,
sin (90 - θ) = cos θ and cos (90 - θ) = sin θ
So, the given can be expressed as
sec 50° sin (90 - 50)° + cos (90 - 50)° cosec 50°
= sec 50° cos 50° + sin 50° cosec 50°
= 1 + 1 [∵ sin θ x cosec θ = 1 and cos θ x sec θ = 1]
= 2

3. Express each one of the following in terms of trigonometric ratios of angles lying between 0º and 45º

(i) sin 59º + cos 56º
(ii) tan 65º + cot 49º
(iii) sec 76º + cosec 52º
(iv) cos 78º + sec 78º
(v) cosec 54º + sin 72º
(vi) cot 85º + cos 75º
(vii) sin 67º + cos 75º

Solution:

Using the below trigonometric ratios of complementary angles, we find the required

$\sin (90 - \theta) = \cos \theta$ $\quad$ $\text{cosec} (90 - \theta) = \sec \theta$
$\cos (90 - \theta) = \sin \theta$ $\quad$ $\sec (90 - \theta) = \text{cosec}\, \theta$
$\tan (90 - \theta) = \cot \theta$ $\quad$ $\cot (90 - \theta) = \tan \theta$

(i) $\sin 59^o + \cos 56^o = \sin (90 - 31)^o + \cos (90 - 34)^o = \cos 31^o + \sin 34^o$
(ii) $\tan 65^o + \cot 49^o = \tan (90 - 25)^o + \cot (90 -41)^o = \cot 25^o + \tan 41^o$
(iii) $\sec 76^o + \text{cosec}\, 52^o = \sec (90 - 14)^o + \text{cosec} (90 - 38)^o = \text{cosec}\, 14^o + \sec 38^o$
(iv) $\cos 78^o + \sec 78^o = \cos (90 - 12)^o + \sec (90 - 12)^o = \sin 12^o + \text{cosec}\, 12^o$
(v) $\text{cosec}\, 54^o + \sin 72^o = \text{cosec} (90 - 36)^o + \sin (90 - 18)^o = \sec 36^o + \cos 18^o$
(vi) $\cot 85^o + \cos 75^o = \cot (90 - 5)^o + \cos (90 - 15)^o = \tan 5^o + \sin 15^o$

4. Express $\cos 75^o + \cot 75^o$ in terms of angles between 0^o and 30^o.
Solution:

Given,

$\cos 75^o + \cot 75^o$

Since, $\cos (90 - \theta) = \sin \theta$ and $\cot (90 - \theta) = \tan \theta$
$\cos 75^o + \cot 75^o = \cos (90 - 15)^o + \cot (90 - 15)^o = \sin 15^o + \tan 15^o$
Hence, $\cos 75^o + \cot 75^o$ can be expressed as $\sin 15^o + \tan 15^o$

5. If $\sin 3A = \cos (A – 26^o)$, where 3A is an acute angle, find the value of A.
Solution:

Given,

$\sin 3A = \cos (A – 26^o)$

Using $\cos (90 - \theta) = \sin \theta$, we have
$\sin 3A = \sin (90^o - (A – 26^o))$
Now, comparing both L.H.S and R.H.S

$3A = 90^o - (A – 26^o)$
$3A + (A – 26^o) = 90^o$
$4A – 26^o = 90^o$
$4A = 116^o$
$A = 116^o/4$
$\therefore A = 29^o$

6. If A, B, C are the interior angles of a triangle ABC, prove that
(i) $\tan ((C + A)/ 2) = \cot (B/2)$ (ii) $\sin ((B + C)/ 2) = \cos (A/2)$
Solution:

We know that, in triangle ABC the sum of the angles i.e $A + B + C = 180^o$
So, $C + A = 180^o – B$ $\quad \Rightarrow (C + A)/2 = 90^o – B/2$ …… (i)
And, $B + C = 180^o – A$ $\quad \Rightarrow (B + C)/2 = 90^o – A/2$ ……. (ii)

(i) L.H.S = $\tan ((C + A)/ 2)$
$\Rightarrow \tan ((C + A)/ 2) = \tan (90^o – B/2)$ [From (i)]
$= \cot (B/2)$ [$\because \tan (90 - \theta) = \cot \theta$]

= R.H.S

- Hence Proved

(ii) L.H.S = sin ((B + C)/2)

⇒ sin ((B + C)/ 2) = sin (90° – A/2) [From (ii)]

= cos (A/2)

= R.H.S

- Hence Proved

7. Prove that:

(i) tan 20° tan 35° tan 45° tan 55° tan 70° = 1

(ii) sin 48° sec 48° + cos 48° cosec 42° = 2

$$\text{(iii)}\ \frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - 2\cos 70^\circ \quad \operatorname{cosec} 42^\circ = 0$$

$$\text{(iv)}\ \frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \quad \operatorname{cosec} 31^\circ = 2$$

Solution:

(i) Taking L.H.S = tan 20° tan 35° tan 45° tan 55° tan 70°

= tan (90° − 70°) tan (90° − 55°) tan 45°tan 55° tan70°

= cot 70°cot 55° tan 45° tan 55° tan 70° [∵ tan (90 - θ) = cot θ]

= (tan 70°cot 70°)(tan 55°cot 55°) tan 45° [∵ tan θ x cot θ = 1]

= 1 × 1 × 1 = 1

- Hence proved

(ii) Taking L.H.S = sin 48° sec 48° + cos 48° cosec 42°

= sin 48° sec (90° − 48°) +cos 48° cosec (90° − 48°)

[∵sec (90 - θ) = cosec θ and cosec (90 - θ) = sec θ]

= sin 48°cosec 48° + cos 48°sec 48° [∵ cosec θ x sin θ = 1 and cos θ x sec θ = 1]

= 1 + 1 = 2

- Hence proved

(iii) Taking the L.H.S,

$$\frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\text{cosec } 20^\circ}{\sec 70^\circ} - 2\cos 70^\circ \text{ cosec } 20^\circ$$

$$= \frac{\sin(90^\circ - 20^\circ)}{\cos 20^\circ} + \frac{\text{cosec}(90^\circ - 70^\circ)}{\sec 70^\circ} - 2\cos(90^\circ - 20^\circ) \text{ cosec } 20^\circ$$

$$= \frac{\cos 20^\circ}{\cos 20^\circ} + \frac{\sec 70^\circ}{\sec 70^\circ} - 2\sin 20^\circ \times \frac{1}{\sin 20^\circ} \quad \begin{bmatrix} \sin(90^\circ - \theta) = \cos\theta \\ \text{cosec}(90^\circ - \theta) = \sec\theta \\ \cos(90^\circ - \theta) = \sin\theta \end{bmatrix}$$

= 1 + 1 – 2
= 2 – 2
= 0

- Hence proved

(iv) Taking L.H.S,

$$\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \text{ cosec } 31^\circ$$

$$= \frac{\cos(90^\circ - 10^\circ)}{\sin 10^\circ} + \cos 59^\circ \text{ cosec}(90^\circ - 59^\circ)$$

$$= \frac{\sin 10^\circ}{\sin 10^\circ} + \cos 59^\circ \sec 59^\circ$$

= 1 + 1
= 2

- Hence proved

8. Prove the following:
(i) sinθ sin (90º - θ) – cos θ cos (90º - θ) = 0
Solution:

Taking the L.H.S,
sinθ sin (90º - θ) – cos θ cos (90º - θ)
= sin θ cos θ - cos θ sin θ [∵ sin (90 - θ) = cos θ and cos (90 - θ) = sin θ]
= 0

(ii) $$\frac{\cos(90^\circ-\theta)\sec(90^\circ-\theta)\tan\theta}{\text{cosec}(90^\circ-\theta)\sin(90^\circ-\theta)\cot(90^\circ-\theta)} + \frac{\tan(90^\circ-\theta)}{\cot\theta} = 2$$

Solution:

Taking the L.H.S,

$$\frac{\cos(90^\circ-\theta)\sec(90^\circ-\theta)\tan\theta}{\text{cosec}(90^\circ-\theta)\sin(90^\circ-\theta)\cot(90^\circ-\theta)} + \frac{\tan(90^\circ-\theta)}{\cot\theta}$$

$$= \frac{\sin\theta\ \text{cosec}\ \theta\ \tan\theta}{\sec\theta\ \cos\theta\ \tan\theta} + \frac{\cot\theta}{\cot\theta}$$

$$= \frac{1\times\tan\theta}{1\times\tan\theta} + 1 = \frac{\tan\theta}{\tan\theta} + 1$$

[∵ cosec θ x sin θ = 1 and cos θ x sec θ = 1]

= 1 + 1

= 2 = R.H.S

- Hence Proved

(iii) $\frac{\tan(90^\circ-A)\cot A}{\text{cosec}^2 A} - \cos^2 A = 0$

Solution:

Taking the L.H.S, [∵ tan (90º - θ) = cot θ]

$$\text{L.H.S.} = \frac{\tan(90^\circ-A)\cot A}{\text{cosec}^2 A} - \cos^2 A$$

$$= \frac{\cot A\ \cot A}{\text{cosec}^2 A} - \cos^2 A$$

$$= \frac{\cot^2 A}{\text{cosec}^2 A} - \cos^2 A = \frac{\frac{\cos^2 A}{\sin^2 \theta}}{\frac{1}{\sin^2 A}} - \cos^2 A$$

$$= \frac{\cos^2 A \times \sin^2 A}{\sin^2 A \times 1} - \cos^2 A = \cos^2 A - \cos^2 A$$

= 0 = R.H.S

- Hence Proved

(iv) $\frac{\cos(90^\circ - A)\sin(90^\circ - A)}{\tan(90^\circ - A)} = \sin^2 A$

Solution:

Taking L.H.S, [∵ sin (90 - θ) = cos θ and cos (90 - θ) = sin θ]

$$\text{L.H.S.} = \frac{\cos(90^\circ - A)\sin(90^\circ - A)}{\tan(90^\circ - A)}$$

$$= \frac{\sin A \cos A}{\cot A} = \frac{\sin A \cos A}{\frac{\cos A}{\sin A}}$$

$$= \frac{\sin A \cos A \times \sin A}{\cos A} = \sin A \times \sin A$$

= $\sin^2 A$ = R.H.S

- Hence Proved

(v) sin (50º + θ) - cos (40º - θ) + tan 1º tan 10º tan 70º tan 80º tan 89º = 1

Solution:

Taking the L.H.S,

= sin (50º + θ) - cos (40º - θ) + tan 1º tan 10º tan 20º tan 70º tan 80º tan 89º

= [sin (90º - (40º - θ)] - cos (40º - θ)) + tan (90 - 89)º tan (90 - 80)º tan (90 - 70)º tan 70º tan 80º tan 89º [∵ sin (90 - θ) = cos θ]

= cos (40º - θ) - cos (40º - θ) + cot 89º cot 80º cot 70º tan 70º tan 80º tan 89º [∵ tan (90º - θ) = cot θ]

= 0 + (cot 89º x tan 89º) (cot 80º x tan 80º) (cot 70º x tan 70º)

= 0 + 1 x 1 x 1 [∵ tan θ x cot θ = 1]

= 1= R.H.S

- Hence Proved