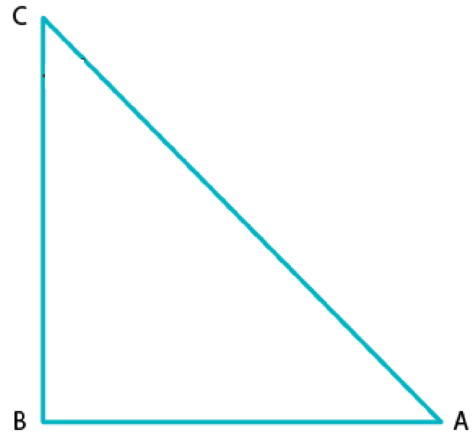
Exercise 5.1 Page No: 5.23

1. In each of the following, one of the six trigonometric ratios s given. Find the values of the other trigonometric ratios.



(i) sin A = 2/3

Solution:

We have,

sin A = 2/3 ……..….. (1)

As we know, by sin definition;

sin A = Perpendicular/ Hypotenuse = 2/3 ….(2)

By comparing eq. (1) and (2), we have

Opposite side = 2 and Hypotenuse = 3

Now, on using Pythagoras theorem in Δ ABC

$AC^2 = AB^{2\ +}\ BC^2$

Putting the values of perpendicular side (BC) and hypotenuse (AC) and for the base side as (AB), we get

⇒ $3^2 = AB^2 + 2^2$

$AB^2 = 3^2 – 2^2$

$AB^2 = 9 – 4$

$AB^{2\ =}\ 5$

$AB = \sqrt{5}$

Hence, Base = √5

By definition,

cos A = Base/Hypotenuse

⇒ cos A = √5/3

Since, cosec A = 1/sin A = Hypotenuse/Perpendicular

⇒ cosec A = 3/2

And, sec A = Hypotenuse/Base

⇒ sec A = 3/√5

And, tan A = Perpendicular/Base

⇒ tan A = 2/√5

And, cot A = 1/ tan A = Base/Perpendicular
⇒ cot A = √5/2

(ii) cos A = 4/5
Solution:

We have,
cos A = 4/5 ……..… (1)
As we know, by cos defination
cos A = Base/Hypotenuse …. (2)
By comparing eq. (1) and (2), we get
Base = 4 and Hypotenuse = 5

Now, using Pythagoras theorem in Δ ABC
$AC^2 = AB^2 + BC^2$
Putting the value of base (AB) and hypotenuse (AC) and for the perpendicular (BC), we get
$5^2 = 4^2 + BC^2$
$BC^2 = 5^2 – 4^2$
$BC^2 = 25 – 16$
$BC^2 = 9$
BC= 3
Hence, Perpendicular = 3

By definition,
sin A = Perpendicular/Hypotenuse
⇒ sin A = 3/5
Then, cosec A = 1/sin A
⇒ cosec A= 1/ (3/5) = 5/3 = Hypotenuse/Perependicular
And, sec A = 1/cos A
⇒ sec A =Hypotenuse/Base
sec A = 5/4
And, tan A = Perpendicular/Base
⇒ tan A = 3/4
Next, cot A = 1/tan A = Base/Perpendicular
∴ cot A = 4/3

(iii) tan θ = 11/1
Solution:

We have, tan θ = 11…..…. (1)
By definition,
tan θ = Perpendicular/ Base…. (2)
On Comparing eq. (1) and (2), we get;
Base = 1 and Perpendicular = 5

Now, using Pythagoras theorem in Δ ABC.

$AC^2 = AB^2 + BC^2$

Putting the value of base (AB) and perpendicular (BC) to get hypotenuse(AC), we get;

$AC^2 = 1^2 + 11^2$

$AC^2 = 1 + 121$

$AC^2 = 122$

$AC = \sqrt{122}$

Hence, hypotenuse = $\sqrt{122}$

By definition,

sin = Perpendicular/Hypotenuse

⇒ $\sin\theta = 11/\sqrt{122}$

And, $\operatorname{cosec}\theta = 1/\sin\theta$

⇒ $\operatorname{cosec}\theta = \sqrt{122}/11$

Next, $\cos\theta$ = Base/ Hypotenuse

⇒ $\cos\theta = 1/\sqrt{122}$

And, $\sec\theta = 1/\cos\theta$

⇒ $\sec\theta = \sqrt{122}/1 = \sqrt{122}$

And, $\cot\theta = 1/\tan\theta$

∴ $\cot\theta = 1/11$

(iv) sin θ = 11/15

Solution:

We have, $\sin\theta = 11/15$ ………. (1)

By definition,

$\sin\theta$ = Perpendicular/ Hypotenuse …. (2)

On Comparing eq. (1) and (2), we get;

Perpendicular = 11 and Hypotenuse= 15

Now, using Pythagoras theorem in Δ ABC

$AC^2 = AB^2 + BC^2$

Putting the value of perpendicular (BC) and hypotenuse (AC) to get the base (AB), we have

$15^2 = AB^2 + 11^2$

$AB^2 = 15^2 - 11^2$

$AB^2 = 225 - 121$

$AB^2 = 104$

$AB = \sqrt{104}$

$AB = \sqrt{(2\times2\times2\times13)}$

$AB = 2\sqrt{(2\times13)}$

$AB = 2\sqrt{26}$

Hence, Base = $2\sqrt{26}$

By definition,

$\cos\theta$ = Base/Hypotenuse

∴ $\cos\theta = 2\sqrt{26}/15$

And, cosec θ = 1/sin θ
∴ cosec θ = 15/11
And, secθ = Hypotenuse/Base
∴ secθ =15/ 2√26
And, tan θ = Perpendicular/Base
∴ tanθ =11/ 2√26
And, cot θ = Base/Perpendicular
∴ cotθ =2√26/ 11

(v) tan α = 5/12

Solution:

We have, tan α = 5/12 …. (1)
By definition,
tan α = Perpendicular/Base…. (2)
On Comparing eq. (1) and (2), we get
Base = 12 and Perpendicular side = 5

Now, using Pythagoras theorem in Δ ABC
$AC^2 = AB^2 + BC^2$
Putting the value of base (AB) and the perpendicular (BC) to get hypotenuse (AC), we have
$AC^2 = 12^2 + 5^2$
$AC^2 = 144 + 25$
$AC^2= 169$
AC = 13 [After taking sq root on both sides]
Hence, Hypotenuse = 13

By definition,
sin α = Perpendicular/Hypotenuse
∴ sin α = 5/13
And, cosec α = Hypotenuse/Perpendicular
∴ cosec α = 13/5
And, cos α = Base/Hypotenuse
∴ cos α = 12/13
And, sec α =1/cos α
∴ sec α = 13/12
And, tan α = sin α/cos α
∴ tan α=5/12
Since, cot α = 1/tan α
∴ cot α =12/5

(vi) sin θ = √3/2

Solution:

We have, sin θ = √3/2 …………. (1)

By definition,
sin θ = Perpendicular/ Hypotenuse….(2)
On Comparing eq. (1) and (2), we get;
Perpendicular = √3 and Hypotenuse = 2

Now, using Pythagoras theorem in Δ ABC
$AC^2 = AB^2 + BC^2$
Putting the value of perpendicular (BC) and hypotenuse (AC) and get the base (AB), we get;
$2^2 = AB^2 + (\sqrt{3})^2$
$AB^2 = 2^2 - (\sqrt{3})^2$
$AB^2 = 4 - 3$
$AB^2 = 1$
AB = 1
Thus, Base = 1

By definition,
cos θ = Base/Hypotenuse
∴ cos θ = 1/2
And, cosec θ = 1/sin θ
Or cosec θ= Hypotenuse/Perpendicualar
∴ cosec θ =2/√3
And, sec θ = Hypotenuse/Base
∴ sec θ = 2/1
And, tan θ = Perpendicula/Base
∴ tan θ = √3/1
And, cot θ = Base/Perpendicular
∴ cot θ = 1/√3

(vii) cos θ = 7/25
Solution:

We have, cos θ = 7/25 ……….. (1)
By definition,
cos θ = Base/Hypotenuse
On Comparing eq. (1) and (2), we get;
Base = 7 and Hypotenuse = 25

Now, using Pythagoras theorem in Δ ABC
$AC^2= AB^2 + BC^2$
Putting the value of base (AB) and hypotenuse (AC) to get the perpendicular (BC)
$25^2 = 7^2 +BC^2$
$BC^2 = 25^2 - 7^2$
$BC^2 = 625 - 49$
$BC^2 = 576$
BC= √576
BC= 24

Hence, Perpendicular side = 24

By definition,

sin θ = perpendicular/Hypotenuse

∴ sin θ = 24/25

Since, cosec θ = 1/sin θ

Also, cosec θ= Hypotenuse/Perpendicualar

∴ cosec θ = 25/24

Since, sec θ = 1/cosec θ

Also, sec θ = Hypotenuse/Base

∴ sec θ = 25/7

Since, tan θ = Perpendicular/Base

∴ tan θ = 24/7

Now, cot = 1/tan θ

So, cot θ = Base/Perpendicular

∴ cot θ = 7/24

(viii) tan θ = 8/15

Solution:

We have, tan θ = 8/15 …………. (1)

By definition,

tan θ = Perpendicular/Base …. (2)

On Comparing eq. (1) and (2), we get;

Base = 15 and Perpendicular = 8

Now, using Pythagoras theorem in Δ ABC

$AC^2 = 15^2 + 8^2$

$AC^2 = 225 + 64$

$AC^2 = 289$

$AC = \sqrt{289}$

AC = 17

Hence, Hypotenuse = 17

By definition,

Since, sin θ = perpendicular/Hypotenuse

∴ sin θ = 8/17

Since, cosec θ = 1/sin θ

Also, cosec θ = Hypotenuse/Perpendicualar

∴ cosec θ = 17/8

Since, cos θ = Base/Hypotenuse

∴ cos θ = 15/17

Since, sec θ = 1/cos θ

Also, sec θ = Hypotenuse/Base

∴ sec θ = 17/15

Since, cot θ = 1/tan θ

Also, cot θ = Base/Perpendicular
∴ cot θ = 15/8

(ix) cot θ = 12/5
Solution:

We have, cot θ = 12/5 …………. (1)
By definition,
cot θ = 1/tan θ
cot θ = Base/Perpendicular ……. (2)
On Comparing eq. (1) and (2), we have
Base = 12 and Perpendicular side = 5

Now, using Pythagoras theorem in Δ ABC
$AC^2 = AB^2 + BC^2$
Putting the value of base (AB) and perpendicular (BC) to get the hypotenuse (AC);
$AC^2 = 12^2 + 5^2$
$AC^2 = 144 + 25$
$AC^2 = 169$
$AC = \sqrt{169}$
$AC = 13$
Hence, Hypotenuse = 13

By definition,
Since, sin θ = perpendicular/Hypotenuse
∴ sin θ= 5/13
Since, cosec θ = 1/sin θ
Also, cosec θ= Hypotenuse/Perpendicualar
∴ cosec θ = 13/5
Since, cos θ = Base/Hypotenuse
∴ cos θ = 12/13
Since, sec θ = 1/cosθ
Also, sec θ = Hypotenuse/Base
∴ sec θ = 13/12
Since, tanθ = 1/cot θ
Also, tan θ = Perpendicular/Base
∴ tan θ = 5/12

(x) sec θ = 13/5
Solution:

We have, sec θ = 13/5………. (1)
By definition,
sec θ = Hypotenuse/Base…………. (2)
On Comparing eq. (1) and (2), we get
Base = 5 and Hypotenuse = 13

Now, using Pythagoras theorem in Δ ABC

$AC^2 = AB^2 + BC^2$

And. putting the value of base side (AB) and hypotenuse (AC) to get the perpendicular side (BC)

$13^2 = 5^2 + BC^2$

$BC^2 = 13^2 – 5^2$

$BC^2=169 – 25$

$BC^2= 144$

$BC= \sqrt{144}$

$BC = 12$

Hence, Perpendicular = 12

By definition,

Since, sin θ = perpendicular/Hypotenuse

∴ sin θ= 12/13

Since, cosec θ= 1/ sin θ

Also, cosec θ= Hypotenuse/Perpendicualar

∴ cosec θ = 13/12

Since, cos θ= 1/sec θ

Also, cos θ = Base/Hypotenuse

∴ cos θ = 5/13

Since, tan θ = Perpendicular/Base

∴ tan θ = 12/5

Since, cot θ = 1/tan θ

Also, cot θ = Base/Perpendicular

∴ cot θ = 5/12

(xi) cosec θ = √10

Solution:

We have, cosec θ = √10/1 ……..… (1)

By definition,

cosec θ = Hypotenuse/ Perpendicualar ……….(2)

And, cosecθ = 1/sin θ

On comparing eq.(1) and(2), we get

Perpendicular side = 1 and Hypotenuse = √10

Now, using Pythagoras theorem in Δ ABC

$AC^2 = AB^2 + BC^2$

Putting the value of perpendicular (BC) and hypotenuse (AC) to get the base side (AB)

$(\sqrt{10})^2 = AB^2 + 1^2$

$AB^2= (\sqrt{10})^2 – 1^2$

$AB^2= 10 – 1$

$AB = \sqrt{9}$

$AB = 3$

So, Base side = 3

By definition,
Since, sin θ = Perpendicular/Hypotenuse
∴ sin θ = 1/√10
Since, cos θ = Base/Hypotenuse
∴ cos θ = 3/√10
Since, sec θ = 1/cos θ
Also, sec θ = Hypotenuse/Base
∴ sec θ = √10/3
Since, tan θ = Perpendicular/Base
∴ tan θ = 1/3
Since, cot θ = 1/tan θ
∴ cot θ = 3/1

(xii) cos θ =12/15
Solution:

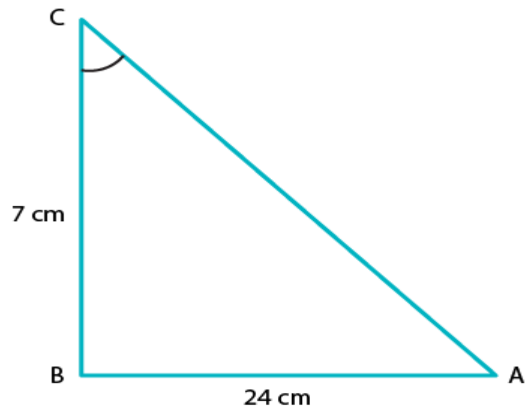
We have; cos θ = 12/15 ………. (1)
By definition,
cos θ = Base/Hypotenuse……… (2)
By comparing eq. (1) and (2), we get;
Base =12 and Hypotenuse = 15

Now, using Pythagoras theorem in Δ ABC, we get
$AC^2 = AB^2 + BC^2$
Putting the value of base (AB) and hypotenuse (AC) to get the perpendicular (BC);
$15^2 = 12^2 + BC^2$
$BC^2 = 15^2 – 12^2$
$BC^2 = 225 – 144$
$BC^2 = 81$
BC = √81
BC = 9
So, Perpendicular = 9

By definition,
Since, sin θ = perpendicular/Hypotenuse
∴ sin θ = 9/15 = 3/5
Since, cosec θ = 1/sin θ
Also, cosec θ = Hypotenuse/Perpendicualar
∴ cosec θ= 15/9 = 5/3
Since, sec θ = 1/cos θ
Also, sec θ = Hypotenuse/Base
∴ sec θ = 15/12 = 5/4
Since, tan θ = Perpendicular/Base
∴ tan θ = 9/12 = 3/4
Since, cot θ = 1/tan θ
Also, cot θ = Base/Perpendicular

∴ $\cot \theta = 12/9 = 4/3$

2. In a △ ABC, right angled at B, AB = 24 cm , BC = 7 cm. Determine
(i) sin A , cos A **(ii) sin C, cos C**
Solution:



(i) Given: In △ABC, AB = 24 cm, BC = 7cm and ∠ABC = 90°
To find: sin A, cos A

By using Pythagoras theorem in △ABC we have

$AC^2 = AB^2 + BC^2$
$AC^2 = 24^2 + 7^2$
$AC^2 = 576 + 49$
$AC^2 = 625$
$AC = \sqrt{625}$
$AC = 25$

Hence, Hypotenuse = 25

By definition,

sin A = Perpendicular side opposite to angle A/ Hypotenuse
sin A = BC/ AC
sin A = 7/ 25

And,

cos A = Base side adjacent to angle A/Hypotenuse
cos A = AB/ AC
cos A = 24/ 25

(ii) Given: In △ABC , AB = 24 cm and BC = 7cm and ∠ABC = 90°
To find: sin C, cos C

By using Pythagoras theorem in △ABC we have

$AC^2 = AB^2 + BC^2$
$AC^2 = 24^2 + 7^2$
$AC^2 = 576 + 49$
$AC^2 = 625$

AC = √625

AC= 25

Hence, Hypotenuse = 25

By definition,

sin C = Perpendicular side opposite to angle C/Hypotenuse

sin C = AB/ AC

sin C = 24/ 25

And,

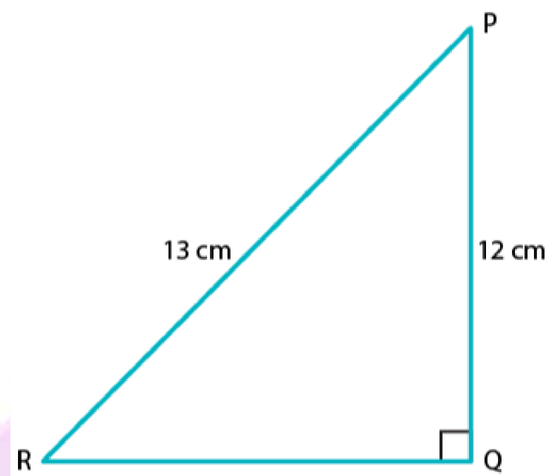
cos C = Base side adjacent to angle C/Hypotenuse

cos A = BC/AC

cos A = 7/25

3. In fig. 5.37, find tan P and cot R. Is tan P = cot R?

Solution:



By using Pythagoras theorem in △PQR, we have

$PR^2 = PQ^2 + QR^2$

Putting the length of given side PR and PQ in the above equation

$13^2 = 12^2 + QR^2$

$QR^2 = 13^2 – 12^2$

$QR^2 = 169 – 144$

$QR^2 = 25$

QR = √25 = 5

By definition,

tan P = Perpendicular side opposite to P/ Base side adjacent to angle P

tan P = QR/PQ

tan P = 5/12 ………. (1)

And,

cot R= Base/Perpendicular

cot R= QR/PQ

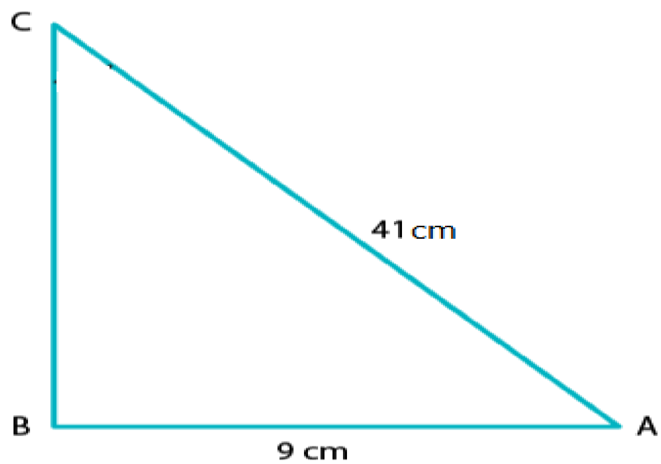
cot R= 5/12 …. (2)

When comparing equation (1) and (2), we can see that R.H.S of both the equation is equal.

Therefore, L.H.S of both equations should also be equal.
∴ tan P = cot R

Yes, tan P = cot R = 5/12

4. If sin A = 9/41, compute cos A and tan A.
Solution:



Given that, sin A = 9/41 …………. (1)
Required to find: cos A, tan A
By definition, we know that
sin A = Perpendicular/ Hypotenuse……………(2)
On Comparing eq. (1) and (2), we get;
Perpendicular side = 9 and Hypotenuse = 41

Let's construct △ABC as shown below,
And, here the length of base AB is unknown.
Thus, by using Pythagoras theorem in △ABC, we get;

$AC^2 = AB^2 + BC^2$
$41^2 = AB^2 + 9^2$
$AB^2 = 41^2 – 9^2$
$AB^2 = 168 – 81$
AB= 1600
AB = √1600
AB = 40

⇒ Base of triangle ABC, AB = 40

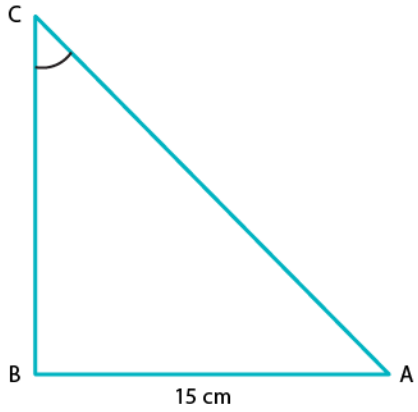
We know that,
cos A = Base/ Hypotenuse
cos A =AB/AC
cos A =40/41

And,
tan A = Perpendicular/ Base
tan A = BC/AB

tan A = 9/40

5. Given 15cot A= 8, find sin A and sec A.

Solution



We have, 15cot A = 8

Required to find: sin A and sec A

As, 15 cot A = 8

⇒ cot A = 8/15 …….(1)

And we know,

cot A = 1/tan A

Also by definition,

Cot A = Base side adjacent to ∠A/ Perpendicular side opposite to ∠A …. (2)

On comparing equation (1) and (2), we get;

Base side adjacent to ∠A = 8

Perpendicular side opposite to ∠A = 15

So, by using Pythagoras theorem to △ABC, we have

$AC^2 = AB^2 + BC^2$

Substituting values for sides from the figure

$AC^2 = 8^2 + 15^2$

$AC^2 = 64 + 225$

$AC^2 = 289$

AC = √289

AC = 17

Therefore, hypotenuse =17

By definition,

sin A = Perpendicular/Hypotenuse

⇒ sin A= BC/AC

sin A= 15/17 (using values from the above)

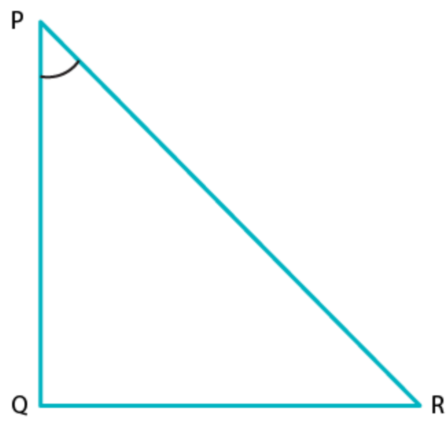
Also,

sec A= 1/ cos A

⇒ secA = Hypotenuse/ Base side adjacent to ∠A

∴ sec A= 17/8

6. In △PQR, right-angled at Q, PQ = 4cm and RQ = 3 cm. Find the value of sin P, sin R, sec P and sec R.
Solution:



Given:
△PQR is right-angled at Q.
PQ = 4cm
RQ = 3cm
Required to find: sin P, sin R, sec P, sec R

Given △PQR,
By using Pythagoras theorem to △PQR, we get
$PR^2 = PQ^2 + RQ^2$
Substituting the respective values,
$PR^2 = 4^2 + 3^2$
$PR^2 = 16 + 9$
$PR^2 = 25$
$PR = \sqrt{25}$
PR = 5
⇒ Hypotenuse =5

By definition,
sin P = Perpendicular side opposite to angle P/ Hypotenuse
sin P = RQ/ PR
⇒ sin P = 3/5
And,
sin R = Perpendicular side opposite to angle R/ Hypotenuse
sin R = PQ/ PR
⇒ sin R = 4/5
And,
sec P=1/cos P
secP = Hypotenuse/ Base side adjacent to ∠P
sec P = PR/ PQ
⇒ sec P = 5/4

Now,

sec R = 1/cos R

secR = Hypotenuse/ Base side adjacent to ∠R

sec R = PR/ RQ

⇒ sec R = 5/3

7. If cot θ = 7/8, evaluate

(i) (1+sin θ)(1–sin θ)/ (1+cos θ)(1–cos θ)

(ii) $\cot^2 \theta$

Solution:

(i) Required to evaluate: $\frac{(1+sin\theta)(1-sin\theta)}{(1+cos\theta)(1-cos\theta)}$, given = cot θ = 7/8

Taking the numerator, we have

(1+sin θ)(1–sin θ) = $1 - \sin^2 \theta$ [Since, (a+b)(a-b) = $a^2 - b^2$]

Similarly,

(1+cos θ)(1–cos θ) = $1 - \cos^2 \theta$

We know that,

$\sin^2 \theta + \cos^2 \theta = 1$

⇒ $1 - \cos^2 \theta = \sin^2 \theta$

And,

$1 - \sin^2 \theta = \cos^2 \theta$

Thus,

(1+sin θ)(1 –sin θ) = $1 - \sin^2 \theta = \cos^2 \theta$

(1+cos θ)(1–cos θ) = $1 - \cos^2 \theta = \sin^2 \theta$

⇒ $\frac{(1+sin\theta)(1-sin\theta)}{(1+cos\theta)(1-cos\theta)}$

= $\cos^2 \theta / \sin^2 \theta$

= $(\cos \theta/\sin \theta)^2$

And, we know that (cos θ/sin θ) = cot θ

⇒ $\frac{(1+sin\theta)(1-sin\theta)}{(1+cos\theta)(1-cos\theta)}$

= $(\cot \theta)^2$

= $(7/8)^2$

= 49/ 64

(ii) Given,

cot θ = 7/8

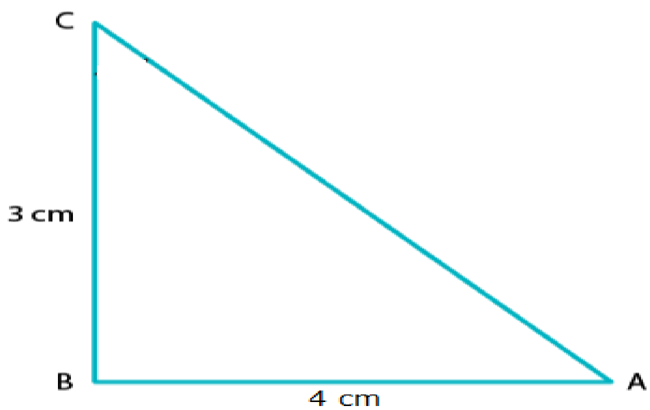
So, by squaring on both sides we get

$(\cot\theta)^2 = (7/8)^2$

$\therefore \cot\theta^2 = 49/64$

8. If 3cot A = 4, check whether $(1–\tan^2A)/(1+\tan^2A) = (\cos^2A – \sin^2A)$ or not.

Solution:



Given,

3cot A = 4

⇒ cot A = 4/3

By definition,

tan A = 1/ Cot A = 1/ (4/3)

⇒ tan A = 3/4

Thus,

Base side adjacent to ∠A = 4

Perpendicular side opposite to ∠A = 3

In ΔABC, Hypotenuse is unknown

Thus, by applying Pythagoras theorem in ΔABC

We get

$AC^2 = AB^2 + BC^2$

$AC^2 = 4^2 + 3^2$

$AC^2 = 16 + 9$

$AC^2 = 25$

$AC = \sqrt{25}$

AC = 5

Hence, hypotenuse = 5

Now, we can find that

sin A = opposite side to ∠A/ Hypotenuse = 3/5

And,

cos A = adjacent side to ∠A/ Hypotenuse = 4/5

Taking the LHS,

L.H.S = $\frac{1-\tan^2 A}{1+\tan^2 A}$

Putting value of tan A

We get,

L.H.S= $\frac{1-(\frac{3}{4})^2}{1+(\frac{3}{4})^2}$

$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-(\frac{3}{4})^2}{1+(\frac{3}{4})^2}$

$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}}$

Take L.C.M of both numerator and denominator;

$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}}$

$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{7}{25}$

Thus, LHS = 7/25

Now, taking RHS

R.H.S = $\cos^2 A - \sin^2 A$

Putting value of sin A and cos A

R.H.S= $(\frac{4}{5})^2 - (\frac{3}{5}^2)$

$\cos^2 A - \sin^2 A = (\frac{4}{5})^2 - (\frac{3}{5}^2)$

$\cos^2 A - \sin^2 A = \frac{16}{25} - \frac{9}{25}$

$\cos^2 A - \sin^2 A = \frac{16-9}{25}$

$\cos^2 A - \sin^2 A = \frac{7}{25}$

Therefore,

$\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

Hence Proved

9. If tan θ = a/b, find the value of (cos θ + sin θ)/ (cos θ - sin θ)

Solution:

Given,

tan θ = a/b

And, we know by definition that

tan θ = opposite side/ adjacent side

Thus, by comparison

Opposite side = a and adjacent side = b

To find the hypotenuse, we know that by Pythagoras theorem that

$\text{Hypotenuse}^2 = \text{opposite side}^2 + \text{adjacent side}^2$

⇒ Hypotenuse = $\sqrt{(a^2 + b^2)}$

So, by definition

sin θ = opposite side/ Hypotenuse

sin θ = $a/\sqrt{(a^2 + b^2)}$

And,

cos θ = adjacent side/ Hypotenuse

cos θ = $b/\sqrt{(a^2 + b^2)}$

Now,

After substituting for cos θ and sin θ, we have

$$\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{(a+b)/\sqrt{(a^2+b^2)}}{(a-b)/\sqrt{(a^2+b^2)}}$$

$$\therefore \quad \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{(a+b)}{(a-b)}$$

Hence Proved.

10. If 3 tan θ = 4, find the value of $\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$

Solution:

Given, 3 tan θ = 4

⇒ tan θ = 4/3

From, $\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$ let's divide the numerator and denominator by cos θ.

We get,

(4 – tan θ) / (2 + tan θ)

⇒ (4 – (4/3)) / (2 + (4/3)) [using the value of tan θ]

⇒ (12 – 4) / (6 + 4) [After taking LCM and cancelling it]

⇒ 8/10 = 4/5

$\therefore \quad \frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$ = 4/5

11. If 3 cot θ = 2, find the value of $\frac{4\sin\theta - 3\cos\theta}{2\sin\theta + 6\cos\theta}$

Solution:

Given, 3 cot θ = 2

⇒ cot θ = 2/3

From, $\frac{4\sin\theta - 3\cos\theta}{2\sin\theta + 6\cos\theta}$ let's divide the numerator and denominator by sin θ.

We get,

(4 –3 cot θ) / (2 + 6 cot θ)

⇒ (4 – 3(2/3)) / (2 + 6(2/3)) [using the value of tan θ]

⇒ (4 – 2) / (2 + 4) [After taking LCM and simplifying it]

⇒ 2/6 = 1/3

$\therefore \quad \frac{4 \sin\theta - 3\cos\theta}{2\sin\theta + 6\cos\theta}$ = 1/3

12. If tan θ = a/b, prove that $\frac{a \sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a^2 - b^2}{a^2 + b^2}$

Solution:

Given, tan θ = a/b

From LHS, $\frac{a \sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$ let's divide the numerator and denominator by cos θ.

And we get,

(a tan θ – b) / (a tan θ + b)

⇒ (a(a/b) – b) / (a(a/b) + b) [using the value of tan θ]

⇒ $(a^2 - b^2)/b^2$ / $(a^2 + b^2)/b^2$ [After taking LCM and simplifying it]

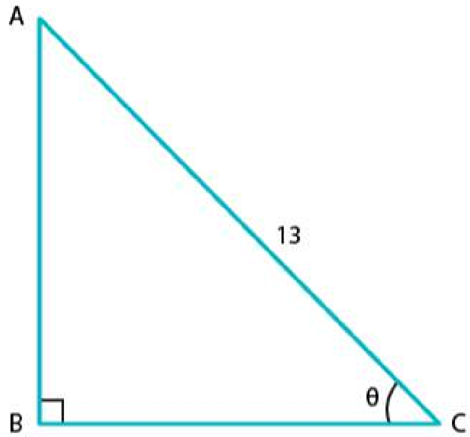
⇒ $(a^2 - b^2)/ (a^2 + b^2)$

= RHS

- Hence Proved

13. If sec θ = 13/5, show that $\frac{2 \sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = 3$

Solution:



Given,

sec θ = 13/5

We know that,

sec θ = 1/ cos θ

⇒ cos θ = 1/ sec θ = 1/ (13/5)

∴ $\cos\theta = 5/13$ ……. (1)

By definition,
$\cos\theta$ = adjacent side/ hypotenuse ….. (2)

Comparing (1) and (2), we have
Adjacent side = 5 and hypotenuse = 13

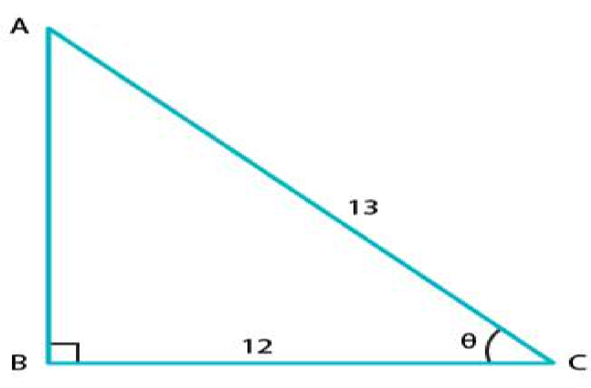
By Pythagoras theorem,
Opposite side = $\sqrt{(\text{hypotenuse})^2 - (\text{adjacent side})^2}$
$= \sqrt{13^2 - 5^2}$
$= \sqrt{169 - 25}$
$= \sqrt{144}$
$= 12$
Thus, opposite side = 12
By definition,
$\tan\theta$ = opposite side/ adjacent side
∴ $\tan\theta = 12/5$

From, $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$ let's divide the numerator and denominator by $\cos\theta$.

We get,
$(2\tan\theta - 3) / (4\tan\theta - 9)$
⇒ $(2(12/5) - 3) / (4(12/5) - 9)$ [using the value of $\tan\theta$]
⇒ $(24 - 15) / (48 - 45)$ [After taking LCM and cancelling it]
⇒ $9/3 = 3$

∴ $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = 3$

14. If $\cos\theta = 12/13$, show that $\sin\theta(1 - \tan\theta) = 35/156$
Solution:



Given, cos θ = 12/13…… (1)
By definition we know that,
cos θ = Base side adjacent to ∠θ / Hypotenuse……. (2)

When comparing equation (1) and (2), we get
Base side adjacent to ∠θ = 12 and Hypotenuse = 13
From the figure,
Base side BC = 12
Hypotenuse AC = 13
Side AB is unknown here and it can be found by using Pythagoras theorem
Thus by applying Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$13^2 = AB^2 + 12^2$

Therefore,

$AB^2 = 13^2 – 12^2$

$AB^2 = 169 – 144$

$AB^2 = 25$

$AB = \sqrt{25}$

AB = 5 …. (3)

Now, we know that
sin θ = Perpendicular side opposite to ∠θ / Hypotenuse
Thus, sin θ = AB/AC [from figure]
⇒ sin θ = 5/13… (4)
And, tan θ = sin θ / cos θ = (5/13) / (12/13)
⇒ tan θ = 12/13… (5)

Taking L.H.S we have
L.H.S = sin θ (1 – tan θ)

Substituting the value of sin θ and tan θ from equation (4) and (5)
We get,

$\Rightarrow$

$$L.H.S = \frac{5}{13}\left(1 - \frac{5}{12}\right)$$

$$L.H.S = \frac{5}{13}\left(\frac{1\times 12}{1\times 12} - \frac{5}{12}\right) \quad \text{[Taking LCM]}$$

$$L.H.S = \frac{5}{13}\left(\frac{12-5}{12}\right)$$

$$L.H.S = \frac{5}{13}\left(\frac{7}{12}\right)$$

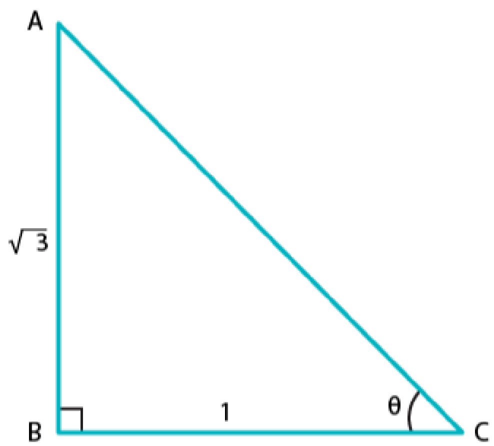
$$L.H.S = \frac{5\times 7}{13\times 12}$$

L.H.S = 35/156

Therefore it's shown that sin θ(1 - tan θ)= 35/156

15. If $\cot\theta = \frac{1}{\sqrt{3}}$, show that $\frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{3}{5}$

Solution:



Given, cot θ = 1/√3....... (1)

By definition we know that,

cot θ = 1/ tan θ

And, since tan θ = perpendicular side opposite to ∠θ / Base side adjacent to ∠θ

⇒ cot θ = Base side adjacent to ∠θ / perpendicular side opposite to ∠θ …… (2)

[Since they are reciprocal to each other]

On comparing equation (1) and (2), we get

Base side adjacent to ∠θ = 1 and Perpendicular side opposite to ∠θ = √3

Therefore, the triangle formed is,
On substituting the values of known sides as AB = √3 and BC = 1

$AC^2 = (\sqrt{3}) + 1$
$AC^2 = 3 + 1$
$AC^2 = 4$
$AC = \sqrt{4}$

Therefore, AC = 2 … (3)
Now, by definition

sin θ = Perpendicular side opposite to ∠θ / Hypotenuse = AB / AC
⇒ sin θ = √3/ 2 ……(4)

And, cos θ = Base side adjacent to ∠θ / Hypotenuse = BC / AC
⇒ cos θ = 1/ 2 ….. (5)

Now, taking L.H.S we have

$$L.H.S = \frac{1-\cos^2\theta}{2-\sin^2\theta}$$

Substituting the values from equation (4) and (5), we have

$$L.H.S = \frac{1-\left(\frac{1}{2}\right)^2}{2-\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$L.H.S = \frac{1-\frac{1}{4}}{2-\frac{3}{4}}$$

Now by taking L.C.M in numerator and denominator, we get

$$L.H.S = \frac{\frac{(4\times 1)-1}{4}}{\frac{(4\times 2)-3}{4}}$$

$$L.H.S = \frac{\frac{4-1}{4}}{\frac{8-3}{4}}$$

$$L.H.S = \frac{3}{4}\times\frac{4}{5}$$

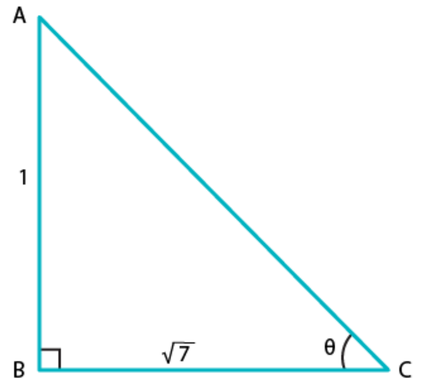
$$L.H.S = \frac{3}{5} = R.H.S$$

Therefore, $\frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{3}{5}$

16. If $\tan\theta = \frac{1}{\sqrt{7}}$, then show that $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} = \frac{3}{4}$

Solution:

Given, tan θ = 1/ √7 …..(1)



By definition, we know that

tan θ = Perpendicular side opposite to ∠θ / Base side adjacent to ∠θ ……(2)

On comparing equation (1) and (2), we have

Perpendicular side opposite to ∠θ = 1

Base side adjacent to ∠θ = √7

Thus, the triangle representing ∠ θ is,

Hypotenuse AC is unknown and it can be found by using Pythagoras theorem

By applying Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$AC^2 = 1^2 + (\sqrt{7})^2$

$AC^2 = 1 + 7$

$AC^2 = 8$

AC = √8

⇒ AC = 2√2

By definition,

sin θ = Perpendicular side opposite to ∠θ / Hypotenuse = AB / AC

⇒ sin θ = 1/ 2√2

And, since cosec θ = 1/sin θ

⇒ cosec θ = 2√2 …….. (3)

Now,

cos θ = Base side adjacent to ∠θ / Hypotenuse = BC / AC

⇒ cos θ = √7/ 2√2

And, since sec θ = 1/ sin θ

⇒ sec θ = 2√2/ √7 ……. (4)

Taking the L.H.S of the equation,

$$L.H.S = \frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$$

Substituting the value of cosec θ and sec θ from equation (3) and (4), we get

$$L.H.S = \frac{[(2\sqrt{2})]^2 - \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{[(2\sqrt{2})]^2 + \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}$$

$$L.H.S = \frac{(8) - \left(\frac{8}{7}\right)}{(8) + \left(\frac{8}{7}\right)} = \frac{\frac{56-8}{7}}{\frac{56+8}{7}}$$ [Taking L.C.M and simplifying]

$$L.H.S = \frac{\frac{48}{7}}{\frac{64}{7}}$$

Therefore,

L.H.S = 48/64 = 3/4 = R.H.S

Hence proved that $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} = \frac{3}{4}$

17. If sec θ = 5/4, find the value of $\frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta}$

Solution:

Given,

sec θ = 5/4

We know that,

sec θ = 1/ cos θ

⇒ cos θ = 1/ (5/4) = 4/5 …… (1)

By definition,

cos θ = Base side adjacent to ∠θ / Hypotenuse …. (2)

On comparing equation (1) and (2), we have

Hypotenuse = 5

Base side adjacent to ∠θ = 4

Thus, the triangle representing ∠ θ is ABC.

Perpendicular side opposite to ∠θ, AB is unknown and it can be found by using Pythagoras theorem

By applying Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$AB^2 = AC^2 + BC^2$

$AB^2 = 5^2 – 4^2$

$AB^2 = 25 – 16$

$AB = \sqrt{9}$

⇒ $AB = 3$

By definition,

sin θ = Perpendicular side opposite to ∠θ / Hypotenuse = AB / AC

⇒ sin θ = 3/ 5 …..(3)

Now, tan θ = Perpendicular side opposite to ∠θ / Base side adjacent to ∠θ

⇒ tan θ = 3/ 4 ……(4)

And, since cot θ = 1/ tan θ

⇒ cot θ = 4/ 3 ……(5)

Now,

Substituting the value of sin θ, cos θ, cot θ and tan θ from the equations (1), (3), (4) and (5) we have,

$$\frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta} = \frac{\frac{3}{5} - 2\left(\frac{4}{5}\right)}{\frac{3}{4} - \frac{4}{3}}$$

= 12/7

Therefore,

$$\frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta} = \frac{12}{7}$$

18. If tan θ = 12/13, find the value of $\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$

Solution:

Given,

tan θ = 12/13 …….. (1)

We know that by definition,

tan θ = Perpendicular side opposite to ∠θ / Base side adjacent to ∠θ …… (2)

On comparing equation (1) and (2), we have

Perpendicular side opposite to ∠θ = 12

Base side adjacent to ∠θ = 13

Thus, in the triangle representing ∠ θ we have,
Hypotenuse AC is the unknown and it can be found by using Pythagoras theorem
So by applying Pythagoras theorem, we have

$AC^2 = 12^2 + 13^2$

$AC^2 = 144 + 169$

$AC^2 = 313$

⇒ $AC = \sqrt{313}$

By definition,

sin θ = Perpendicular side opposite to ∠θ / Hypotenuse = AB / AC

⇒ $\sin\theta = 12/\sqrt{313}$ …..(3)

And, cos θ = Base side adjacent to ∠θ / Hypotenuse = BC / AC

⇒ $\cos\theta = 13/\sqrt{313}$ …..(4)

Now, substituting the value of sin θ and cos θ from equation (3) and (4) respectively in the equation below

$$\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} = \frac{2 \times \frac{13}{\sqrt{313}} \times \frac{12}{\sqrt{313}}}{\left(\frac{13}{\sqrt{313}}\right)^2 - \left(\frac{12}{\sqrt{313}}\right)^2}$$

$$= \frac{\frac{312}{313}}{\frac{25}{313}}$$

$$= \frac{312}{25}$$

Therefore,

$$\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} = \frac{312}{25}$$

Exercise 5.2

Evaluate each of the following:

1. sin 45° sin 30° + cos 45° cos 30°

Solution:

Sin 45° sin 30° + cos 45° cos 30°

Value of trigonometric ratios are:

$sin45^\circ = \frac{1}{\sqrt{2}}\ sin30^\circ = \frac{1}{2}$

$cos45^\circ = \frac{1}{\sqrt{2}}\ cos30^\circ = \frac{\sqrt{3}}{2}$

Substituting in the given equation, we get

$\frac{1}{\sqrt{2}} \cdot \frac{1}{2} + \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2}$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

2. sin 60° cos 30° + cos 60° sin 30°

Solution:

sin 60° cos 30° + cos 60° sin 30°

By trigonometric ratios we have ,

$sin60^\circ = \frac{\sqrt{3}}{2}\ sin30^\circ = \frac{1}{2}$

$cos30^\circ = \frac{\sqrt{3}}{2}\ cos60^\circ = \frac{1}{2}$

Substituting the values in given equation

$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2}$

$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

3. cos 60° cos 45° – sin 60° sin 45°
Solution:

cos 60° cos 45° – sin 60° sin 45°

We know that by trigonometric ratios

$cos60^\circ = \frac{1}{2}\ cos45^\circ = \frac{1}{\sqrt{2}}$

$sin60^\circ = \frac{\sqrt{3}}{2}\ sin45^\circ = \frac{1}{\sqrt{2}}$

Substituting the values in given equation

$\frac{1}{2}\cdot\frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}$

$= \frac{1-\sqrt{3}}{2\sqrt{2}}$

4. sin^2 30° + sin^2 45° + sin^2 60° + sin^2 90°
Solution:

$sin^2 30^\circ + sin^2 45^\circ + sin^2 60^\circ + sin^2 90^\circ$

We know that by trigonometric ratios

$sin30^\circ = \frac{1}{2}\ sin45^\circ = \frac{1}{\sqrt{2}}$

$sin60^\circ = \frac{\sqrt{3}}{2}\ sin90^\circ = 1$

Substituting the values in given equation, we get

$= \left[\frac{1}{2}\right]^2 + \left[\frac{1}{\sqrt{2}}\right]^2 + \left[\frac{\sqrt{3}}{2}\right]^2 + 1$

$= \frac{1}{4} + \frac{1}{2} + \frac{3}{4} + 1$

$= \frac{5}{2}$

5. $\cos^2 30^\circ + \cos^2 45^\circ + \cos^2 60^\circ + \cos^2 90^\circ$

Solution:

$$cos^2 30^\circ + cos^2 45^\circ + cos^2 60^\circ + cos^2 90^\circ$$

We know that by trigonometric ratios

$$cos30^\circ = \frac{\sqrt{3}}{2} \; cos45^\circ = \frac{1}{\sqrt{2}}$$

$$cos60^\circ = \frac{1}{2} \; cos90^\circ = 0$$

Substituting the values in given equation

$$\left[\frac{\sqrt{3}}{2}\right]^2 + \left[\frac{1}{\sqrt{2}}\right]^2 + \left[\frac{1}{2}\right]^2 + 0$$

$$= \frac{3}{4} + \frac{1}{2} + \frac{1}{4}$$

$$= \frac{3}{2}$$

6. $\tan^2 30^\circ + \tan^2 45^\circ + \tan^2 60^\circ$

Solution:

$$tan^2 30^\circ + tan^2 45^\circ + tan^2 60^\circ$$

We know that by trigonometric ratios

$$tan30^\circ = \frac{1}{\sqrt{3}} \; tan60^\circ = \sqrt{3}$$

$$tan45^\circ = 1$$

Substituting the values in given equation

$$\left[\frac{1}{\sqrt{3}}\right]^2 + \left[\sqrt{3}\right]^2 + 1$$

$$= \frac{1}{3} + 3 + 1$$

$$= \frac{13}{3}$$

7. $2\sin^2 30^\circ - 3\cos^2 45^\circ + \tan^2 60^\circ$
Solution:

$$2sin^2 30^\circ - 3cos^2 45^\circ + tan^2 60^\circ$$

We know that by trigonometric ratios

$sin30^\circ = \frac{1}{2}\ cos45^\circ = \frac{1}{\sqrt{2}}$

$tan60^\circ = \sqrt{3}$

Substituting the values in given equation

$= 2\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\sqrt{3}\right)^2$

$= 2\left(\frac{1}{4}\right) - 3\left(\frac{1}{2}\right) + 3$

$= \frac{1-3+6}{2}$

$= 2$

8. $\sin^2 30^\circ \cos^2 45^\circ + 4\tan^2 30^\circ + (1/2) \sin^2 90^\circ - 2\cos^2 90^\circ + (1/24) \cos 20^\circ$
Solution:

$$sin^2 30^\circ cos^2 45^\circ + 4tan^2 30^\circ + \frac{1}{2}sin^2 90^\circ - 2cos^2 90^\circ + \frac{1}{24}cos^2 0^\circ$$

We know that by trigonometric ratios

$sin30^\circ = \frac{1}{2}$

$cos45^\circ = \frac{1}{\sqrt{2}}$

$tan30^\circ = \frac{1}{\sqrt{3}}$

$sin90^\circ = 1$

$cos90^\circ = 0$

$cos0^\circ = 1$

Substituting the values in given equation

$$\left[\frac{1}{2}\right]^2 \cdot \left[\frac{1}{\sqrt{2}}\right]^2 + 4\left[\frac{1}{\sqrt{3}}\right]^2 + \frac{1}{2}[1]^2 - 2[0]^2 + \frac{1}{24}[1]^2$$

$= \frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24}$

$= \frac{48}{24}$

$= 2$

9. $4(\sin^4 60^\circ + \cos^4 30^\circ) - 3(\tan^2 60^\circ - \tan^2 45^\circ) + 5\cos^2 45^\circ$

Solution:

$$4\left(\sin^4 60^\circ + \cos^4 30^\circ\right) - 3\left(\tan^2 60^\circ - \tan^2 45^\circ\right) + 5\cos^2 45^\circ$$

We know that by trigonometric ratios we have ,

$\sin 60^\circ = \frac{\sqrt{3}}{2}\ \cos 45^\circ = \frac{1}{\sqrt{2}}$

$\tan 60^\circ = \sqrt{3}\ \cos 30^\circ = \frac{\sqrt{3}}{2}$

Substituting the values in given equation

$= 4 \cdot \frac{18}{16} - 6 + \frac{5}{2}$

$= \frac{1}{4} - 6 + \frac{5}{2}$

$= \frac{14}{2} - 6 = 7 - 6 = 1$

10. $(\operatorname{cosec}^2 45^\circ \sec^2 30^\circ)(\sin^2 30^\circ + 4\cot^2 45^\circ - \sec^2 60^\circ)$

Solution:

$\left(cosec^2 45^\circ sec^2 30^\circ\right)\left(sin^2 30^\circ + 4cot^2 45^\circ - sec^2 60^\circ\right)$

We know that by trigonometric ratios,

$cosec45^\circ = \sqrt{2}\ sec30^\circ = \frac{2}{\sqrt{3}}$

$sin30^\circ = \frac{1}{2}\ cot45^\circ$ = 1

$sec60^\circ$ = 2

Substituting the values in given equation

$\left(\left[\sqrt{2}\right]^2 . \left[\frac{2}{\sqrt{3}}\right]^2\right)\left(\left[\frac{1}{2}\right]^2 + 4(1)\text{-}(2)^2\right)$

= (2 . (4/3))[(1/4) + 4 - 4] = (8/3) . (1/4)
= 2/3

11. $cosec^3 30^\circ\ cos60^\circ\ tan^3 45^\circ\ sin^2 90^\circ\ sec^2 45^\circ\ cot30^\circ$
Solution:

$cosec^3 30^\circ cos60^\circ tan^3 45^\circ sin^2 90^\circ sec^2 45^\circ cot30^\circ$

Using trigonometric values, we have

$= (2)^3 \times \left(\frac{1}{2}\right) \times \left(1^3\right) \times \left(1^2\right) \times \left(\sqrt{2}^2\right) \times \left(\sqrt{3}\right)$

$= 8 \times \left(\frac{1}{2}\right) \times (1) \times (1) \times (2) \times \left(\sqrt{3}\right)$

$= 8\sqrt{3}$

12. $cot^2 30^\circ - 2cos^2 60^\circ - (3/4)sec^2 45^\circ - 4sec^2 30^\circ$
Solution:

Using trigonometric values, we have

$cot^2 30° - 2cos^2 60° - \frac{3}{4}sec^2 45° - 4sec^2 30°$

$= (\sqrt{3}^2) - 2(\frac{1}{2})^2 - (\frac{3}{4} \times \sqrt{2}^2) - (4 \times (\frac{2}{\sqrt{3}})^2)$

$= 3 - \frac{1}{2} - \frac{3}{2} - \frac{16}{3}$

$= \frac{-13}{3}$

13. (cos0° + sin45° + sin30°)(sin90° − cos45° + cos60°)
Solution:

(cos0° + sin45° + sin30°)(sin90° − cos45° + cos60°)

Using trigonometric values, we have

$= \left(1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\left(1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$

$= \left(\frac{3}{2} + \frac{1}{\sqrt{2}}\right)\left(\frac{3}{2} - \frac{1}{\sqrt{2}}\right)$

$= \left(\left(\frac{3}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2\right)$

$= \frac{9}{4} - \frac{1}{2}$

$= \frac{7}{4}$

14. $\frac{\sin 30^\circ - \sin 90^\circ + 2\cos 0^\circ}{\tan 30^\circ \tan 60^\circ}$

Solution:

Given,

$$\frac{\sin 30^\circ - \sin 90^\circ + 2\cos 0^\circ}{\tan 30^\circ \tan 60^\circ}$$

Using trigonometric values, we have

$$= \frac{\frac{1}{2} - 1 + 2}{\frac{1}{\sqrt{3}} \times \sqrt{3}}$$

$$= \frac{3}{2}$$

15. $4/\cot^2 30^\circ + 1/\sin^2 60^\circ - \cos^2 45^\circ$

Solution:

$$\frac{4}{\cot^2 30^\circ} + \frac{1}{\sin^2 60^\circ} - \cos^2 45^\circ$$

$$= \frac{4}{(\sqrt{3})^2} + \frac{1}{(\frac{\sqrt{3}}{2})^2} - (\frac{1}{\sqrt{2}})^2$$

$$= \frac{4}{3} + \frac{4}{3} - \frac{1}{2}$$

$$= \frac{16-3}{6}$$

$$= \frac{13}{6}$$

16. $4(\sin^4 30^\circ + \cos^2 60^\circ) - 3(\cos^2 45^\circ - \sin^2 90^\circ) - \sin^2 60^\circ$

Solution:

Using trigonometric values, we have

$$4(sin^4 30° + cos^2 60°) - 3(cos^2 45° - sin^2 90°) - sin^2 60°$$
$$= 4((\tfrac{1}{2})^4 + (\tfrac{1}{2})^2) - 3((\tfrac{1}{\sqrt{2}})^2 - 1) - (\tfrac{\sqrt{3}}{2})^2$$
$$= 4(\tfrac{1}{16} + \tfrac{1}{4}) + \tfrac{3}{2} - \tfrac{3}{4}$$
$$= \tfrac{8}{4} = 2$$

17. $\frac{tan^2 60° + 4cos^2 45° + 3sec^2 30° + 5cos^2 90°}{cosec 30° + sec 60° - cot^2 30°}$

Solution:

Using trigonometric values, we have

$$\frac{tan^2 60° + 4cos^2 45° + 3sec^2 30° + 5cos^2 90°}{cosec 30° + sec 60° - cot^2 30°}$$
$$= \frac{(\sqrt{3})^2 + 4(\frac{1}{\sqrt{2}})^2 + 3(\frac{2}{\sqrt{3}})^2 + 5(0)}{2 + 2 - (\sqrt{3})^2}$$
$$= 3 + 2 + 4$$
$$= 9$$

18. $\frac{\sin 30°}{\sin 45°} + \frac{\tan 45°}{\sec 60°} - \frac{\sin 60°}{\cot 45°} - \frac{\cos 30°}{\sin 90°}$

Solution:

Using trigonometric values, we have

$$\frac{\sin 30°}{\sin 45°} + \frac{\tan 45°}{\sec 60°} - \frac{\sin 60°}{\cot 45°} - \frac{\cos 30°}{\sin 90°}$$
$$= \frac{\frac{1}{2}}{\frac{1}{\sqrt{2}}} + \frac{1}{2} - \frac{\frac{\sqrt{3}}{2}}{1} - \frac{\frac{\sqrt{3}}{2}}{1}$$
$$= \frac{\sqrt{2}}{2} + \frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}$$
$$= \frac{\sqrt{2} + 1 - 2\sqrt{3}}{2}$$

19. $\frac{\tan 45°}{\text{cosec } 30°} + \frac{\sec 60°}{\cot 45°} - \frac{5\sin 90°}{2\cos 0°}$

Solution:

Using trigonometric values, we have

$\frac{\tan 45°}{\text{cosec } 30°} + \frac{\sec 60°}{\cot 45°} - \frac{5\sin 90°}{2\cos 0°}$

$= \frac{1}{2} + \frac{2}{1} - \frac{5(1)}{2(1)}$

$= \frac{5}{2} - \frac{5}{2}$

$= 0$

Find the value of x in each of the following: (20-25)
20. 2sin 3x = √3
Solution:

Given,

2 sin 3x = √3
sin 3x = √3/2
sin 3x = sin 60°
3x = 60°
x = 20°

Exercise 5.3

1. Evalute the following:
(i) sin 20º/ cos 70º
(ii) cos 19º/ sin 71º
(iii) sin 21º/ cos 69º
(iv) tan 10º/ cot 80º
(v) sec 11º/ cosec 79º
Solution:

(i) We have,
sin 20º/ cos 70º = sin (90º – 70º)/ cos 70º = cos 70º/ cos70º = 1 [∵ sin (90 - θ) = cos θ]

(ii) We have,
cos 19º/ sin 71º = cos (90º – 71º)/ sin 71º = sin 71º/ sin 71º = 1 [∵ cos (90 - θ) = sin θ]

(iii) We have,
sin 21º/ cos 69º = sin (90º – 69º)/ cos 69º = cos 69º/ cos69º = 1 [∵ sin (90 - θ) = cos θ]

(iv) We have,
tan 10º/ cot 80º = tan (90º – 10º) / cot 80º = cot 80º/ cos80º = 1 [∵ tan (90 - θ) = cot θ]

(v) We have,
sec 11º/ cosec 79º = sec (90º – 79º)/ cosec 79º = cosec 79º/ cosec 79º = 1
[∵ sec (90 - θ) = cosec θ]

2. Evaluate the following:

(i) $\left(\frac{\sin 49°}{\cos 41°}\right)^2 + \left(\frac{\cos 41°}{\sin 49°}\right)^2$

Solution:

We have, [∵ sin (90 - θ) = cos θ and cos (90 - θ) = sin θ]

$$\left(\frac{\sin 49°}{\cos 41°}\right)^2 + \left(\frac{\cos 41°}{\sin 49°}\right)^2$$

$$= \left(\frac{\sin(90° - 41°)}{\cos 41°}\right)^2 + \left(\frac{\cos(90° - 49°)}{\sin 49°}\right)^2$$

$$= \left(\frac{\cos 41°}{\cos 41°}\right)^2 + \left(\frac{\sin 49°}{\sin 49°}\right)^2$$

$= 1^2 + 1^2 = 1 + 1$
$= 2$

(ii) cos 48°- sin 42°
Solution:

We know that, cos (90° − θ) = sin θ.
So,
cos 48° - sin 42° = cos (90° − 42°) – sin 42° = sin 42° - sin 42°= 0
Thus the value of cos 48° - sin 42° is 0.

(iii) $\frac{\cot 40^\circ}{\tan 50^\circ} - \frac{1}{2}\left(\frac{\cos 35^\circ}{\sin 55^\circ}\right)$

Solution:
We have, [∵ cot (90 - θ) = tan θ and cos (90 - θ) = sin θ]

$$\frac{\cot 40^\circ}{\tan 50^\circ} - \frac{1}{2}\left(\frac{\cos 35^\circ}{\sin 55^\circ}\right)$$

$$= \frac{\cot(90^\circ - 50^\circ)}{\tan 50^\circ} - \frac{1}{2}\left(\frac{\cos(90^\circ - 55^\circ)}{\sin 55^\circ}\right)$$

$$= \frac{\tan 50^\circ}{\tan 50^\circ} - \frac{1}{2}\left(\frac{\sin 55^\circ}{\sin 55^\circ}\right)$$

= 1 – 1/2(1)
= 1/2

(iv) $\left(\frac{\sin 27^\circ}{\cos 63^\circ}\right)^2 - \left(\frac{\cos 63^\circ}{\sin 27^\circ}\right)^2$

Solution:
We have, [∵ sin (90 - θ) = cos θ and cos (90 - θ) = sin θ]

$$\left(\frac{\sin 27^\circ}{\cos 63^\circ}\right)^2 - \left(\frac{\cos 63^\circ}{\sin 27^\circ}\right)^2$$

$$= \left(\frac{\sin(90^\circ - 63^\circ)}{\cos 63^\circ}\right)^2 - \left(\frac{\cos(90^\circ - 27^\circ)}{\sin 27^\circ}\right)^2$$

$$= \left(\frac{\cos 63^\circ}{\cos 63^\circ}\right)^2 - \left(\frac{\sin 27^\circ}{\sin 27^\circ}\right)^2$$

= 1 – 1
= 0

(v) $\frac{\tan 35°}{\cot 55°} + \frac{\cot 78°}{\tan 12°} - 1$

Solution:

We have, [∵ cot (90 - θ) = tan θ and tan (90 - θ) = cot θ]

$$\frac{\tan 35°}{\cot 55°} + \frac{\cot 78°}{\tan 12°} - 1$$

= tan (90° – 35°)/ cot 55° + cot (90° – 12°)/ tan 12° – 1
= cot 55°/ cot 55° + tan 12°/ tan 12° – 1
= 1 + 1 – 1
= 1

(vi) $\frac{\sec 70°}{\text{cosec } 20°} + \frac{\sin 59°}{\cos 31°}$

Solution:

We have , [∵ sin (90 - θ) = cos θ and sec (90 - θ) = cosec θ]

$$\frac{\sec 70°}{\text{cosec } 20°} + \frac{\sin 59°}{\cos 31°}$$

= sec (90° – 20°)/ cosec 20° + sin (90° – 31°)/ cos 31°
= cosec 20°/ cosec 20° + cos 12°/ cos 12°
= 1 + 1
= 2

(vii) cosec 31° - sec 59°
Solution:

We have,
cosec 31° - sec 59°
Since, cosec (90 - θ) = cos θ
So,
cosec 31° - sec 59° = cosec (90° - 59°) – sec 59° = sec 59° - sec 59° = 0
Thus,
cosec 31° - sec 59° = 0

(viii) (sin 72° + cos 18°) (sin 72° - cos 18°)
Solution:

We know that,
sin (90 - θ) = cos θ

So, the given can be expressed as

(sin 72° + cos 18°) (sin (90 - 18)° - cos 18°)
= (sin 72° + cos 18°) (cos 18° - cos 18°)
= (sin 72° + cos 18°) x 0
= 0

(ix) sin 35° sin 55° - cos 35° cos 55°
Solution:

We know that,
sin (90 - θ) = cos θ

So, the given can be expressed as
sin (90 - 55)° sin (90 - 35)° - cos 35° cos 55°
= cos 55° cos 35° - cos 35° cos 55°
= 0

(x) tan 48° tan 23° tan 42° tan 67°
Solution:

We know that,
tan (90 - θ) = cot θ
So, the given can be expressed as
tan (90 - 42)° tan (90 - 67)° tan 42° tan 67°
= cot 42° cot 67° tan 42° tan 67°
= (cot 42° tan 42°)(cot 67° tan 67°)
= 1 x 1 [∵ tan θ x cot θ = 1]
= 1

(xi) sec 50° sin 40° + cos 40° cosec 50°
Solution:

We know that,
sin (90 - θ) = cos θ and cos (90 - θ) = sin θ
So, the given can be expressed as
sec 50° sin (90 - 50)° + cos (90 - 50)° cosec 50°
= sec 50° cos 50° + sin 50° cosec 50°
= 1 + 1 [∵ sin θ x cosec θ = 1 and cos θ x sec θ = 1]
= 2

3. Express each one of the following in terms of trigonometric ratios of angles lying between 0º and 45º
(i) sin 59º + cos 56º **(ii) tan 65º + cot 49º** **(iii) sec 76º + cosec 52º**
(iv) cos 78º + sec 78º **(v) cosec 54º + sin 72º** **(vi) cot 85º + cos 75º**
(vii) sin 67º + cos 75º
Solution:

Using the below trigonometric ratios of complementary angles, we find the required

$\sin (90 - \theta) = \cos \theta$ $\quad$ $\text{cosec} (90 - \theta) = \sec \theta$

$\cos (90 - \theta) = \sin \theta$ $\quad$ $\sec (90 - \theta) = \text{cosec}\, \theta$

$\tan (90 - \theta) = \cot \theta$ $\quad$ $\cot (90 - \theta) = \tan \theta$

(i) $\sin 59^o + \cos 56^o = \sin (90 - 31)^o + \cos (90 - 34)^o = \cos 31^o + \sin 34^o$

(ii) $\tan 65^o + \cot 49^o = \tan (90 - 25)^o + \cot (90 -41)^o = \cot 25^o + \tan 41^o$

(iii) $\sec 76^o + \text{cosec}\, 52^o = \sec (90 - 14)^o + \text{cosec} (90 - 38)^o = \text{cosec}\, 14^o + \sec 38^o$

(iv) $\cos 78^o + \sec 78^o = \cos (90 - 12)^o + \sec (90 - 12)^o = \sin 12^o + \text{cosec}\, 12^o$

(v) $\text{cosec}\, 54^o + \sin 72^o = \text{cosec} (90 - 36)^o + \sin (90 - 18)^o = \sec 36^o + \cos 18^o$

(vi) $\cot 85^o + \cos 75^o = \cot (90 - 5)^o + \cos (90 - 15)^o = \tan 5^o + \sin 15^o$

4. Express cos 75º + cot 75º in terms of angles between 0º and 30º.

Solution:

Given,

$\cos 75^o + \cot 75^o$

Since, $\cos (90 - \theta) = \sin \theta$ and $\cot (90 - \theta) = \tan \theta$

$\cos 75^o + \cot 75^o = \cos (90 - 15)^o + \cot (90 - 15)^o = \sin 15^o + \tan 15^o$

Hence, $\cos 75^o + \cot 75^o$ can be expressed as $\sin 15^o + \tan 15^o$

5. If sin 3A = cos (A – 26º), where 3A is an acute angle, find the value of A.

Solution:

Given,

$\sin 3A = \cos (A – 26^o)$

Using $\cos (90 - \theta) = \sin \theta$, we have

$\sin 3A = \sin (90^o - (A – 26^o))$

Now, comparing both L.H.S and R.H.S

$3A = 90^o - (A – 26^o)$

$3A + (A – 26^o) = 90^o$

$4A – 26^o = 90^o$

$4A = 116^o$

$A = 116^o/4$

$\therefore A = 29^o$

6. If A, B, C are the interior angles of a triangle ABC, prove that

(i) tan ((C + A)/ 2) = cot (B/2) (ii) sin ((B + C)/ 2) = cos (A/2)

Solution:

We know that, in triangle ABC the sum of the angles i.e $A + B + C = 180^o$

So, $C + A = 180^o – B$ $\Rightarrow (C + A)/2 = 90^o – B/2$ …… (i)

And, $B + C = 180^o – A$ $\Rightarrow (B + C)/2 = 90^o – A/2$ ……. (ii)

(i) L.H.S = $\tan ((C + A)/ 2)$

$\Rightarrow \tan ((C + A)/ 2) = \tan (90^o – B/2)$ [From (i)]

$= \cot (B/2)$ [$\because \tan (90 - \theta) = \cot \theta$]

= R.H.S

- Hence Proved

(ii) L.H.S = sin ((B + C)/2)

⇒ sin ((B + C)/ 2) = sin (90º – A/2) [From (ii)]

= cos (A/2)

= R.H.S

- Hence Proved

7. Prove that:

(i) tan 20° tan 35° tan 45° tan 55° tan 70° = 1

(ii) sin 48° sec 48° + cos 48° cosec 42° = 2

(iii) $\frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - 2\cos 70^\circ \quad \operatorname{cosec} 42^\circ = 0$

(iv) $\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \quad \operatorname{cosec} 31^\circ = 2$

Solution:

(i) Taking L.H.S = tan 20° tan 35° tan 45° tan 55° tan 70°

= tan (90° − 70°) tan (90° − 55°) tan 45°tan 55° tan70°

= cot 70°cot 55° tan 45° tan 55° tan 70° [∵ tan (90 - θ) = cot θ]

= (tan 70°cot 70°)(tan 55°cot 55°) tan 45° [∵ tan θ x cot θ = 1]

= 1 × 1 × 1 = 1

- Hence proved

(ii) Taking L.H.S = sin 48° sec 48° + cos 48° cosec 42°

= sin 48° sec (90° − 48°) + cos 48° cosec (90° − 48°)

[∵sec (90 - θ) = cosec θ and cosec (90 - θ) = sec θ]

= sin 48°cosec 48° + cos 48°sec 48° [∵ cosec θ x sin θ = 1 and cos θ x sec θ = 1]

= 1 + 1 = 2

- Hence proved

(iii) Taking the L.H.S,

$$\frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\text{cosec } 20^\circ}{\sec 70^\circ} - 2\cos 70^\circ \text{ cosec } 20^\circ$$

$$= \frac{\sin(90^\circ - 20^\circ)}{\cos 20^\circ} + \frac{\text{cosec}(90^\circ - 70^\circ)}{\sec 70^\circ} - 2\cos(90^\circ - 20^\circ) \text{ cosec } 20^\circ$$

$$= \frac{\cos 20^\circ}{\cos 20^\circ} + \frac{\sec 70^\circ}{\sec 70^\circ} - 2\sin 20^\circ \times \frac{1}{\sin 20^\circ} \quad \begin{bmatrix} \sin(90^\circ - \theta) = \cos\theta \\ \text{cosec}(90^\circ - \theta) = \sec\theta \\ \cos(90^\circ - \theta) = \sin\theta \end{bmatrix}$$

= 1 + 1 – 2
= 2 – 2
= 0

- Hence proved

(iv) Taking L.H.S,

$$\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \text{ cosec } 31^\circ$$

$$= \frac{\cos(90^\circ - 10^\circ)}{\sin 10^\circ} + \cos 59^\circ \text{ cosec}(90^\circ - 59^\circ)$$

$$= \frac{\sin 10^\circ}{\sin 10^\circ} + \cos 59^\circ \sec 59^\circ$$

= 1 + 1
= 2

- Hence proved

8. Prove the following:
(i) sinθ sin (90º - θ) – cos θ cos (90º - θ) = 0
Solution:

Taking the L.H.S,
sinθ sin (90º - θ) – cos θ cos (90º - θ)
= sin θ cos θ - cos θ sin θ [∵ sin (90 - θ) = cos θ and cos (90 - θ) = sin θ]
= 0

(ii)

$$\frac{\cos(90^\circ - \theta)\sec(90^\circ - \theta)\tan\theta}{\text{cosec}(90^\circ - \theta)\sin(90^\circ - \theta)\cot(90^\circ - \theta)} + \frac{\tan(90^\circ - \theta)}{\cot\theta} = 2$$

Solution:

Taking the L.H.S,

$$\frac{\cos(90^\circ-\theta)\sec(90^\circ-\theta)\tan\theta}{\operatorname{cosec}(90^\circ-\theta)\sin(90^\circ-\theta)\cot(90^\circ-\theta)}+\frac{\tan(90^\circ-\theta)}{\cot\theta}$$

$$=\frac{\sin\theta\operatorname{cosec}\theta\tan\theta}{\sec\theta\cos\theta\tan\theta}+\frac{\cot\theta}{\cot\theta}$$

$$=\frac{1\times\tan\theta}{1\times\tan\theta}+1=\frac{\tan\theta}{\tan\theta}+1$$

[∵ cosec θ x sin θ = 1 and cos θ x sec θ = 1]

= 1 + 1

= 2 = R.H.S

- Hence Proved

(iii) $\frac{\tan(90^\circ-A)\cot A}{\operatorname{cosec}^2 A}-\cos^2 A=0$

Solution:

Taking the L.H.S, [∵ tan (90º - θ) = cot θ]

$$\text{L.H.S.}=\frac{\tan(90^\circ-A)\cot A}{\operatorname{cosec}^2 A}-\cos^2 A$$

$$=\frac{\cot A\cot A}{\operatorname{cosec}^2 A}-\cos^2 A$$

$$=\frac{\cot^2 A}{\operatorname{cosec}^2 A}-\cos^2 A=\frac{\frac{\cos^2 A}{\sin^2\theta}}{\frac{1}{\sin^2 A}}-\cos^2 A$$

$$=\frac{\cos^2 A\times\sin^2 A}{\sin^2 A\times 1}-\cos^2 A=\cos^2 A-\cos^2 A$$

= 0 = R.H.S

- Hence Proved

(iv) $\dfrac{\cos(90^\circ - A)\sin(90^\circ - A)}{\tan(90^\circ - A)} = \sin^2 A$

Solution:

Taking L.H.S, [∵ sin (90 - θ) = cos θ and cos (90 - θ) = sin θ]

$$\text{L.H.S.} = \frac{\cos(90^\circ - A)\sin(90^\circ - A)}{\tan(90^\circ - A)}$$

$$= \frac{\sin A \cos A}{\cot A} = \frac{\sin A \cos A}{\frac{\cos A}{\sin A}}$$

$$= \frac{\sin A \cos A \times \sin A}{\cos A} = \sin A \times \sin A$$

= $\sin^2 A$ = R.H.S

- Hence Proved

(v) sin (50º + θ) - cos (40º - θ) + tan 1º tan 10º tan 70º tan 80º tan 89º = 1
Solution:

Taking the L.H.S,
= sin (50º + θ) - cos (40º - θ) + tan 1º tan 10º tan 20º tan 70º tan 80º tan 89º
= [sin (90º - (40º - θ)] - cos (40º – (40º - θ)) + tan (90 - 89)º tan (90 - 80)º tan (90 - 70)º tan 70º tan 80º tan 89º ∵ sin (90 - θ)[= cos θ]
= cos (40º - θ) - cos (40º - θ) + cot 89º cot 80º cot 70º tan 70º tan 80º tan 89º
[∵ tan (90º - θ) = cot θ]
= 0 + (cot 89º x tan 89º) (cot 80º x tan 80º) (cot 70º x tan 70º)
= 0 + 1 x 1 x 1 [∵ tan θ x cot θ = 1]
= 1= R.H.S

- Hence Proved