Exercise 8.1

1. Which of the following are quadratic equations?

(i) $x^2 + 6x - 4 = 0$

Solution:

Let $p(x) = x^2 + 6x - 4$,

It's clearly seen that $p(x) = x^2 + 6x - 4$ is a quadratic polynomial. Thus, the given equation is a quadratic equation.

(ii) $\sqrt{3}x^2 - 2x + 1/2 = 0$

Solution:

Let $p(x) = \sqrt{3}x^2 - 2x + 1/2$,

It's clearly seen that $p(x) = \sqrt{3}x^2 - 2x + 1/2$ having real coefficients is a quadratic polynomial. Thus, the given equation is a quadratic equation.

(iii) $x^2 + 1/x^2 = 5$

Solution:

Given,

$x^2 + 1/x^2 = 5$

On multiplying by x^2 on both sides we have,

$x^4 + 1 = 5x^2$

$\Rightarrow$ $x^4 - 5x^2 + 1 = 0$

It's clearly seen that $x^4 - 5x^2 + 1$ is not a quadratic polynomial as its degree is 4. Thus, the given equation is not a quadratic equation.

(iv) $x - 3/x = x^2$

Solution:

Given,

$x - 3/x = x^2$

On multiplying by x on both sides we have,

$x^2 - 3 = x^3$

$\Rightarrow$ $x^3 - x^2 + 3 = 0$

It's clearly seen that $x^3 - x^2 + 3$ is not a quadratic polynomial as its degree is 3. Thus, the given equation is not a quadratic equation.

(v) $2x^2 - \sqrt{(3x)} + 9 = 0$

Solution:

It's clearly seen that $2x^2 - \sqrt{(3x)} + 9$ is not a polynomial because it contains a term involving $x^{1/2}$, where 1/2 is not an integer. Thus, the given equation is not a quadratic equation.

(vi) $x^2 - 2x - \sqrt{x} - 5 = 0$
Solution:

It's clearly seen that $x^2 - 2x - \sqrt{x} - 5$ is not a polynomial because it contains a term involving $x^{1/2}$, where 1/2 is not an integer. Thus, the given equation is not a quadratic equation.

(vii) $3x^2 - 5x + 9 = x^2 - 7x + 3$
Solution:

Given,
$3x^2 - 5x + 9 = x^2 - 7x + 3$
On simplifying the equation, we have
$2x^2 + 2x + 6 = 0$
$\Rightarrow$ $x^2 + x + 3 = 0$ (dividing by 2 on both sides)
Now, it's clearly seen that $x^2 + x + 3$ is a quadratic polynomial. Thus, the given equation is a quadratic equation.

(viii) $x + 1/x = 1$
Solution:

Given,
$x + 1/x = 1$
On multiplying by x on both sides we have,
$x^2 + 1 = x$
$\Rightarrow$ $x^2 - x + 1 = 0$
It's clearly seen that $x^2 - x + 1$ is a quadratic polynomial. Thus, the given equation is a quadratic equation.

(ix) $x^2 - 3x = 0$
Solution:

Let $p(x) = x^2 - 3x$,
It's clearly seen that $p(x) = x^2 - 3x$ is a quadratic polynomial. Thus, the given equation is a quadratic equation.

(x) $(x + 1/x)^2 = 3(x + 1/x) + 4$
Solution:

Given,
$(x + 1/x)^2 = 3(x + 1/x) + 4$
$\Rightarrow$ $x^2 + 1/x^2 + 2 = 3x + 3/x + 4$
$\Rightarrow$ $x^4 + 1 + 2x^2 = 3x^3 + 3x + 4x^2$
$\Rightarrow$ $x^4 - 3x^3 - 2x^2 - 3x + 1 = 0$
Now, it's clearly seen that $x^4 - 3x^3 - 2x^2 - 3x + 1$ is not a quadratic polynomial since its degree is 4. Thus, the given equation is not a quadratic equation.

(xi) $(2x + 1)(3x + 2) = 6(x – 1)(x – 2)$
Solution:

Given,
$(2x + 1)(3x + 2) = 6(x – 1)(x – 2)$
$\Rightarrow$ $6x^2 + 4x + 3x + 2 = 6x^2 - 12x - 6x + 12$
$\Rightarrow$ $7x + 2 = -18x + 12$
$\Rightarrow$ $25x – 10 = 0$
Now, it's clearly seen that $25x – 10$ is not a quadratic polynomial since its degree is 1. Thus, the given equation is not a quadratic equation.

(xii) $x + 1/x = x^2, x \neq 0$
Solution:

Given,
$x + 1/x = x^2$
On multiplying by x on both sides we have,
$x^2 + 1 = x^3$
$\Rightarrow$ $x^3 – x^2 – 1 = 0$
Now, it's clearly seen that $x^3 – x^2 – 1$ is not a quadratic polynomial since its degree is 3. Thus, the given equation is not a quadratic equation.

(xiii) $16x^2 – 3 = (2x + 5)(5x - 3)$
Solution:

Given,
$16x^2 – 3 = (2x + 5)(5x - 3)$
$16x^2 – 3 = 10x^2 – 6x + 25x – 15$
$\Rightarrow$ $6x^2 – 19x + 12 = 0$
Now, it's clearly seen that $6x^2 – 19x + 12$ is a quadratic polynomial. Thus, the given equation is a quadratic equation.

(xiv) $(x + 2)^3 = x^3 – 4$
Solution:

Given,
$(x + 2)^3 = x^3 – 4$
On expanding, we get
$\Rightarrow$ $x^3 + 6x^2 + 8x + 8 = x^3 – 4$
$\Rightarrow$ $6x^2 + 8x + 12 = 0$
Now, it's clearly seen that $6x^2 + 8x + 12$ is a quadratic polynomial. Thus, the given equation is a quadratic equation.

(xv) $x(x + 1) + 8 = (x + 2)(x - 2)$
Solution:

Given,

$x(x + 1) + 8 = (x + 2)(x - 2)$

$x^2 + x + 8 = x^2 - 4$

$\Rightarrow$ $x + 12 = 0$

Now, it's clearly seen that $x + 12$ is a not quadratic polynomial since its degree is 1. Thus, the given equation is not a quadratic equation.

2. In each of the following, determine whether the given values are solutions of the given equation or not:

(i) $x^2 - 3x + 2 = 0$, $x = 2$, $x = - 1$

Solution:

Here we have,

LHS = $x^2 - 3x + 2$

Substituting $x = 2$ in LHS, we get

$(2)^2 - 3(2) + 2$

$\Rightarrow$ $4 - 6 + 2 = 0$ = RHS

$\Rightarrow$ LHS = RHS

Thus, $x = 2$ is a solution of the given equation.

Similarly,

Substituting $x = -1$ in LHS, we get

$(-1)^2 - 3(-1) + 2$

$\Rightarrow$ $1 + 3 + 2 = 6 \neq$ RHS

$\Rightarrow$ LHS $\neq$ RHS

Thus, $x = -1$ is not a solution of the given equation.

(ii) $x^2 + x + 1 = 0$, $x = 0$, $x = 1$

Solution:

Here we have,

LHS = $x^2 + x + 1$

Substituting $x = 0$ in LHS, we get

$(0)^2 + 0 + 1$

$\Rightarrow$ $1 \neq$ RHS

$\Rightarrow$ LHS $\neq$ RHS

Thus, $x = 0$ is not a solution of the given equation.

Similarly,

Substituting $x = 1$ in LHS, we get

$(1)^2 + 1 + 1$

$\Rightarrow$ $3 \neq$ RHS

$\Rightarrow$ LHS $\neq$ RHS

Thus, $x = 1$ is not a solution of the given equation.

(iii) $x^2 - 3\sqrt{3}x + 6 = 0$, $x = \sqrt{3}$ and $x = -2\sqrt{3}$
Solution:

Here we have,
LHS = $x^2 - 3\sqrt{3}x + 6$
Substituting $x = \sqrt{3}$ in LHS, we get
$(\sqrt{3})^2 - 3\sqrt{3}(\sqrt{3}) + 6$
⇒ $3 - 9 + 6 = 0$ = RHS
⇒ LHS = RHS
Thus, $x = \sqrt{3}$ is a solution of the given equation.

Similarly,
Substituting $x = -2\sqrt{3}$ in LHS, we get
$(-2\sqrt{3})^2 - 3\sqrt{3}(-2\sqrt{3}) + 6$
⇒ $12 + 18 + 6 = 36 \neq$ RHS
⇒ LHS ≠ RHS
Thus, $x = -2\sqrt{3}$ is not a solution of the given equation.

(iv) $x + 1/x = 13/6$, $x = 5/6$, $x = 4/3$
Solution:

Here we have,
LHS = $x + 1/x$
Substituting $x = 5/6$ in LHS, we get
$(5/6) + 1/(5/6) = 5/6 + 6/5$
⇒ $61/30 \neq$ RHS
⇒ LHS ≠ RHS
Thus, $x = 5/6$ is not a solution of the given equation.
Similarly,
Substituting $x = 4/3$ in LHS, we get
$(4/3) + 1/(4/3) = 4/3 + 3/4$
⇒ $25/12 \neq$ RHS
⇒ LHS ≠ RHS
Thus, $x = 4/3$ is not a solution of the given equation.

(v) $2x^2 - x + 9 = x^2 + 4x + 3$, $x = 2$, $x = 3$
Solution:

Here we have,
$2x^2 - x + 9 = x^2 + 4x + 3$
⇒ $x^2 - 5x + 6 = 0$

LHS = $x^2 - 5x + 6$
Substituting $x = 2$ in LHS, we get
$(2)^2 - 5(2) + 6$

⇒ $4 - 10 + 6 = 0 =$ RHS

⇒ LHS = RHS

Thus, $x = 2$ is a solution of the given equation.

Similarly,

Substituting $x = 3$ in LHS, we get

$(3)^2 - 5(3) + 6$

⇒ $9 - 15 + 6 = 0 =$ RHS

⇒ LHS = RHS

Thus, $x = 3$ is a solution of the given equation.

(vi) $x^2 - \sqrt{2}x - 4 = 0, x = -\sqrt{2}, x = -2\sqrt{2}$

Solution:

Here we have,

LHS $= x^2 - \sqrt{2}x - 4$

Substituting $x = -\sqrt{2}$ in LHS, we get

$(-\sqrt{2})^2 - \sqrt{2}(-\sqrt{2}) - 4$

⇒ $4 + 2 - 4 = 2 \neq$ RHS

⇒ LHS ≠ RHS

Thus, $x = -\sqrt{2}$ is a solution of the given equation.

Similarly,

Substituting $x = -2\sqrt{2}$ in LHS, we get

$(-2\sqrt{2})^2 - \sqrt{2}(-2\sqrt{2}) - 4$

⇒ $8 + 4 - 4 = 8 \neq$ RHS

⇒ LHS ≠ RHS

Thus, $x = -2\sqrt{2}$ is not a solution of the given equation.

(vii) $a^2x^2 - 3abx + 2b^2 = 0, x = a/b, x = b/a$

Solution:

We have,

LHS $= a^2x^2 - 3abx + 2b^2$ and RHS $= 0$

Substituting the $x = \frac{a}{b}$ and $x = \frac{b}{a}$ in

$$\text{LHS} = a^2\left(\frac{a}{b}\right)^2 - 3ab\left(\frac{a}{b}\right) + 2b^2$$

$$= \frac{a^4}{b^2} - 3a^2 + 2b^2$$

≠ RHS

And, for x = b/a

$$\text{LHS} = a^2\left(\frac{b}{a}\right)^2 - 3ab\left(\frac{b}{a}\right) + 2b^2$$

$$= b^2 - 3b^2 + 2b^2 = 0$$

= RHS

Therefore, x = b/a is a solution of the given equation.