Exercise 5(B) Page No: 56

1. Without solving, comment upon the nature of roots of each of the following equations:

(i) $7x^2 - 9x + 2 = 0$ **(ii) $6x^2 - 13x + 4 = 0$**

(iii) $25x^2 - 10x + 1 = 0$ **(iv) $x^2 + 2\sqrt{3}x - 9 = 0$**

(v) $x^2 - ax - b^2 = 0$ **(vi) $2x^2 + 8x + 9 = 0$**

Solution:

(i) Given quadratic equation, $7x^2 - 9x + 2 = 0$

Here, a = 7, b = -9 and c = 2

So, the Discriminant (D) = $b^2 - 4ac$

$D = (-9)^2 - 4(7)(2)$

$= 81 - 56$

$= 25$

As D > 0, the roots of the equation is real and unequal.

(ii) Given quadratic equation, $6x^2 - 13x + 4 = 0$

Here, a = 6, b = -13 and c = 4

So, the Discriminant (D) = $b^2 - 4ac$

$D = (-13)^2 - 4(6)(4)$

$= 169 - 96$

$= 73$

As D > 0, the roots of the equation is real and unequal.

(iii) Given quadratic equation, $25x^2 - 10x + 1 = 0$

Here, a = 25, b = -10 and c = 1

So, the Discriminant (D) = $b^2 - 4ac$

$D = (-10)^2 - 4(25)(1)$

$= 100 - 100$

$= 0$

As D = 0, the roots of the equation is real and equal.

(iv) Given quadratic equation, $x^2 + 2\sqrt{3}x - 9 = 0$

Here, a = 1, b = $2\sqrt{3}$ and c = -9

So, the Discriminant (D) = $b^2 - 4ac$

$D = (2\sqrt{3})^2 - 4(1)(-9)$

$= 12 + 36$

$= 48$

As D > 0, the roots of the equation is real and unequal.

(v) Given quadratic equation, $x^2 - ax - b^2 = 0$

Here, a = 1, b = -a and c = $-b^2$

So, the Discriminant (D) = $b^2 - 4ac$

$D = (a)^2 - 4(1)(-b^2)$

$= a^2 + 4b^2$

$a^2 + 4b^2$ is always positive value.

Thus D > 0, and the roots of the equation is real and unequal

(vi) Given quadratic equation, $2x^2 + 8x + 9 = 0$
Here, a = 2, b = 8 and c = 9
So, the Discriminant (D) = $b^2 – 4ac$
$D = (8)^2 – 4(2)(9)$
$= 64 – 72$
$= -8$
As D < 0, the equation has no roots.

2. Find the value of 'p', if the following quadratic equations has equal roots:
(i) $4x^2 - (p - 2)x + 1 = 0$
(ii) $x^2 + (p - 3)x + p = 0$
Solution:

(i) $4x^2 - (p - 2)x + 1 = 0$
Here, a = 4, b = -(p - 2), c = 1
Given that the roots are equal,
So, Discriminant = $0 \Rightarrow b^2 - 4ac = 0$
$D = (-(p - 2))^2 - 4(4)(1) = 0$
$\Rightarrow p^2 + 4 - 4p - 16 = 0$
$\Rightarrow p^2 - 4p - 12 = 0$
$\Rightarrow p^2 - 6p + 2p - 12 = 0$
$\Rightarrow p(p – 6) + 2(p – 6) = 0$
$\Rightarrow (p + 2)(p - 6) = 0$
$\Rightarrow p + 2 = 0$ or $p - 6 = 0$
Hence, p = -2 or p = 6

(ii) $x^2 + (p - 3)x + p = 0$
Here, a = 1, b = (p - 3), c = p
Given that the roots are equal,
So, Discriminant = $0 \Rightarrow b^2 - 4ac = 0$
$D = (p - 3)^2 - 4(1)(p) = 0$
$\Rightarrow p^2 + 9 - 6p - 4p = 0$
$\Rightarrow p^2 - 10p + 9 = 0$
$\Rightarrow p^2 - 9p - p + 9 = 0$
$\Rightarrow p(p - 9) - 1(p - 9) = 0$
$\Rightarrow (p -9)(p - 1) = 0$
$\Rightarrow p - 9 = 0$ or $p - 1 = 0$
Hence, p = 9 or p = 1