Exercise 6(D)   Page No: 78

1. The sum S of n successive odd numbers starting from 3 is given by the relation: S = n(n + 2). Determine n, if the sum is 168.
Solution:

From the question, we have
n(n + 2) = 168
n^2 + 2n - 168 = 0
n^2 + 14n - 12n - 168 = 0
n(n + 14) - 12(n + 14) = 0
(n + 14) (n - 12) = 0
n = -14, 12
Since, n cannot be negative.
Thus, n = 12

2. A stone is thrown vertically downwards and the formula d = $16t^2$ + 4t gives the distance, d metres, that it falls in t seconds. How long does it take to fall 420 metres?
Solution:

According to the question,
$16t^2$ + 4t = 420
$4t^2$ + t - 105 = 0
$4t^2$ - 20t + 21t - 105 = 0
4t(t - 5) + 21(t - 5) = 0
(4t + 21)(t - 5) = 0
t = -21/4 or 5
As, time cannot be negative.
Therefore, the required time taken by the stone to fall 420 meter is 5 seconds.

3. The product of the digits of a two digit number is 24. If its unit's digit exceeds twice its ten's digit by 2; find the number.
Solution:

Let's assume the ten's and unit's digit of the required number to be x and y respectively.
Then, from the question we have
x × y = 24
y = 24/x …. (1)
Also,
y = 2x + 2
Using (1) in the above equation,
24/x = 2x + 2
24 = $2x^2$ + 2x
$2x^2$ + 2x – 24 = 0
x^2 + x – 12 = 0   [Dividing by 2]
(x + 4) (x - 3) = 0

x = -4, 3
As the digit of a number cannot be negative, x = -4 is neglected
Thus, x = 3
So, y = 24/ 3 = 8
Therefore, the required number is 38.

4. The ages of two sisters are 11 years and 14 years. In how many years' time will the product of their ages be 304?
Solution:

Given, the ages of two sisters are 11 years and 14 years.
Let x be the number of years later when their product of their ages become 304.
So, (11 + x) (14 + x) = 304
$154 + 11x + 14x + x^2 = 304$
$x^2 + 25x - 150 = 0$
(x + 30) (x - 5) = 0
x = -30, 5
As, the number of years cannot be negative. We only consider, x = 5.
Therefore, the required number of years is 5 years.

5. One year ago, a man was 8 times as old as his son. Now his age is equal to the square of his son's age. Find their present ages.
Solution:

Let's consider the present age of the son to be x years.
So, the present age of the man = x^2 years
One year ago,
Son's age = (x - 1) years
Man's age = $(x^2 - 1)$ years
From the question, it's given that one year ago; the man was 8 times as old as his son.
$(x^2 - 1) = 8(x - 1)$
$x^2 - 8x - 1 + 8 = 0$
$x^2 - 8x + 7 = 0$
(x - 7) (x - 1) = 0
x = 7, 1
When x = 1, then x^2 = 1, which is not possible as father's age cannot be equal to son's age.
Hence, x = 7 is taken
Therefore,
The present age of son = x years = 7 years
The present age of man = x^2 years = 49 years

6. The age of the father is twice the square of the age of his son. Eight years hence, the age of the father will be 4 years more than three times the age of the son. Find their present ages.
Solution:

Let's assume the present age of the son to be x years.

So, the present age of the father $= 2x^2$ years
Eight years hence,
Son's age $= (x + 8)$ years
Father's age $= (2x^2 + 8)$ years
From the question, it's given that eight years hence, the age of the father will be 4 years more than three times the age of the son.
$2x^2 + 8 = 3(x + 8) + 4$
$2x^2 + 8 = 3x + 24 + 4$
$2x^2 - 3x - 20 = 0$
$2x^2 - 8x + 5x - 20 = 0$
$2x(x - 4) + 5(x - 4) = 0$
$(x - 4)(2x + 5) = 0$
$x = 4, -5/2$
As, age cannot be negative. $x = 4$ is considered.
Therefore,
The present age of the son $= 4$ years
The present age of the father $= 2(4)^2$ years $= 32$ years