BYJU'S
The Learning App
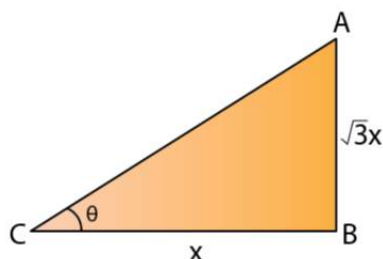
Exercise 22(A)   Page No: 336

1. The height of a tree is √3 times the length of its shadow. Find the angle of elevation of the sun.
Solution:

Let's assume the length of the shadow of the tree to be x m.
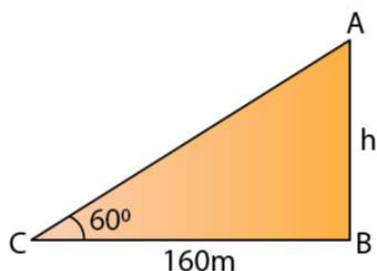So, the height of the tree = √3 x m
If θ is the angle of elevation of the sun, then we have
tan θ = √3 x/ x = √3 = tan 60º
Therefore, θ = 60º

2. The angle of elevation of the top of a tower from a point on the ground and at a distance of 160 m from its foot, is found to be 60º. Find the height of the tower.
Solution:

Let's take the height of the tower to be h m.
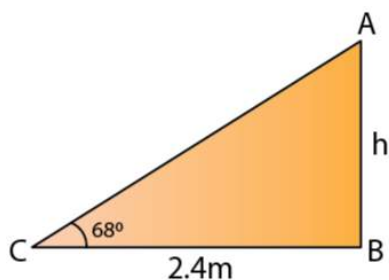Given that, the angle of elevation is 60º
So, tan 60º = h/160
√3 = h/160
h = 160√3 = 277.12 m   [For √3 = 1.732]
Thus, the height of the tower is 277.12 m.

3. A ladder is placed along a wall such that its upper end is resting against a vertical wall. The foot of the ladder is 2.4 m from the wall and the ladder is making an angle of 68º with the ground. Find the height, up to which the ladder reaches.
Solution:

Let's take the height upto which the ladder reaches as 'h' m.
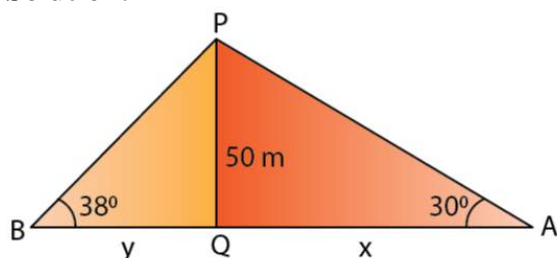Given that, the angle of elevation is 68°
So, tan 68° = h/ 2.4
2.475 = h/2.4
h = 2.475 x 2.4 = 5.94 m
Thus, the ladder reaches upto a height of 5.94 m.

4. Two persons are standing on the opposite sides of a tower. They observe the angles of elevation of the top of the tower to be 30° and 38° respectively. Find the distance between them, if the height of the tower is 50 m.
Solution:

Let one of the persons, A be at a distance of 'x' m and the second person B be at a distance of 'y' m from the foot of the tower.
Given that angle of elevation of A is 30°
tan 30° = 50/x
1/√3 = 50/x
x = 50√3 = 86.60 m
And the angle of elevation of B is 38°
So, tan 38° = 50/y
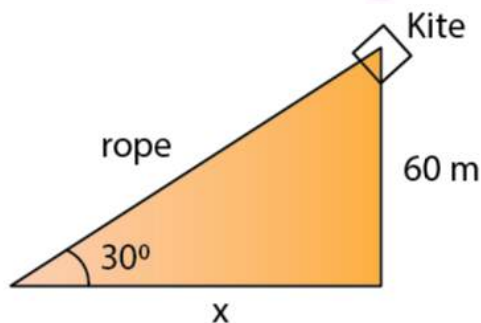0.7813 = 50/ y
y ≈ 64 m
Thus, the distance between A and B is x + y = 150.6 m

5. A kite is attached to a string. Find the length of the string, when the height of the kite is 60 m and the string makes an angle 30° with the ground.
Solution:

Let's assume the length of the rope to be x m.
Now, we have
sin 30° = 60/x
½ = 60/x

x = 120 m
Thus, the length of the rope is 120 m.

6. A boy, 1.6 m tall, is 20 m away from a tower and observes the angle of elevation of the top of the tower to be (i) 45º, (ii) 60º. Find the height of the tower in each case.
Solution:

Let's consider the height of the tower to be 'h' m.
(i) Here, θ = 45º
tan 45º = (h – 1.6)/ 20
1 = (h – 1.6)/ 20
h = 21.6 m
Thus, the height of the tower is 21.6 m.

(ii) Here, θ = 60º
tan 60º = (h – 1.6)/ 20
√3 = (h – 1.6)/ 20
h = 20 x √3 + 1.6 = 36.24 m
Thus, the height of the tower is 36.24 m.

Exercise 22(B)   Page No: 341

1. In the figure, given below, it is given that AB is perpendicular to BD and is of length X metres. DC = 30 m, ∠ADB = 30° and ∠ACB = 45°. Without using tables, find X.
Solution:

In ΔABC,
AB/BC = tan 45° = 1
So, BC = AB = X
In ΔABD,
AB/BC = tan 45° = 1
So, BC = AB = X

In ΔABD,
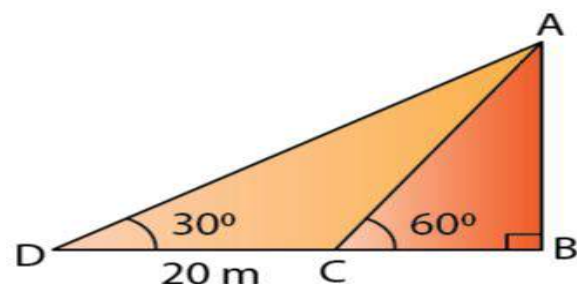AB/BD = tan 30°
X/(30 + X) = 1/ √3
30 + X = √3X
X = 30/ (√3 - 1) = 30/ (1.732 - 1) = 30/ 0.732
Thus, X = 40.98 m
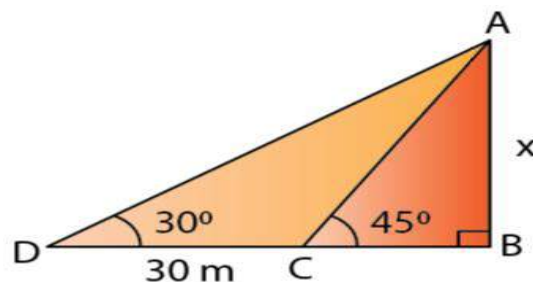
2. Find the height of a tree when it is found that on walking away from it 20 m, in a horizontal line through its base, the elevation of its top changes from 60° to 30°.
Solution:

Let's assume AB to be the height of the tree, h m.
Let the two points be C and D be such that CD = 20 m, ∠ADB = 30° and ∠ACB = 60°
In ΔABC,
AB/BC = tan 60° = √3
BC = AB/ √3 = h/ √3 ….. (i)
In ΔABD,
AB/BD = tan 30°
h/ (20 + BC) = 1/ √3
√3 h = 20 + BC
√3 h = 20 + h/ √3 …… [From (i)]
h (√3 – 1/√3) = 20
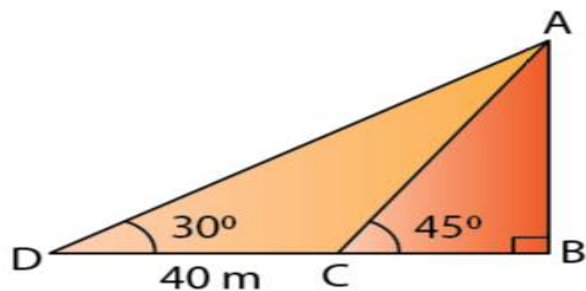h = 20/ (√3 – 1/√3) = 20/ 1.154 = 17.32 m
Therefore, the height of the tree is 17.32 m.

3. Find the height of a building, when it is found that on walking towards it 40 m in a horizontal

line through its base the angular elevation of its top changes from 30º to 45º.
Solution:

Let's assume AB to be the building of height h m.
Let the two points be C and D be such that CD = 40 m, ∠ADB = 30º and ∠ACB = 45º
In ΔABC,
AB/BC = tan 45º = 1
BC = AB = h
And, in ΔABD,
AB/ BD = tan 30º
h/ (40 + h) = 1/√3
√3h = 40 + h
h = 40/ (√3 - 1) = 40/ 0.732 = 54.64 m
Therefore, the height of the building is 54.64 m.

4. From the top of a light house 100 m high, the angles of depression of two ships are observed as 48º and 36º respectively. Find the distance between the two ships (in the nearest metre) if:
(i) the ships are on the same side of the light house.
(ii) the ships are on the opposite sides of the light house.
Solution:
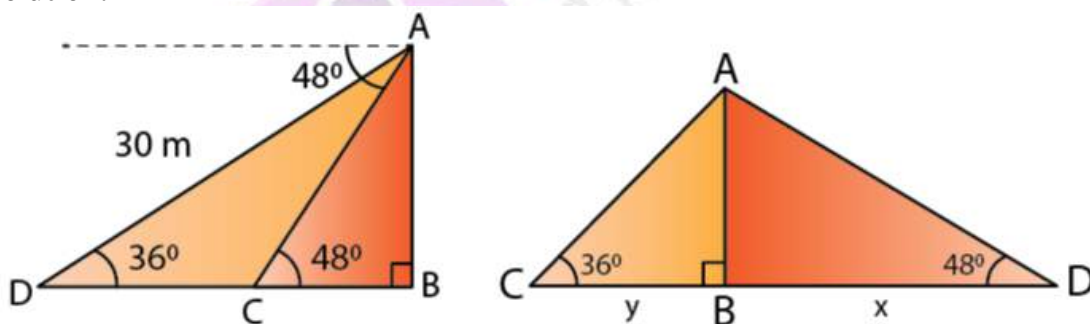
Let's consider AB to be the lighthouse.
And, let the two ships be C and D such that ∠ADB = 36º and ∠ACB = 48º
In ΔABC,
AB/BC = tan 48º
BC = 100/ 1.1106 = 90.04 m

In ΔABD,
AB/BD = tan 36º
BD = 100/ 0.7265 = 137.64 m
Now,

(i) If the ships are on the same side of the light house,
Then, the distance between the two ships = BD - BC = 48 m
(ii) If the ships are on the opposite sides of the light house,
Then, the distance between the two ships = BD + BC = 228 m

5. Two pillars of equal heights stand on either side of a roadway, which is 150 m wide. At a point in the roadway between the pillars the elevations of the tops of the pillars are 60° and 30°; find the height of the pillars and the position of the point.
Solution:
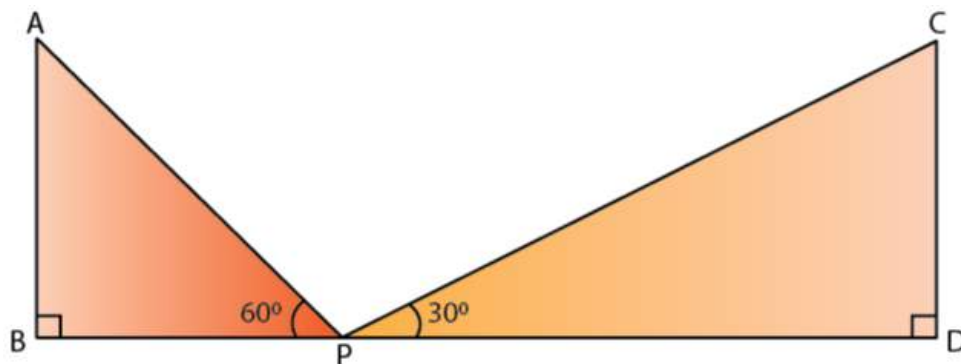
Let AB and CD be the two towers of height h m each.
And, let P be a point in the roadway BD such that BD = 150 m, ∠APB = 60° and ∠CPD = 30°
In ΔABP,
AB/BP = tan 60°
BP = h/ tan 60°
BP = h/ √3
In ΔCDP,
CD/DP = tan 30°
PD = √3 h
Now, 150 = BP + PD
150 = √3h + h/√3
h = 150/(√3 + 1/√3) = 150/ 2.309
h = 64.95 m
Thus, the height of the pillars are 64.95 m each.
Now,
The point is BP/ √3 from the first pillar.
Which is a distance of 64.95/ √3 from the first pillar.
Thus, the position of the point is 37.5 m from the first pillar.

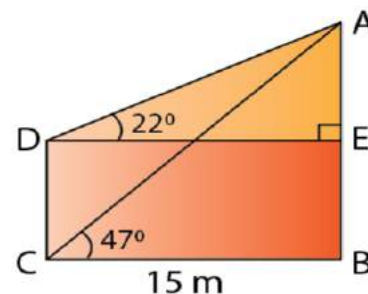**6. From the figure, given below, calculate the length of CD.**
Solution:

In ΔAED,
AE/ DE = tan 22°
AE = DE tan 22° = 15 x 0.404 = 6.06 m
In ΔABC,
AB/BC = tan 47°

AB = BC tan 47º = 15 x 1.072 = 16.09 m
Thus,
CD = BE = AB – AE = 10.03 m

7. The angle of elevation of the top of a tower is observed to be 60º. At a point, 30 m vertically above the first point of observation, the elevation is found to be 45º. Find:
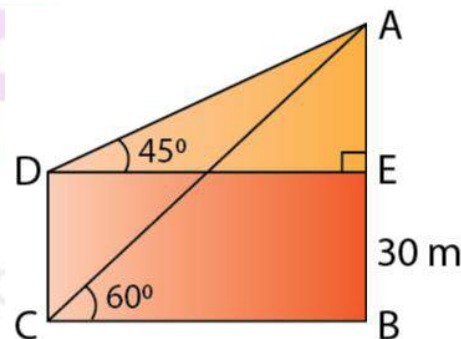**(i) the height of the tower,**
(ii) its horizontal distance from the points of observation.
Solution:

Let's consider AB to be the tower of height h m.
And let the two points be C and D be such that CD = 30 m, ∠ADE = 45º and ∠ACB = 60º

(i)   In ΔADE,
        AE/DE = tan 45º = 1
        AE = DE
        In ΔABC,
        AB/BC = tan 60º = √3
        AE + 30 = √3 BC
        BC + 30 = √3 BC   [Since, AE = DE = BC]
        BC = 30/(√3 -1) = 30/ 0.732 = 40.98 m
        Thus,
        AB = 30 + 40.98 = 70.98 m
        Thus, the height of the tower is 70.98 m
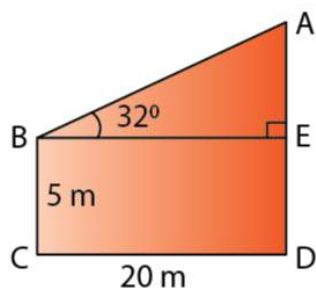
(ii)   The horizontal distance from the points of observation is BC = 40.98 m

Exercise 22(C)   Page No: 342
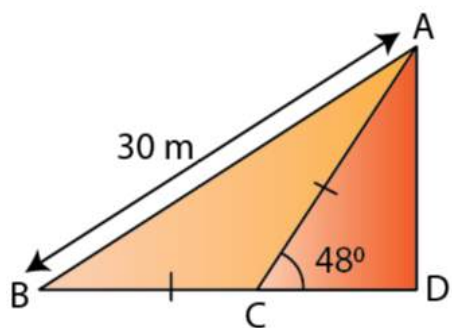
1. Find AD.

(i)

Solution:

In ΔAEB,
AE/BE = tan 32°
AE = 20 x 0.6249 = 12.50 m
AD = AE + ED = 12.50 + 5 = 17.50 m

(ii)

Solution:
In ΔABC,
∠ACD = ∠ABC + ∠BAC
and ∠ABC = ∠BAC   [Since, AC = BC]
∠ABC = ∠BAC = 48°/ 2 = 24°
Now,
AD/AB = sin 24°
AD = 30 x 0.4067 = 12.20 m

2. In the following diagram, AB is a floor-board; PQRS is a cubical box with each edge = 1 m and ∠B = 60°. Calculate the length of the board AB.
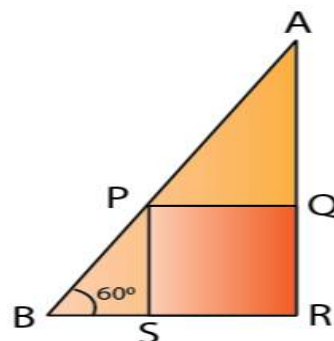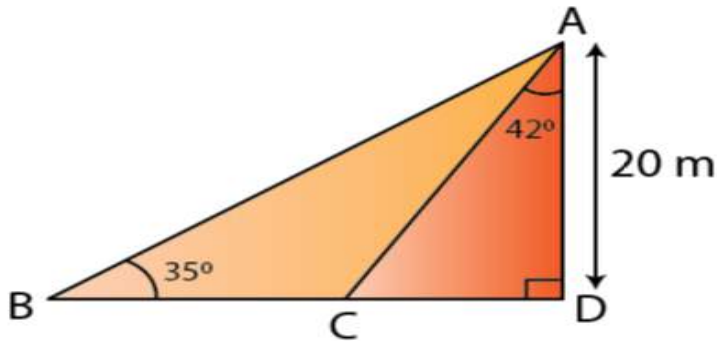**Solution:**

In ΔPSB,
PS/PB = sin 60°
PB = 2/ √3 = 1.155 m
In ΔAPQ,

∠APQ = 60º
PQ/AP = cos 60º
AP = 1/ (1/2) = 2 m
Thus,
AB = AP + PB = 2 + 1.155 = 3.155 m

3. Calculate BC.

Solution:

In ΔADC,
CD/AD = tan 42º
CD = 20 x 0.9004 = 18.008 m
In ΔADB,
AD/BD = tan 35º
BD = AD/ tan 35º = 20/ 0.7002 = 28.563 m
Thus, BC = BD – CD = 10.55 m

4. Calculate AB.
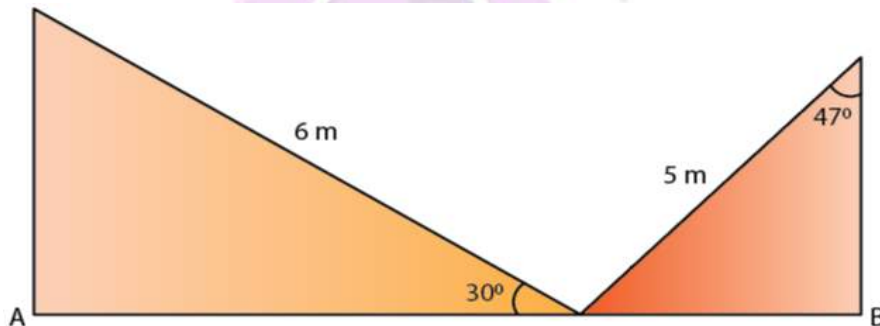
Solution:

In ΔAMOB,
cos 30º = AO/MO
√3/2 = AO/6
AO = 5.20 m
In ΔBNO,
sin 47º = OB/NO
0.73 = OB/5

OB = 3.65 m

So, AB = OA + OB
AB = 5.20 + 3.65
AB = 8.85 m

**5. The radius of a circle is given as 15 cm and chord AB subtends an angle of 131º at the centre C
of the circle. Using trigonometry, calculate:**
(i) the length of AB;
(ii) the distance of AB from the centre C.
Solution:

Given, CA = CB = 15 cm and ∠ACB = 131º
Construct a perpendicular CP from centre C to the chord AB.
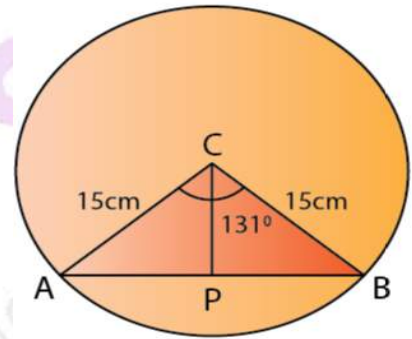Then, CP bisects ∠ACB as well as chord AB.
So, ∠ACP = 65.5º
In ΔACP,
AP/AC = sin (65.5º)
AP = 15 x 0.91 = 13.65 cm

(i) AB = 2 AP = 2 x 13.65 = 27.30 cm
(ii) CP = AP cos (65.5º) = 15 x 0.415 = 6.22 cm

**6. At a point on level ground, the angle of elevation of a vertical tower is found to be such that its
tangent is 5/12. On walking 192 meters towards the tower, the tangent of the angle is found to
be 3/4. Find the height of the tower.**
Solution:
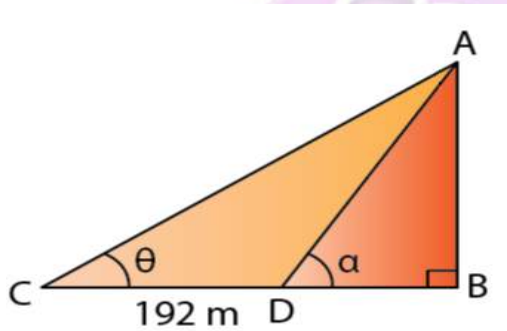
Let's assume AB to be the vertical tower and C and D be the two points such that CD = 192 m.
And let ∠ACB = θ and ∠ADB = α
Given,
tan θ = 5/12
AB/BC = 5/12
AB = 5/12 BC …… (i)
Also, tan α = ¾
AB/BD = ¾
(5/12 x BC)/ BD = ¾
(192 + BD)/ BD = ¾ x 12/5

BD = 240 m
BC = (192 + 240) = 432 m
By (i), AB = 5/12 x 432 = 180 m
Therefore, the height of the tower is 180 m.

7. A vertical tower stands on a horizontal plane and is surmounted by a vertical flagstaff of height h meter. At a point on the plane, the angle of elevation of the bottom of the flagstaff is α and at the top of the flagstaff is β. Prove that the height of the tower is h tan α/ (tan β - tan α).
Solution:

Let AB be the tower of height x metre, surmounted by a vertical
flagstaff AD. Let C be a point on the plane such that ∠ACB = α,
∠ACB = β and AD = h.
In ΔABC,
AB/BC = tan α
BC = x/ tan α ……. (i)
In ΔDBC,
BD/BC = tan β
BD = (x/tan α) x tan β …… [From (i)]
(h + x) tan α = x tan β
x tan β – x tan α = h tan α
Therefore, height of the tower is h tan α/ (tan β - tan α)
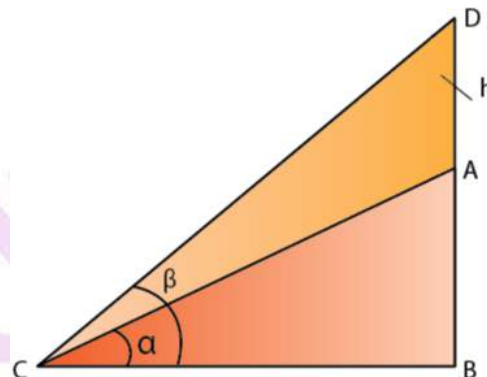
8. With reference to the given figure, a man stands on the ground at point A, which is on the same horizontal plane as B, the foot of the vertical pole BC. The height of the pole is 10 m. The man's eye s 2 m above the ground. He observes the angle of elevation of C, the top of the pole, as x⁰, where tan x⁰ = 2/5. Calculate:
(i) the distance AB in metres;
(ii) angle of elevation of the top of the pole when he is standing 15 metres from the pole. Give your answer to the nearest degree.
Solution:

Let take AD to be the height of the man, AD = 2 m.
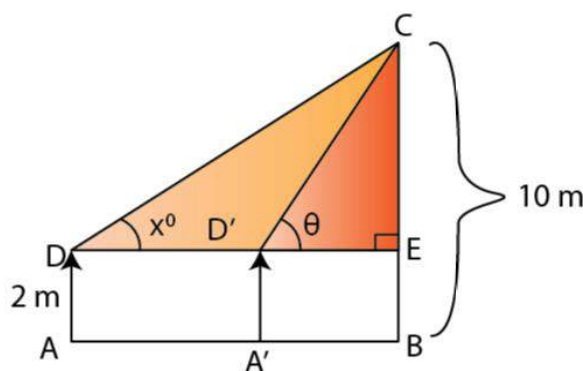So, CE = (10 - 2) = 8 m
(i) In ΔCED,
CE/DE = tan x = 2/5
8/DE = 2/5
⇒ DE = 20 m
And, as AB = DE we get
AB = 20 m

(ii) Let A'D' be the new position of the man and θ be the angle of elevation of the top of the tower.
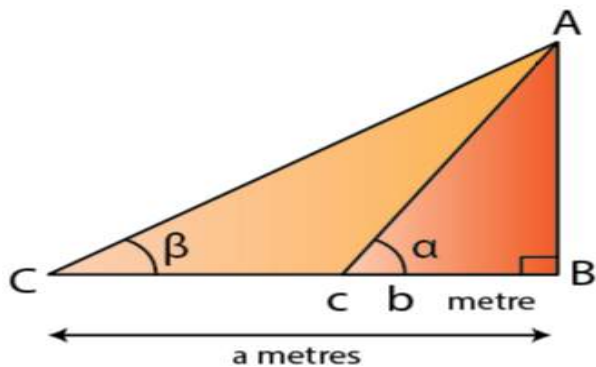So, D'E = 15 m
In ΔCED,
tan θ = CE/ D'E = 8/15 = 0.533
θ = 28⁰

9. The angles of elevation of the top of a tower from two points on the ground at distances *a* and *b* meters from the base of the tower and in the same line are complementary. Prove that the height of the tower is √ab meter.
Solution:

Let's assume AB to be the tower of height h meters.
And, let C and D be two points on the level ground such that BC = b meters, BD = a meters, ∠ACB = α, ∠ADB = β.
Given, α + β = 90º
In ΔABC,
AB/BC = tan α
h/b = tan α ….. (i)
In ΔABD,
AB/BD = tan β
h/a = tan (90º - α) = cot α ….. (ii)
Now, multiplying (i) by (ii), we get
(h/a) x (h/b) = 1
h^2 = ab
So, h = √ab meter
Therefore, height of the tower is √ab meter.

10. From a window A, 10 m above the ground the angle of elevation of the top C of a tower is xº, where tan xº = 5/2 and the angle of depression of the foot D of the tower is yº, where tan yº = 1/4. Calculate the height CD of the tower in metres.
Solution:

We have, AB = DE = 10 m
So, in ΔABC
DE/AE = tan y = ¼
AE = 4 DE = 4 x 10 = 40 m
In ΔABC,
CE/AE = tan x = 5/2
CE = 40 x 5/2 = 100 m
So, CD = DE + EC = 10 + 100 = 110 m
Therefore, the height of the tower CD is 110 m.