Exercise 7(B)

1. Find the fourth proportional to:
(i) 1.5, 4.5 and 3.5 (ii) 3a, $6a^2$ and $2ab^2$
Solution:

(i) Let's assume the fourth proportional to 1.5, 4.5 and 3.5 be x. 1.5:
4.5 = 3.5: x
1.5 × x = 3.5 × 4.5
x = (3.5 x 4.5)/ 1.5
x = 10.5

(ii) Let's assume the fourth proportional to 3a, $6a^2$ and $2ab^2$ be x.
$3a: 6a^2 = 2ab^2: x$
$3a \times x = 2ab^2 \times 6a^2$
$3a \times x = 12a^3b^2$
$x = 4a^2b^2$

2. Find the third proportional to:
(i) $2\frac{2}{3}$ and 4 (ii) a - b and $a^2 - b^2$
Solution:

(i) Let's take the third proportional to $2\frac{2}{3}$ and 4 be x.
So, $2\frac{2}{3}$, 4, x are in continued proportion.
8/3: 4 = 4: x
(8/3)/ 4 = 4/x
x = 16 x 3/8 = 6

(ii) Let's take the third proportional to a - b and $a^2 - b^2$ be x.
So, a - b, $a^2 - b^2$, x are in continued proportion.
$a - b: a^2 - b^2 = a^2 - b^2: x$

$$\frac{a-b}{a^2-b^2} = \frac{a^2-b^2}{x}$$

$$x = \frac{(a^2-b^2)^2}{a-b}$$

$$x = \frac{(a+b)(a-b)(a^2-b^2)}{a-b}$$

$$x = (a+b)(a^2-b^2)$$

3. Find the mean proportional between:
(i) 6 + 3√3 and 8 - 4√3

(ii) a - b and $a^3 - a^2b$
Solution:
(i) Let the mean proportional between $6 + 3\sqrt{3}$ and $8 - 4\sqrt{3}$ be x.
So, $6 + 3\sqrt{3}$, x and $8 - 4\sqrt{3}$ are in continued proportion.
$6 + 3\sqrt{3} : x = x : 8 - 4\sqrt{3}$
$x \times x = (6 + 3\sqrt{3})(8 - 4\sqrt{3})$
$x^2 = 48 + 24\sqrt{3} - 24\sqrt{3} - 36$
$x^2 = 12$
$x = 2\sqrt{3}$

(ii) Let the mean proportional between a - b and $a^3 - a^2b$ be x.
a - b, x, $a^3 - a^2b$ are in continued proportion.
$a – b: x = x: a^3 - a^2b$
$x \times x = (a - b)(a^3 - a^2b)$
$x^2 = (a - b)a^2(a - b) = [a(a - b)]^2$
$x = a(a - b)$

4. If x + 5 is the mean proportional between x + 2 and x + 9; find the value of x.
Solution:

Given, x + 5 is the mean proportional between x + 2 and x + 9.
So, (x + 2), (x + 5) and (x + 9) are in continued proportion.
$(x + 2): (x + 5) = (x + 5): (x + 9)$
$(x + 2)/ (x + 5) = (x + 5)/ (x + 9)$
$(x + 5)^2 = (x + 2)(x + 9)$
$x^2 + 25 + 10x = x^2 + 2x + 9x + 18$
$25 - 18 = 11x - 10x$
$x = 7$

5. If x^2, 4 and 9 are in continued proportion, find x.
Solution:

Given, x^2, 4 and 9 are in continued proportion
So, we have
$x^2/4 = 4/9$
$x^2 = 16/9$
Thus, $x = 4/3$

6. What least number must be added to each of the numbers 6, 15, 20 and 43 to make them proportional?
Solution:

Let assume the number added to be x.
So, $(6 + x): (15 + x) :: (20 + x): (43 + x)$
$(6 + x)/ (15 + x) = (20 + x)/ (43 + x)$
$(6 + x)(43 + x) = (20 + x)(43 + x)$

$258 + 6x + 43x + x^2 = 300 + 20x = 15x + x^2$
$49x – 35x = 300 – 258$
$14x = 42$
$x = 3$
Therefore, the required number which should be added is 3.

7. (i) If a, b, c are in continued proportion,
Show that: $\frac{a^2+b^2}{b(a+c)} = \frac{b(a+c)}{b^2+c^2}$

Solution:

Given,
a, b, c are in continued proportion.
So, we have
$a/b = b/c$
$\Rightarrow b^2 = ac$
Now,
$(a^2 + b^2)(b^2 + c^2) = (a^2 + ac)(ac + c^2)$ [As $b^2 = ac$]
$= a(a + c) c(a + c)$
$= ac(a + c)^2$
$= b^2(a + c)^2$
$(a^2 + b^2)(b^2 + c^2) = [b(a + c)][b(a + c)]$
Thus, L.H.S = R.H.S

$$\frac{a^2+b^2}{b(a+c)} = \frac{b(a+c)}{b^2+c^2}$$

- Hence Proved

(ii) If a, b, c are in continued proportion and a(b - c) = 2b, prove that: a – c = 2(a + b)/ a
Solution:

Given,
a, b, c are in continued proportion.
So, we have
$a/b = b/c$
$\Rightarrow b^2 = ac$
And, given $a(b - c) = 2b$
$ab – ac = 2b$
$ab – b^2 = 2b$
$ab = 2b + b^2$
$ab = b(2 + b)$
$a = b + 2$
$a – b = 2$
Now, taking the L.H.S we have
L.H.S = $a – c$
$= a(a - c)/ a$ [Multiply and divide by a]

$= a^2 – ac/ a$
$= a^2 – b^2/ a$
$= (a - b) (a + b)/a$
$= 2(a + b)/a$
= R.H.S
- Hence Proved

(iii) If a/b = c/d, show that: $\frac{a^3c + ac^3}{b^3d + bd^3} = \frac{(a+c)^4}{(b+d)^4}$

Solution:

Let's take a/b = c/d = k
So, a = bk and c = dk
Taking L.H.S,

$$\text{L.H.S.} = \frac{a^3c + ac^3}{b^3d + bd^3} = \frac{ac(a^2 + c^2)}{bd(b^2 + d^2)}$$

$$= \frac{(bk \times dk)(b^2k^2 + d^2k^2)}{bd(b^2 + d^2)}$$

$$= \frac{k^2 \times k^2(b^2 + d^2)}{(b^2 + d^2)} = k^4$$

Now, taking the R.H.S

$$\text{R.H.S.} = \frac{(a+c)^4}{(b+d)^4} = \frac{(bk + dk)^4}{(b+d)^4} = \left[\frac{k(b+d)}{b+d}\right]^4 = k^4$$

Thus, L.H.S = R.H.S
- Hence Proved

8. What least number must be subtracted from each of the numbers 7, 17 and 47 so that the remainders are in continued proportion?
Solution:

Let's assume the number subtracted to be x.
So, we have
(7 - x): (17 - x):: (17 - x): (47 - x)

$$\frac{7 - x}{17 - x} = \frac{17 - x}{47 - x}$$

$(7 - x)(17 - x) = (17 - x)^2$
$329 – 47x – 7x + x^2 = 289 – 34x + x^2$
$329 – 289 = -34x + 54x$
$20x = 40$
$x = 2$
Therefore, the required number which must be subtracted is 2.