## Exercise 7(A)



**1. If a: b = 5: 3, find: 5a – 3b/ 5a + 3b**
**Solution:**

Given, a: b = 5: 3
So, a/b = 5/3
Now,

$$\frac{5a-3b}{5a+3b} = \frac{5\left(\frac{a}{b}\right)-3}{5\left(\frac{a}{b}\right)+3} \quad \text{(Dividing each term by b)}$$

$$= \frac{5\left(\frac{5}{3}\right)-3}{5\left(\frac{5}{3}\right)+3} = \frac{\frac{25}{3}-3}{\frac{25}{3}+3} = \frac{25-9}{25+9}$$

$$= \frac{16}{34} = \frac{8}{17}$$

**2. If x: y = 4: 7, find the value of (3x + 2y): (5x + y).**
**Solution:**

Given, x: y = 4: 7
So, x/y = 4/7

$$\frac{3x+2y}{5x+y} = \frac{3\left(\frac{x}{y}\right)+2}{5\left(\frac{x}{y}\right)+1} \quad \text{(Dividing each term by y)}$$

$$= \frac{3\left(\frac{4}{7}\right)+2}{5\left(\frac{4}{7}\right)+1} = \frac{\frac{12}{7}+2}{\frac{20}{7}+1} = \frac{12+14}{20+7}$$

$$= \frac{26}{27}$$

**3. If a: b = 3: 8, find the value of 4a + 3b/ 6a – b.**
**Solution:**

Given, a: b = 3: 8
So, a/b = 3/8

$$\frac{4a+3b}{6a-b} = \frac{4\left(\frac{a}{b}\right)+3}{6\left(\frac{a}{b}\right)-1} \text{ (Dividing each term by b)}$$

$$= \frac{4\left(\frac{3}{8}\right)+3}{6\left(\frac{3}{8}\right)-1} = \frac{\frac{3}{2}+3}{\frac{9}{4}-1} = \frac{\frac{9}{2}}{\frac{5}{4}}$$

$$= \frac{18}{5}$$

**4. If (a - b): (a + b) = 1: 11, find the ratio (5a + 4b + 15): (5a – 4b + 3).**
**Solution:**

Given,
(a - b)/ (a + b) = 1/ 11
11a – 11b = a + b
10a = 12b
a/b = 12/10 = 6/5
Now, lets take a = 6k and b = 5k
So,

$$\frac{5a+4b+15}{5a-4b+3} = \frac{5(6k)+4(5k)+15}{5(6k)-4(5k)+3}$$

$$= \frac{30k+20k+15}{30k-20k+3}$$

$$= \frac{50k+15}{10k+3}$$

$$= \frac{5(10k+3)}{10k+3}$$

$$= 5$$

Therefore, (5a + 4b + 15): (5a – 4b + 3 = 5: 1

**5. Find the number which bears the same ratio to 7/33 that 8/21 does to 4/9.**
**Solution:**

Let consider the required number to be x/y
Now, given that
Ratio of 8/21 to 4/9 = (8/21)/ (4/9) = (8/21) x (9/4) = 6/7
Hence, we have
(x/y)/ (7/33) = 6/7
x/y = (6/7)/ (7/33)
= (6/7) x (7/33)
= 2/11

Therefore, the required number is 2/11.

**6.** $If \frac{m+n}{m+3n} = \frac{2}{3}, find : \frac{2n^2}{3m^2+mn}.$

**Solution:**

Given,

$$\frac{m+n}{m+3n} = \frac{2}{3}$$

3(m + n) = 2(m + 3n)

3m + 3n = 2m + 6n

m = 3n

m/n = 3/1

Now,

$$\frac{2n^2}{3m^2+mn} = \frac{2}{3\left(\frac{m}{n}\right)^2 + \left(\frac{m}{n}\right)} \text{ (Dividing each term by } n^2)$$

$$= \frac{2}{3\left(\frac{3}{1}\right)^2 + \left(\frac{3}{1}\right)}$$

$$= \frac{2}{27+3} = \frac{1}{15}$$

**7. Find x/y; when $x^2 + 6y^2 = 5xy$**

**Solution:**

Given,

$x^2 + 6y^2 = 5xy$

Dividing by $y^2$ both side, we have

$$\frac{x^2}{y^2} + \frac{6y^2}{y^2} = \frac{5xy}{y^2}$$

$$\left(\frac{x}{y}\right)^2 + 6 = 5\left(\frac{x}{y}\right)$$

$$\left(\frac{x}{y}\right)^2 - 5\left(\frac{x}{y}\right) + 6 = 0$$

Let x/y = a

So,

$a^2 – 5a + 6 = 0$

(a - 2) (a - 3) = 0

a = 2 or a = 3

Therefore, x/y = 2 or 3

**8. If the ratio between 8 and 11 is the same as the ratio of 2x - y to x + 2y, find the value of 7x/ 9y.**
**Solution:**

Given,
(2x - y)/ (x + 2y) = 8/11
On cross multiplying, we get
11(2x - y) = 8(x + 2y)
22x – 11y = 8x + 16y
14x = 27y
x/y = 27/14
So,
7x / 9y = (7 x 27)/ (9 x 14) = 3/2

**9. Divide Rs 1290 into A, B and C such that A is 2/5 of B and B: C = 4: 3.**
**Solution:**

Given,
B: C = 4: 3 so, B/C = 4/3 ⇒ C = (3/4) B
And, A = (2/5) B
We know that,
A + B + C = Rs 1290
(2/5) B + B + (3/4) B = 1290
Taking L.C.M,
(8B + 20B + 15B)/ 20 = 1290
43B = 1290 x 20
B = 1290 x 20/ 43 = 600
So,
A = (2/5) x 600 = 240
And,
C = (3/4) x 600 = 450

Therefore,
A gets Rs 600, B gets Rs 240 and C gets Rs 450

**10. A school has 630 students. The ratio of the number of boys to the number of girls is 3: 2. This ratio changes to 7: 5 after the admission of 90 new students. Find the number of newly admitted boys.**
**Solution:**

Let's consider the number of boys be 3x.
Then, the number of girls = 2x
⇒ 3x + 2x = 630
5x = 630
x = 126
So, the number of boys = 3x = 3 x 126 = 378
And, number of girls = 2x = 2 x 126 = 252

After admission of 90 new students,
Total number of students = 630 + 90 = 720
Here, let take the number of boys to be 7x
And, the number of girls = 5x
⇒ 7x + 5x = 720
12x = 720
x = 720/12
x = 60
So, the number of boys = 7x = 7 x 60 = 420
And, the number of girls = 5x = 5 x 60 = 300
Therefore, the number of newly admitted boys = 420 – 378 = 42

**11. What quantity must be subtracted from each term of the ratio 9: 17 to make it equal to 1: 3?**
**Solution:**

Let x be subtracted from each term of the ratio 9: 17.

$$\frac{9-x}{17-x} = \frac{1}{3}$$

27 – 3x = 17 – x
10 = 2x
x = 5
Therefore, the required number which should be subtracted is 5.

**12. The monthly pocket money of Ravi and Sanjeev are in the ratio 5: 7. Their expenditures are in the ratio 3: 5. If each saves Rs. 80 every month, find their monthly pocket money.**
**Solution:**

Given,
The pocket money of Ravi and Sanjeev are in the ratio 5: 7
So, we can assume the pocket money of Ravi as 5k and that of Sanjeev as 7k.
Also, give that
The expenditure of Ravi and Snajeev are in the ratio 3: 5
So, it can be taken as the expenditure of Ravi as 3m and that of Sanjeev as 5m.
And, each of them saves Rs 80
This can be expressed as below:
5k – 3m = 80 ….. (a)
7k – 5m = 80 …... (b)
Solving equations (a) and (b), we have
k = 40 and m = 40
Therefore, the monthly pocket money of Ravi is Rs 5k = Rs 5 x 40 = Rs 200 and that of Sanjeev is Rs 7k = Rs 7 x 40 = Rs 280.

**13. The work done by (x - 2) men in (4x + 1) days and the work done by (4x + 1) men in (2x - 3) days are in the ratio 3: 8. Find the value of x.**
**Solution:**

On assuming that the same amount of work is done one day by all the men and one day work of each man = 1 units, we have
Amount of work done by (x - 2) men in (4x + 1) days
= Amount of work done by (x - 2)*(4x + 1) men in one day
= (x - 2)(4x + 1) units of work

Similarly, we have
Amount of work done by (4x + 1) men in (2x - 3) days
= (4x + 1)*(2x - 3) units of work
Then according to the question, we have

$$\frac{(x-2)(4x+1)}{(4x+1)(2x-3)} = \frac{3}{8}$$

$$\frac{x-2}{2x-3} = \frac{3}{8}$$

8x – 16 = 6x – 9
2x = 7
x = 7/2

**14. The bus fare between two cities is increased in the ratio 7: 9. Find the increase in the fare, if:**
**(i) the original fare is Rs 245;**
**(ii) the increased fare is Rs 207.**
**Solution:**

From the question we have,
Increased (new) bus fare = (9/7) x original bus fare
(i) We have,
Increased (new) bus fare = 9/7 x Rs 245 = Rs 315
Thus, the increase in fare = Rs 315 - Rs 245 = Rs 70
(ii) Here we have,
Rs 207 = (9/7) x original bus fare
Original bus fare = Rs 207 x 7/9 = Rs 161
Thus, the increase in fare = Rs 207 - Rs 161 = Rs 46

**15. By increasing the cost of entry ticket to a fair in the ratio 10: 13, the number of visitors to the fair has decreased in the ratio 6: 5. In what ratio has the total collection increased or decreased?**
**Solution:**

Let's take the cost of the entry ticket initially and at present to be 10x and 13x respectively.
And let the number of visitors initially and at present be 6y and 5y respectively.
So,
Initially, the total collection = 10x × 6y = 60 xy
And at present, the total collection = 13x × 5y = 65 xy
Hence,
The ratio of total collection = 60 xy: 65 xy = 12: 13
Therefore, it's seen that the total collection has been increased in the ratio 12: 13.

## Exercise 7(B)



**1. Find the fourth proportional to:**
**(i) 1.5, 4.5 and 3.5 (ii) 3a, $6a^2$ and $2ab^2$**
**Solution:**

(i) Let's assume the fourth proportional to 1.5, 4.5 and 3.5 be x.
1.5: 4.5 = 3.5: x
1.5 × x = 3.5 × 4.5
x = (3.5 x 4.5)/ 1.5
x = 10.5

(ii) Let's assume the fourth proportional to 3a, $6a^2$ and $2ab^2$ be x.
$3a: 6a^2 = 2ab^2: x$
$3a \times x = 2ab^2 \times 6a^2$
$3a \times x = 12a^3b^2$
$x = 4a^2b^2$

**2. Find the third proportional to:**
**(i) $2\frac{2}{3}$ and 4 (ii) a - b and $a^2 - b^2$**
**Solution:**

(i) Let's take the third proportional to $2\frac{2}{3}$ and 4 be x.
So, $2\frac{2}{3}$, 4, x are in continued proportion.
8/3: 4 = 4: x
(8/3)/ 4 = 4/x
x = 16 x 3/8 = 6

(ii) Let's take the third proportional to a - b and $a^2 - b^2$ be x.
So, a - b, $a^2 - b^2$, x are in continued proportion.
$a - b: a^2 - b^2 = a^2 - b^2: x$

$$\frac{a-b}{a^2-b^2} = \frac{a^2-b^2}{x}$$

$$x = \frac{(a^2-b^2)^2}{a-b}$$

$$x = \frac{(a+b)(a-b)(a^2-b^2)}{a-b}$$

$$x = (a+b)(a^2-b^2)$$

**3. Find the mean proportional between:**
**(i) 6 + 3√3 and 8 - 4√3**

**(ii) a - b and $a^3 - a^2b$**
**Solution:**

(i) Let the mean proportional between $6 + 3\sqrt{3}$ and $8 - 4\sqrt{3}$ be x.
So, $6 + 3\sqrt{3}$, x and $8 - 4\sqrt{3}$ are in continued proportion.
$6 + 3\sqrt{3} : x = x : 8 - 4\sqrt{3}$
$x \times x = (6 + 3\sqrt{3})(8 - 4\sqrt{3})$
$x^2 = 48 + 24\sqrt{3} - 24\sqrt{3} - 36$
$x^2 = 12$
$x = 2\sqrt{3}$

(ii) Let the mean proportional between a - b and $a^3 - a^2b$ be x.
a - b, x, $a^3 - a^2b$ are in continued proportion.
$a – b: x = x: a^3 - a^2b$
$x \times x = (a - b)(a^3 - a^2b)$
$x^2 = (a - b)a^2(a - b) = [a(a - b)]^2$
$x = a(a - b)$

**4. If x + 5 is the mean proportional between x + 2 and x + 9; find the value of x.**
**Solution:**

Given, x + 5 is the mean proportional between x + 2 and x + 9.
So, (x + 2), (x + 5) and (x + 9) are in continued proportion.
$(x + 2): (x + 5) = (x + 5): (x + 9)$
$(x + 2)/ (x + 5) = (x + 5)/ (x + 9)$
$(x + 5)^2 = (x + 2)(x + 9)$
$x^2 + 25 + 10x = x^2 + 2x + 9x + 18$
$25 - 18 = 11x - 10x$
$x = 7$

**5. If $x^2$, 4 and 9 are in continued proportion, find x.**
**Solution:**

Given, $x^2$, 4 and 9 are in continued proportion
So, we have
$x^2/4 = 4/9$
$x^2 = 16/9$
Thus, $x = 4/3$

**6. What least number must be added to each of the numbers 6, 15, 20 and 43 to make them proportional?**
**Solution:**

Let assume the number added to be x.
So, $(6 + x): (15 + x) :: (20 + x): (43 + x)$
$(6 + x)/ (15 + x) = (20 + x)/ (43 + x)$
$(6 + x)(43 + x) = (20 + x)(43 + x)$

$258 + 6x + 43x + x^2 = 300 + 20x = 15x + x^2$
$49x – 35x = 300 – 258$
$14x = 42$
$x = 3$
Therefore, the required number which should be added is 3.

**7. (i) If a, b, c are in continued proportion,**
**Show that:** $\frac{a^2+b^2}{b(a+c)} = \frac{b(a+c)}{b^2+c^2}$

**Solution:**

Given,
a, b, c are in continued proportion.
So, we have
a/b = b/c
$\Rightarrow b^2 = ac$
Now,
$(a^2 + b^2)(b^2 + c^2) = (a^2 + ac)(ac + c^2)$ [As $b^2 = ac$]
$= a(a + c)\, c(a + c)$
$= ac(a + c)^2$
$= b^2(a + c)^2$
$(a^2 + b^2)(b^2 + c^2) = [b(a + c)][b(a + c)]$
Thus, L.H.S = R.H.S

$$\frac{a^2+b^2}{b(a+c)} = \frac{b(a+c)}{b^2+c^2}$$

- Hence Proved

**(ii) If a, b, c are in continued proportion and a(b - c) = 2b, prove that: a – c = 2(a + b)/ a**
**Solution:**

Given,
a, b, c are in continued proportion.
So, we have
a/b = b/c
$\Rightarrow b^2 = ac$
And, given a(b - c) = 2b
ab – ac = 2b
$ab – b^2 = 2b$
$ab = 2b + b^2$
ab = b(2 + b)
a = b + 2
a – b = 2
Now, taking the L.H.S we have
L.H.S = a – c
= a(a - c)/ a [Multiply and divide by a]

$= a^2 – ac/ a$
$= a^2 – b^2/ a$
$= (a - b) (a + b)/a$
$= 2(a + b)/a$
= R.H.S
- Hence Proved

**(iii) If a/b = c/d, show that:** $\frac{a^3c + ac^3}{b^3d + bd^3} = \frac{(a+c)^4}{(b+d)^4}$

**Solution:**

Let's take a/b = c/d = k
So, a = bk and c = dk
Taking L.H.S,

$$\text{L.H.S.} = \frac{a^3c + ac^3}{b^3d + bd^3} = \frac{ac(a^2 + c^2)}{bd(b^2 + d^2)}$$

$$= \frac{(bk \times dk)(b^2k^2 + d^2k^2)}{bd(b^2 + d^2)}$$

$$= \frac{k^2 \times k^2(b^2 + d^2)}{(b^2 + d^2)} = k^4$$

Now, taking the R.H.S

$$\text{R.H.S.} = \frac{(a+c)^4}{(b+d)^4} = \frac{(bk + dk)^4}{(b+d)^4} = \left[\frac{k(b+d)}{b+d}\right]^4 = k^4$$

Thus, L.H.S = R.H.S
- Hence Proved

**8. What least number must be subtracted from each of the numbers 7, 17 and 47 so that the remainders are in continued proportion?**
**Solution:**

Let's assume the number subtracted to be x.
So, we have
(7 - x): (17 - x):: (17 - x): (47 - x)

$$\frac{7 - x}{17 - x} = \frac{17 - x}{47 - x}$$

$(7 - x)(17 - x) = (17 - x)^2$
$329 – 47x – 7x + x^2 = 289 – 34x + x^2$
$329 – 289 = -34x + 54x$
$20x = 40$
$x = 2$
Therefore, the required number which must be subtracted is 2.

## Exercise 7(C)



**1. If a : b = c : d, prove that:**
**(i) 5a + 7b : 5a - 7b = 5c + 7d : 5c - 7d.**
**(ii) (9a + 13b) (9c - 13d) = (9c + 13d) (9a - 13b).**
**(iii) xa + yb : xc + yd = b : d.**
**Solution:**

(i) Given, a/b = c/d

$$\frac{5a}{7b} = \frac{5c}{7d}$$ (Multiplying each by 5/7)

$$\frac{5a+7b}{5a-7b} = \frac{5c+7d}{5c-7d}$$ (By componendo and Dividendo)

(ii) Given, a/b = c/d

$$\frac{9a}{13b} = \frac{9c}{13d}$$ (Multiplying each by 9/13)

$$\frac{9a+13b}{9a-13b} = \frac{9c+13d}{9c-13d}$$ (By componendo and Dividendo)

On cross-multiplication we have,

(9a + 13b)(9c – 13d) = (9c + 13d)(9a – 13b)

(iii) Given, a/b = c/d

$$\frac{xa}{yb} = \frac{xc}{yd}$$ (Multiplying each by x/y)

$$\frac{xa+yb}{yb} = \frac{xc+yd}{yd}$$ (By compenendo)

$$\frac{xa+yb}{xc+yd} = \frac{yb}{yd}$$

$$\frac{xa+yb}{xc+yd} = \frac{b}{d}$$

- Hence Proved

**2. If a : b = c : d, prove that:**
**(6a + 7b) (3c - 4d) = (6c + 7d) (3a - 4b).**
**Solution:**

Given, a/b = c/d

$\frac{6a}{7b} = \frac{6c}{7d}$ (Multiplying each by 6/7)

$\frac{6a+7b}{7b} = \frac{6c+7d}{7d}$ (By compenendo)

$\frac{6a+7b}{6c+7d} = \frac{7b}{7d} = \frac{b}{d}$ .......... (1)

Also, a/b = c/d

$\frac{3a}{4b} = \frac{3c}{4d}$ (Multiplying each by 3/4)

$\frac{3a-4b}{4b} = \frac{3c-4d}{4d}$ (By dividendo)

$\frac{3a-4b}{3c-4d} = \frac{4b}{4d} = \frac{b}{d}$ ....... (2)

Fromo (1) and (2), we have

$\frac{6a+7b}{6c+7d} = \frac{3a-4b}{3c-4d}$

(6a + 7b)(3c – 4d) = (3a – 4b)(6c + 7d)

- Hence Proved

**3. Given, a/b = c/d, prove that:**
**(3a – 5b)/ (3a + 5b) = (3c – 5d)(3c + 5d)**
**Solution:**

Given,

$\frac{a}{b} = \frac{c}{d}$

$\frac{3a}{5b} = \frac{3c}{5d}$ (Multiplying both by 3/5)

$\frac{3a+5b}{3a-5b} = \frac{3c+5d}{3c-5d}$ (By compnendo and Dividendo)

$\frac{3a-5b}{3a+5b} = \frac{3c-5d}{3c+5d}$ (By alternendo)

**4. If** $\frac{5x+6y}{5u+6v} = \frac{5x-6y}{5u-6v}$ **;**
**Then prove that x: y = u: v**
**Solution:**

$$\frac{5x+6y}{5u+6v} = \frac{5x-6y}{5u-6v} \text{ (By alternendo)}$$

$$\frac{5x+6y}{5x-6y} = \frac{5u+6v}{5u-6v}$$

$$\frac{5x+6y+5x-6y}{5x+6y-5x+6y} = \frac{5u+6v+5u-6v}{5u+6v-5u+6v}$$

(By componendo and dividendo)

10x/ 12y = 10u/ 12v

Thus,

x/y = u/v ⇒ x: y = u: v

**5. If (7a + 8b) (7c – 8d) = (7a – 8b) (7c + 8d);**
**Prove that a: b = c: d**
**Solution:**

The given can the rewritten as,

$$\frac{7a+8b}{7a-8b} = \frac{7c+8d}{7c-8d}$$

Applying componendo and dividendo, we have

$$\frac{7a+8b+7a-8b}{7a+8b-7a+8b} = \frac{7c+8d+7c-8d}{7c+8d-7c+8d}$$

$$\frac{14a}{16b} = \frac{14c}{16d}$$

$$\frac{a}{b} = \frac{c}{d}$$

**6. (i) If x = 6ab/ (a + b), find the value of:**

$$\frac{x+3a}{x-3a} + \frac{x+3b}{x-3b}$$

**Solution:**

Given, x = 6ab/ (a + b)

⇒ x/3a = 2b/ a + b

Now, applying componendo and dividendo we have

$$\frac{x+3a}{x-3a} = \frac{2b+a+b}{2b-a-b}$$

$$\frac{x+3a}{x-3a} = \frac{3b+a}{b-a} \quad \ldots (1)$$

Again, x = 6ab/ (a + b)

⇒ x/3b = 2a/ a + b

Now, applying componendo and dividendo we have

$$\frac{x+3b}{x-3b} = \frac{2a+a+b}{2a-a-b}$$

$$\frac{x+3b}{x-3b} = \frac{3a+b}{a-b} \quad \ldots (2)$$

From (1) and (2), we get

$$\frac{x+3a}{x-3a} + \frac{x+3b}{x-3b} = \frac{3b+a}{b-a} + \frac{3a+b}{a-b}$$

$$\frac{x+3a}{x-3a} + \frac{x+3b}{x-3b} = \frac{-3b-a+3a+b}{a-b}$$

$$\frac{x+3a}{x-3a} + \frac{x+3b}{x-3b} = \frac{2a-2b}{a-b} = 2$$

**(ii) If a = 4√6/ (√2 + √3), find the value of:**

$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}} + \frac{a+2\sqrt{3}}{a-2\sqrt{3}}$$

**Solution:**

Given, a = 4√6/ (√2 + √3)
a/2√2 = 2√3/ (√2 + √3)
Now, applying componendo and dividendo we have

$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}} = \frac{2\sqrt{3}+\sqrt{2}+\sqrt{3}}{2\sqrt{3}-\sqrt{2}-\sqrt{3}}$$

$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}} = \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \quad \ldots (1)$$

Again, a = 4√6/ (√2 + √3)
a/2√3 = 2√2/ (√2 + √3)
Now, applying componendo and dividendo we have

$$\frac{a+2\sqrt{3}}{a-2\sqrt{3}} = \frac{2\sqrt{2}+\sqrt{2}+\sqrt{3}}{2\sqrt{2}-\sqrt{2}-\sqrt{3}}$$

$$\frac{a+2\sqrt{3}}{a-2\sqrt{3}} = \frac{3\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} \quad \ldots (2)$$

From (1) and (2), we have

$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}} + \frac{a+2\sqrt{3}}{a-2\sqrt{3}} = \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{3\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$

$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}} + \frac{a+2\sqrt{3}}{a-2\sqrt{3}} = \frac{3\sqrt{2}+\sqrt{3}-3\sqrt{3}-\sqrt{2}}{\sqrt{2}-\sqrt{3}}$$

$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}} + \frac{a+2\sqrt{3}}{a-2\sqrt{3}} = \frac{2\sqrt{2}-2\sqrt{3}}{\sqrt{2}-\sqrt{3}} = 2$$

**7. If (a + b + c + d) (a - b - c + d) = (a + b - c - d) (a - b + c - d), prove that a: b = c: d.**
**Solution:**

Rewriting the given, we have

$$\frac{a+b+c+d}{a+b-c-d} = \frac{a-b+c-d}{a-b-c+d}$$

Now, applying componendo and dividendo

$$\frac{(a+b+c+d)+(a+b-c-d)}{(a+b+c+d)-(a+b-c-d)} = \frac{(a-b+c-d)+(a-b-c+d)}{(a-b+c-d)-(a-b-c+d)}$$

$$\frac{2(a+b)}{2(c+d)} = \frac{2(a-b)}{2(c-d)}$$

$$\frac{a+b}{c+d} = \frac{a-b}{c-d}$$

$$\frac{a+b}{a-b} = \frac{c+d}{c-d}$$

Applying componendo and dividendo again, we get

$$\frac{a+b+a-b}{a+b-a+b} = \frac{c+d+c-d}{c+d-c+d}$$

$$\frac{2a}{2b} = \frac{2c}{2d}$$

$$\frac{a}{b} = \frac{c}{d}$$

- Hence Proved

## Exercise 7(D)



**1. If a: b = 3: 5, find:**
**(10a + 3b): (5a + 2b)**
**Solution:**

Given, a/b = 3/5
(10a + 3b)/ (5a + 2b)

$$= \frac{10(a/b)+3}{5(a/b)+2}$$

$$= \frac{10(3/5)+3}{5(3/5)+2}$$

$$= \frac{6+3}{3+2}$$

$$= \frac{9}{5}$$

**2. If 5x + 6y: 8x + 5y = 8: 9, find x: y.**
**Solution:**

Given, $\frac{5x+6y}{8x+5y} = \frac{8}{9}$

On cross multiplying, we get
45x + 54y = 64x + 40y
14y = 19x
Thus,
x/y = 14/19

**3. If (3x - 4y): (2x - 3y) = (5x - 6y): (4x - 5y), find x: y.**
**Solution:**

Given, (3x - 4y): (2x - 3y) = (5x - 6y): (4x - 5y)
This can be rewritten as,

$$\frac{3x-4y}{2x-3y} = \frac{5x-6y}{4x-5y}$$

Applying componendo and dividendo,

$$\frac{3x-4y+2x-3y}{3x-4y-2x+3y} = \frac{5x-6y+4x-5y}{5x-6y-4x+5y}$$

$$\frac{5x-7y}{x-y} = \frac{9x-11y}{x-y}$$

5x – 7y = 9x – 11y
4y = 4x

x/y = 1/1
Thus,
x: y = 1: 1

**4. Find the:**
**(i) duplicate ratio of 2√2: 3√5**
**(ii) triplicate ratio of 2a: 3b**
**(iii) sub-duplicate ratio of $9x^2a^4 : 25y^6b^2$**
**(iv) sub-triplicate ratio of 216: 343**
**(v) reciprocal ratio of 3: 5**
**(vi) ratio compounded of the duplicate ratio of 5: 6, the reciprocal ratio of 25: 42 and the sub-duplicate ratio of 36: 49.**
**Solution:**

(i) Duplicate ratio of 2√2: 3√5 = $(2\sqrt{2})^2: (3\sqrt{5})^2$ = 8: 45
(ii) Triplicate ratio of 2a: 3b = $(2a)^3: (3b)^3 = 8a^3: 27b^3$
(iii) Sub-duplicate ratio of $9x^2a^4: 25y^6b^2 = \sqrt{(9x^2a^4)}: \sqrt{(25y^6b^2)} = 3xa^2: 5y^3b$
(iv) Sub-triplicate ratio of 216: 343 = $(216)^{1/3}: (343)^{1/3}$ = 6: 7
(v) Reciprocal ratio of 3: 5 = 5: 3
(vi) Duplicate ratio of 5: 6 = 25: 36
Reciprocal ratio of 25: 42 = 42: 25
Sub-duplicate ratio of 36: 49 = 6: 7
Required compound ratio = $\frac{25 \times 42 \times 6}{36 \times 25 \times 7} = 1 : 1$

**5. Find the value of x, if:**
**(i) (2x + 3): (5x - 38) is the duplicate ratio of √5: √6.**
**(ii) (2x + 1): (3x + 13) is the sub-duplicate ratio of 9: 25.**
**(iii) (3x - 7): (4x + 3) is the sub-triplicate ratio of 8: 27.**
**Solution:**

(i) (2x + 3): (5x - 38) is the duplicate ratio of √5: √6
And, the duplicate ratio of √5: √6 = 5: 6
So,
(2x + 3)/ (5x - 38) = 5/6
12x + 18 = 25x – 190
25x – 12x = 190 + 18
13x = 208
x = 208/13 = 16

(ii) (2x + 1): (3x + 13) is the sub-duplicate ratio of 9: 25
Then the sub-duplicate ratio of 9: 25 = 3: 5
(2x + 1)/ (3x + 13) = 3/5
10x + 5 = 9x + 39
x = 34

(iii) (3x - 7): (4x + 3) is the sub-triplicate ratio of 8: 27
And the sub-triplicate ratio of 8: 27 = 2: 3
(3x – 7)/ (4x + 3) = 2/3
9x – 8x = 6 + 21
x = 27

**6. What quantity must be added to each term of the ratio x: y so that it may become equal to c: d?**
**Solution:**

Let's assume the required quantity which has to be added be p.
So, we have
$$\frac{x+p}{y+p}=\frac{c}{d}$$
dx + pd = cy + cp
pd – cp = cy – dx
p(d - c) = cy – dx
p = cy –dx/ (d - c)

**7. A woman reduces her weight in the ratio 7: 5. What does her weight become if originally it was 84 kg?**
**Solution:**

Let's consider the woman's reduced weight as x.
Given, the original weight = 84 kg
So, we have
84: x = 7: 5
84/x = 7/5
84 x 5 = 7x
x = (84 x 5)/ 7
x = 60
Therefore, the reduced weight of the woman is 60 kg.

**8. If $15(2x^2 - y^2) = 7xy$, find x: y; if x and y both are positive.**
**Solution:**

$15(2x^2 - y^2) = 7xy$
$$\frac{2x^2-y^2}{xy}=\frac{7}{15}$$
$$\frac{2x}{y}-\frac{y}{x}=\frac{7}{15}$$
Let's take the substitution as x/y = a
2a – 1/a = 7/15
$(2a^2 - 1)/ a = 7/15$
$30a^2 – 15 = 7a$
$30a^2 – 7a – 15 = 0$

$30a^2 – 25a + 18a – 15 = 0$
$5a(6a - 5) + 3(6a - 5) = 0$
$(6a - 5) (5a + 3) = 0$
So, $6a – 5 = 0$ or $5a + 3 = 0$
$a = 5/6$ or $a = -3/5$
As, a cannot be taken negative (ratio)
Thus, $a = 5/6$
$x/y = 5/6$
Hence, x: y = 5: 6

**9. Find the:**
**(i) fourth proportional to 2xy, $x^2$ and $y^2$.**
**(ii) third proportional to $a^2 - b^2$ and a + b.**
**(iii) mean proportional to (x - y) and ($x^3 - x^2y$).**
**Solution:**

(i) Let the fourth proportional to 2xy, $x^2$ and $y^2$ be n.
$2xy: x^2 = y^2: n$
$2xy \times n = x^2 \times y^2$
$n = \frac{x^2y^2}{2xy} = \frac{xy}{2}$

(ii) Let the third proportional to $a^2 - b^2$ and a + b be n.
$a^2 - b^2$, a + b and n are in continued proportion.
$a^2 - b^2: a + b = a + b: n$
$n = \frac{(a+b)^2}{a^2 - b^2} = \frac{(a+b)^2}{(a+b)(a-b)} = \frac{a+b}{a-b}$

(iii) Let the mean proportional to (x - y) and ($x^3 - x^2y$) be n.
(x - y), n, ($x^3 - x^2y$) are in continued proportion
$(x - y): n = n: (x^3 - x^2y)$
$n^2 = (x -y) (x^3 – x^2y)$
$n^2 = (x -y) x^2(x – y)$
$n^2 = x^2 (x – y)^2$
$n = x(x - y)$

**10. Find two numbers such that the mean proportional between them is 14 and third proportional to them is 112.**
**Solution:**

Let's assume the required numbers be a and b.
Given, 14 is the mean proportional between a and b.
a: 14 = 14: b
ab = 196
a = 196/b …. (1)

Also, given, third proportional to a and b is 112.
a: b = b: 112
$b^2 = 112a$ …. (2)

Using (1), we have:
$b^2 = 112 \times (196/b)$
$b^3 = 14^3 \times 2^3$
b = 28
From (1),
a = 196/ 28 = 7
Therefore, the two numbers are 7 and 28.

**11. If x and y be unequal and x: y is the duplicate ratio of x + z and y + z, prove that z is mean proportional between x and y.**
**Solution:**

Given, $\frac{x}{y} = \frac{(x+z)^2}{(y+z)^2}$
$x(y^2 + z^2 + 2yz) = y(x^2 + z^2 + 2xz)$
$xy^2 + xz^2 + 2yzx = x^2y + z^2y + 2xzy$
$xy^2 + xz^2 = x^2y + z^2y$
$xy(y - x) = z^2(y - x)$
$xy = z^2$
Therefore, z is mean proportional between x and y.

**12. If $x = \frac{2ab}{a+b}$, find the value of $\frac{x+a}{x-a} + \frac{x+b}{x-b}$.**
**Solution:**

x = 2ab/ (a + b)
x/a = 2b/(a + b)
Applying componendo and dividendo,
$$\frac{x+a}{x-a} = \frac{2b+a+b}{2b-a-b}$$
$$\frac{x+a}{x-a} = \frac{3b+a}{b-a} \quad \ldots (1)$$
Also, x = 2ab/ (a + b)
x/b = 2a/ (a + b)
Applying componendo and dividendo, we have
$$\frac{x+b}{x-b} = \frac{2a+a+b}{2a-a-b}$$
$$\frac{x+b}{x-b} = \frac{3a+b}{a-b} \quad \ldots (2)$$
Now, comparing (1) and (2) we have

$$\frac{x+a}{x-a}+\frac{x+b}{x-b}=\frac{3b+a}{b-a}+\frac{3a+b}{a-b}$$
$$\frac{x+a}{x-a}+\frac{x+b}{x-b}=\frac{-3b-a+3a+b}{a-b}$$
$$\frac{x+a}{x-a}+\frac{x+b}{x-b}=\frac{2a-2b}{a-b}=2$$

**13. If (4a + 9b) (4c - 9d) = (4a - 9b) (4c + 9d), prove that:**
**a: b = c: d.**
**Solution:**

Given, $\frac{4a+9b}{4a-9b}=\frac{4c+9d}{4c-9d}$

Applying componendo and dividendo, we get

$$\frac{4a+9b+4a-9b}{4a+9b-4a+9b}=\frac{4c+9d+4c-9d}{4c+9d-4c+9d}$$

8a/18b = 8c/18d

a/b = c/d

- Hence Proved