Exercise 9(D)

1. Find x and y, if:

$$\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 2x \\ 1 \end{bmatrix} + 2\begin{bmatrix} -4 \\ 5 \end{bmatrix} = 4\begin{bmatrix} 2 \\ y \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 2x \\ 1 \end{bmatrix} + 2\begin{bmatrix} -4 \\ 5 \end{bmatrix} = 4\begin{bmatrix} 2 \\ y \end{bmatrix}$$
$$\begin{bmatrix} 6x-2 \\ -2x+4 \end{bmatrix} + \begin{bmatrix} -8 \\ 10 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$$
$$\begin{bmatrix} 6x-2-8 \\ -2x+4+10 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$$
$$\begin{bmatrix} 6x-10 \\ -2x+14 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$$

On comparing the corresponding terms, we have

6x – 10 = 8 and -2x + 14 = 4y
6x = 18 and y = (14 - 2x)/4
x = 3 and y = (14 – 2(3))/4
y = (14 - 6)/ 4
y = 8/4 = 2

Thus, x = 3 and y = 2

2. Find x and y, if:

$$\begin{bmatrix} 3x & 8 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 3 & 7 \end{bmatrix} - 3\begin{bmatrix} 2 & -7 \end{bmatrix} = 5\begin{bmatrix} 3 & 2y \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 3x & 8 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 3 & 7 \end{bmatrix} - 3\begin{bmatrix} 2 & -7 \end{bmatrix} = 5\begin{bmatrix} 3 & 2y \end{bmatrix}$$
$$\begin{bmatrix} 3x+24 & 12x+56 \end{bmatrix} - \begin{bmatrix} 6 & -21 \end{bmatrix} = \begin{bmatrix} 15 & 10y \end{bmatrix}$$
$$\begin{bmatrix} 3x+24-6 & 12x+56+21 \end{bmatrix} = \begin{bmatrix} 15 & 10y \end{bmatrix}$$
$$\begin{bmatrix} 3x+18 & 12x+77 \end{bmatrix} = \begin{bmatrix} 15 & 10y \end{bmatrix}$$

On comparing the corresponding terms, we have

3x + 18 = 15 and 12x + 77 = 10y
3x = -3 and y = (12x + 77)/10
x = -1 and y = (12(-1) + 77)/10
y = 65/10 = 6.5

Thus, x = -1 and y = 6.5

3. If; $\begin{bmatrix} x & y \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 25 \end{bmatrix}$ and $\begin{bmatrix} -x & y \end{bmatrix}\begin{bmatrix} 2x \\ y \end{bmatrix} = \begin{bmatrix} -2 \end{bmatrix}$ **; find x and y, if:**

(i) x, y ∈ W (whole numbers)
(ii) x, y ∈ Z (integers)
Solution:

From the question, we have
$x^2 + y^2 = 25$ and $-2x^2 + y^2 = -2$

(i) x, y ∈ W (whole numbers)
It can be observed that the above two equations are satisfied when x = 3 and y = 4.

(ii) x, y ∈ Z (integers)
It can be observed that the above two equations are satisfied when x = ± 3 and y = ± 4.

4. $Given \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} .X = \begin{bmatrix} 7 \\ 6 \end{bmatrix} .Write :$

(i) The order of the matrix X.
(ii) The matrix X.
Solution:

(i) Let the order of the matrix be a x b.
Then, we know that

$$\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}_{2X\underline{2}} .X_{\underline{a}Xb} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}_{2X1}$$

Thus, for multiplication of matrices to be possible
a = 2
And, form noticing the order of the resultant matrix
b = 1

(ii)

$$\text{Let} X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x + y \\ -3x + 4y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

On comparing the corresponding terms, we have
2x + y = 7 and
-3x + 4y = 6
Solving the above two equations, we have

x = 2 and y = 3

Thus, the matrix X is $X = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

5. Evaluate:

$$\begin{bmatrix} cos45^o & sin30^o \\ \sqrt{2}cos0^o & sin0^o \end{bmatrix} \begin{bmatrix} sin45^o & cos90^o \\ sin90^o & cot45^o \end{bmatrix}$$

Solution:

$$= \begin{bmatrix} cos45^o & sin30^o \\ \sqrt{2}cos0^o & sin0^o \end{bmatrix} \begin{bmatrix} sin45^o & cos90^o \\ sin90^o & cot45^o \end{bmatrix}$$

$$= \begin{bmatrix} cos45^o sin45^o + sin30^o sin90^o & cos45^o cos90^o + sin30^o cot45^o \\ \sqrt{2}cos0^o sin45^o + sin0^o sin90^o & \sqrt{2}cos0^o cos90^o + sin0^o cot45^o \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 + 1/2 & 0 + 1/2 \\ 1 & 0 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0.5 \\ 1 & 0 \end{bmatrix}$$

6. $If A = \begin{bmatrix} 0 & -1 \\ 4 & -3 \end{bmatrix}, B = \begin{bmatrix} -5 \\ 6 \end{bmatrix}$ and **3A x M = 2B; find matrix M.**

Solution:

Given,

3A x M = 2B

And let the order of the matric of M be (a x b)

$$3\begin{bmatrix} 0 & -1 \\ 4 & -3 \end{bmatrix}_{2\times 2} \times M_{a\times b} = 2\begin{bmatrix} -5 \\ 6 \end{bmatrix}_{2\times 1}$$

Now, it's clearly seen that

a = 2 and b = 1

So, the order of the matrix M is (2 x 1)

$$3\begin{bmatrix} 0 & -1 \\ 4 & -3 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = 2\begin{bmatrix} -5 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -3 \\ 12 & -9 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -10 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} -3y \\ 12x - 9y \end{bmatrix} = \begin{bmatrix} -10 \\ 12 \end{bmatrix}$$

Now, on comparing with corresponding elements we have

-3y = -10 and 12x – 9y = 12

y = 10/3 and 12x – 9(10/3) = 12
12x – 30 = 12
12x = 42
x = 42/12 = 7/2

Therefore,

Matrix M = $\begin{bmatrix} 7/2 \\ 10/3 \end{bmatrix}$

7. $If \begin{bmatrix} a & 3 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & b \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix},$

find the values of a, b and c.
Solution:

$$\begin{bmatrix} a & 3 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & b \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$
$$\begin{bmatrix} a+2 & 3+b \\ 4+1 & 1-2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$
$$\begin{bmatrix} a+2-1 & 3+b-1 \\ 4+1+2 & 1-2-c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$
$$\begin{bmatrix} a+1 & b+2 \\ 7 & -1-c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$

On comparing the corresponding elements, we have
a + 1 = 5 ⇒ a = 4
b + 2 = 0 ⇒ b = -2
-1 – c = 3 ⇒ c = -4

8. $If A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} and\ B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}; find:$

(i) A (BA) **(ii) (AB) B.**
Solution:

(i) A (BA) $= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \left(\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \right)$

$$= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \left(\begin{bmatrix} 2+2 & 4+1 \\ 1+4 & 2+2 \end{bmatrix} \right)$$
$$= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$
$$= \begin{bmatrix} 4+10 & 5+8 \\ 8+5 & 10+4 \end{bmatrix}$$
$$= \begin{bmatrix} 14 & 13 \\ 13 & 14 \end{bmatrix}$$

(ii) (AB) B $= \left(\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}\right)\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}$

$$= \begin{bmatrix}2+2 & 1+4\\4+1 & 2+2\end{bmatrix}\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}$$

$$= \begin{bmatrix}4 & 5\\5 & 4\end{bmatrix}\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix} = \begin{bmatrix}4+10 & 8+5\\5+8 & 10+4\end{bmatrix}$$

$$= \begin{bmatrix}14 & 13\\13 & 14\end{bmatrix}$$

9. Find x and y, if: $\begin{bmatrix}x & 3x\\y & 4y\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}5\\12\end{bmatrix}$

Solution:

$$\begin{bmatrix}x & 3x\\y & 4y\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}5\\12\end{bmatrix}$$

$$\begin{bmatrix}2x+3x\\2y+4y\end{bmatrix} = \begin{bmatrix}5\\12\end{bmatrix}$$

Thus, on comparing the corresponding terms, we have

2x + 3x = 5 and 2y + 4y = 12

5x = 5 and 6y = 12

x = 1 and y = 2

10. If matrix $X = \begin{bmatrix}-3 & 4\\2 & -3\end{bmatrix}\begin{bmatrix}2\\-2\end{bmatrix}$ $and\ 2X - 3Y = \begin{bmatrix}10\\-8\end{bmatrix}$, **find the matrix 'X' and matrix 'Y'.**

Solution:

$$X = \begin{bmatrix}-3 & 4\\2 & -3\end{bmatrix}\begin{bmatrix}2\\-2\end{bmatrix} = \begin{bmatrix}-6-8\\4+6\end{bmatrix} = \begin{bmatrix}-14\\10\end{bmatrix}$$

Now,

$$Let\ Y = \begin{bmatrix}x\\y\end{bmatrix}$$

$$2X - 3Y = 2\begin{bmatrix}-14\\10\end{bmatrix} - 3\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}10\\-8\end{bmatrix}$$

$$\begin{bmatrix}-28\\20\end{bmatrix} - \begin{bmatrix}3x\\3y\end{bmatrix} = \begin{bmatrix}10\\-8\end{bmatrix}$$

$$\begin{bmatrix}-28-3x\\20-3y\end{bmatrix} = \begin{bmatrix}10\\-8\end{bmatrix}$$

On comparing with the corresponding terms, we have

-28 – 3x = 10

3x = -38
x = -38/3
And,
20 – 3y = -8
3y = 28
y = 28/3
Therefore,

$$Y = 1/3\begin{bmatrix}-38\\28\end{bmatrix}$$

11. Given $A = \begin{bmatrix}2 & -1\\2 & 0\end{bmatrix}, B = \begin{bmatrix}-3 & 2\\4 & 0\end{bmatrix}\ and\ \ C = \begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}$ **find the matrix X such that:**

A + X = 2B + C
Solution:

$$\begin{bmatrix}2 & -1\\2 & 0\end{bmatrix} + X = 2\begin{bmatrix}-3 & 2\\4 & 0\end{bmatrix} + \begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}$$
$$\begin{bmatrix}2 & -1\\2 & 0\end{bmatrix} + X = \begin{bmatrix}-6 & 4\\8 & 0\end{bmatrix} + \begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}$$
$$\begin{bmatrix}2 & -1\\2 & 0\end{bmatrix} + X = \begin{bmatrix}-5 & 4\\8 & 2\end{bmatrix}$$
$$X = \begin{bmatrix}-5 & 4\\8 & 2\end{bmatrix} - \begin{bmatrix}2 & -1\\2 & 0\end{bmatrix}$$
$$X = \begin{bmatrix}-7 & 5\\6 & 2\end{bmatrix}$$

12. Find the value of x, given that A^2 = B,

$$A = \begin{bmatrix}2 & 12\\0 & 1\end{bmatrix}\ and\ B = \begin{bmatrix}4 & x\\0 & 1\end{bmatrix}$$

Solution:

$$A^2 = \begin{bmatrix}2 & 12\\0 & 1\end{bmatrix}\begin{bmatrix}2 & 12\\0 & 1\end{bmatrix} = \begin{bmatrix}4 & x\\0 & 1\end{bmatrix} = B$$
$$A^2 = \begin{bmatrix}4+0 & 24+12\\0 & 1\end{bmatrix} = \begin{bmatrix}4 & x\\0 & 1\end{bmatrix} = B$$
$$A^2 = \begin{bmatrix}4 & 36\\0 & 1\end{bmatrix} = \begin{bmatrix}4 & x\\0 & 1\end{bmatrix} = B$$

Thus, on comparing the terms we get x = 36.