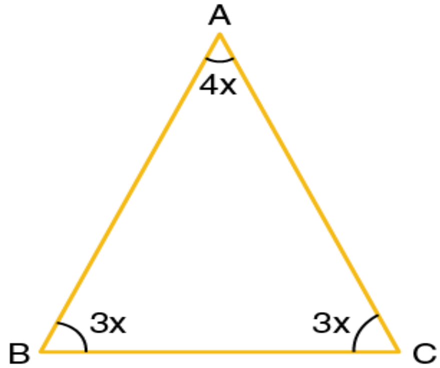
1. Find the angles of an isosceles triangle whose equal angles and the non-equal angles are in the ratio 3:4.

Solution:



Given that,

The equal angles and the non-equal angles are in the ratio 3:4

Let the equal angles be 3x each,

So, non- equal angle is 4x

We know that,

Sum of angles of a triangle = 180^0

Hence,

$3x + 3x + 4x = 180^0$

$10x = 180^0$

We get,

$x = 18^0$

Therefore,

$3x = 3 \times 18^0 = 54^0$ and

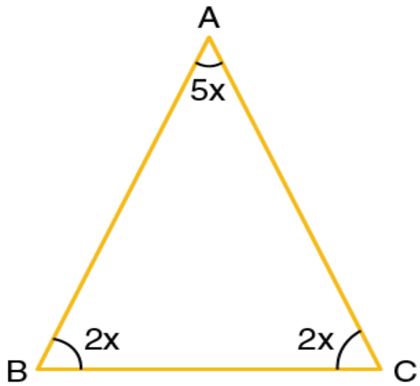
$4x = 4 \times 18^0 = 72^0$

Hence,

The angles of a triangle are 54^0, 54^0 and 72^0

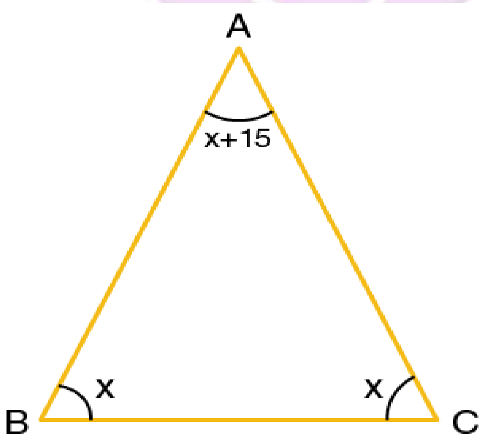
2. Find the angles of an isosceles triangle which are in the ratio 2:2:5

Solution:



The equal angles and the non-equal angle are in the ratio 2:2:5
Let equal angles be 2x each
So, non-equal angle is 5x
We know that,
Sum of angles of a triangle = 180^0
$2x + 2x + 5x = 180^0$
$9x = 180^0$
$x = 20^0$
Therefore,
$2x = 2 \times 20^0 = 40^0$
$5x = 5 \times 20^0 = 100^0$
Hence, the angles of a triangle are 40^0, 40^0 and 100^0

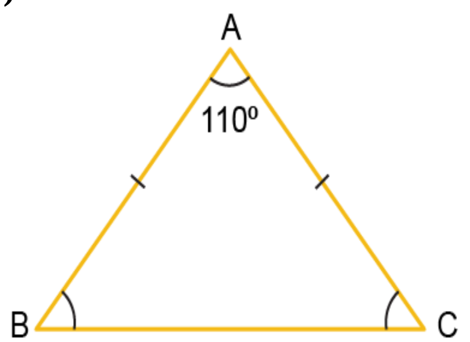
3. Each equal angle of an isosceles triangle is less than the third angle by 15^0. Find the angles.
Solution:



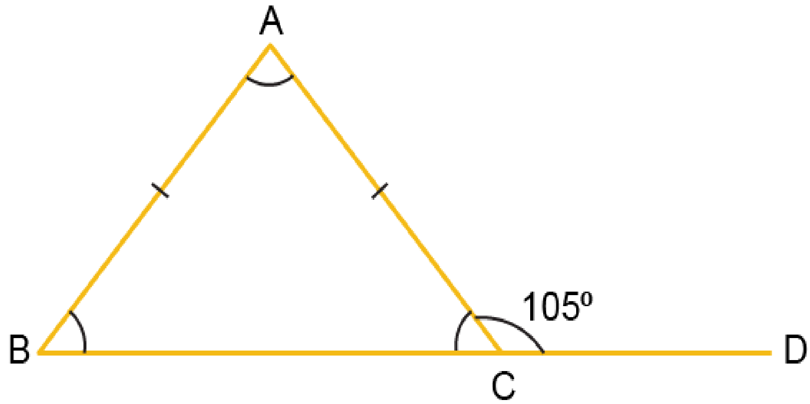
Let equal angles of the isosceles triangle be x each
Therefore, non-equal angle is $x + 15^0$
We know that,
Sum of angles of a triangle = 180^0

$x + x + (x + 15^0) = 180^0$
$3x + 15^0 = 180^0$
$3x = 180^0 - 15^0$
$3x = 165^0$
We get,
$x = 55^0$
So,
$(x + 15^0) = 55^0 + 15^0 = 70^0$
Hence, the angles of a triangle are 55^0, 55^0 and 70^0

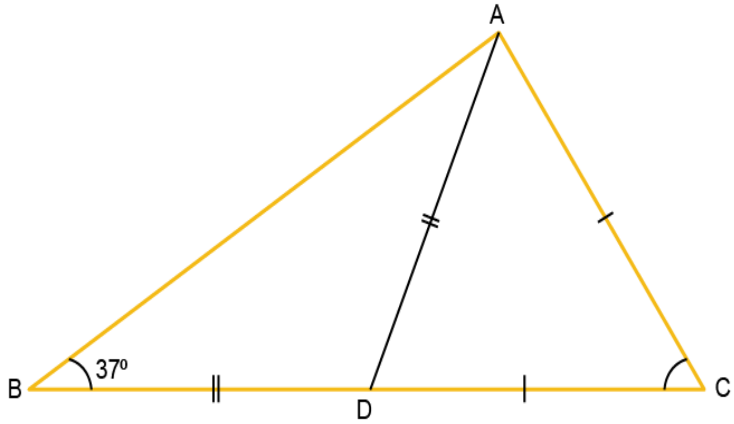
4. Find the interior angles of the following triangles
(a)



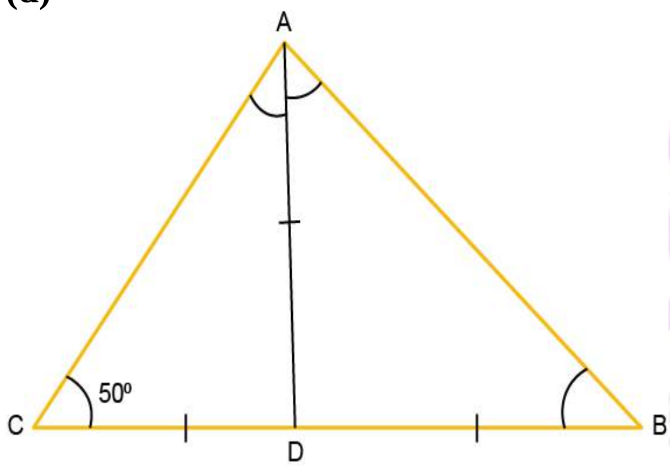
(b)



(c)

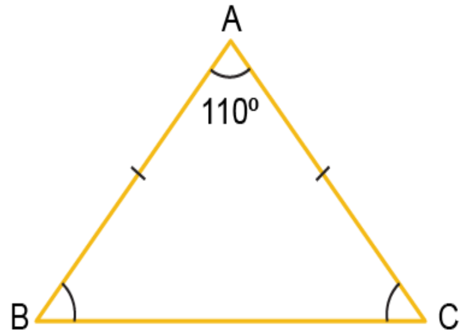


(d)



Solution:

(a)



In △ABC,

$\angle A = 110^0$

AB = AC

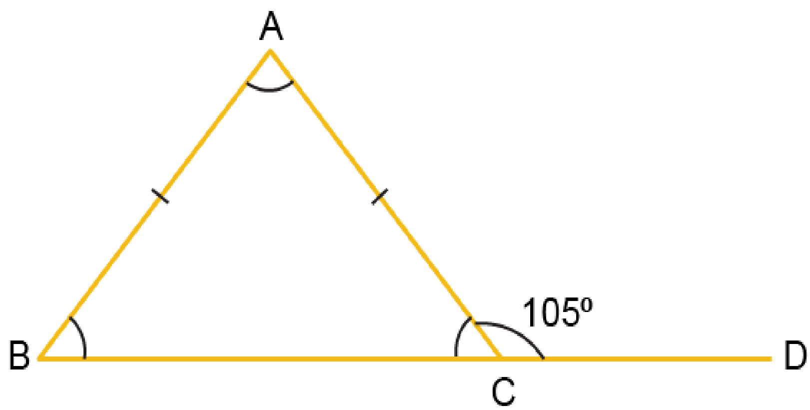
$\angle C = \angle B$ (angles opposite to two equal sides are equal)

Now,

By angle sum property,

$\angle A + \angle B + \angle C = 180^0$
$\angle A + \angle B + \angle B = 180^0$
$110^0 + 2\angle B = 180^0$
$2\angle B = 180^0 - 110^0$
$2\angle B = 70^0$
We get,
$\angle B = 35^0$
$\angle C = 35^0$
Hence,
The interior angles are $\angle B = 35^0$ and $\angle C = 35^0$

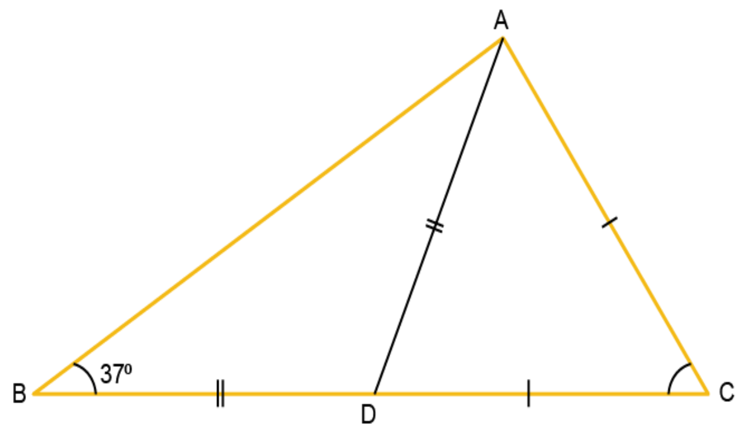
(b)



In $\triangle ABC$,
$AB = AC$
$\angle ACB = \angle ABC$ ....(1) [∵ angles opposite to two equal sides are equal]
Now,
$\angle ACB + \angle ACD = 180^0$ [linear pair]
$\angle ACB = 180^0 - \angle ACD$
$\angle ACB = 180^0 - 105^0$
$\angle ACB = 75^0$
So,
$\angle ABC = 75^0$ [from equation (1)]
Now, in $\triangle ABC$,
By angle sum property,
$\angle ABC + \angle ACB + \angle BAC = 180^0$
$75^0 + 75^0 + \angle BAC = 180^0$
$150^0 + \angle BAC = 180^0$
$\angle BAC = 180^0 - 150^0$

We get,
∠BAC = 30^0
Hence,
In △ABC,
∠A = 30^0
∠B = 75^0
∠C = 75^0

(c)



In △ABD,
Given that,
AD = BD
∠ABD = ∠BAD … (angles opposite to two equal sides are equal)
Now,
∠ABD = 37^0 …. (given)
Hence,
∠BAD = 37^0

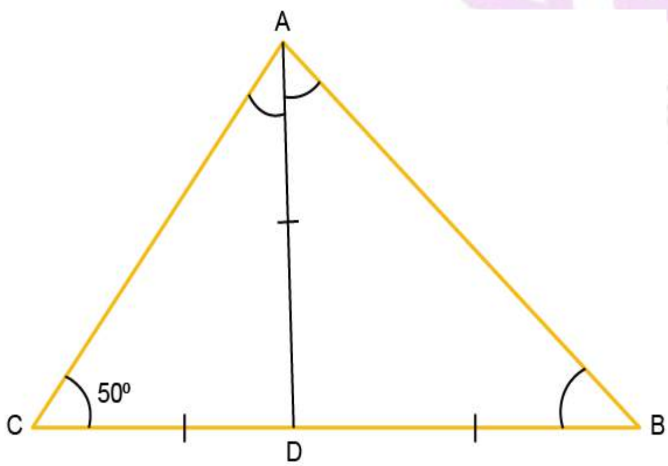
By exterior angle property,
∠ADC = ∠ABD + ∠BAD
∠ADC = $37^0 + 37^0$
We get,
∠ADC = 74^0

In △ADC,
AC = DC …. (given)
∠ADC = ∠DAC …. (angles opposite to two equal sides are equal)
∠DAC = 74^0

Now,
∠BAC = ∠BAD + ∠DAC
∠BAC = $37^0 + 74^0$
We get,
∠BAC = 111^0

In △ABC,
∠BAC + ∠ABC + ∠ACB = 180^0
$111^0 + 37^0$ + ∠ACB = 180^0
∠ACB = $180^0 - 111^0 - 37^0$
We get,
∠ACB = 32^0
Therefore,
The interior angles of △ABC are 37^0, 111^0 and 32^0

(d)



In △ACD,
AD = CD …… (given)
∠ACD = ∠CAD … (angles opposite to two equal sides are equal)
Now,
∠ACD = 50^0 …… (given)
∠CAD = 50^0

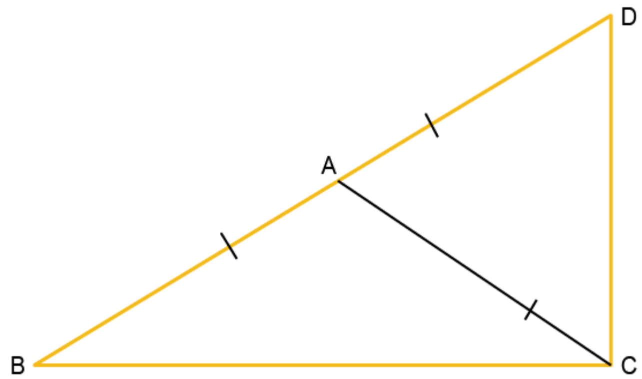
By exterior angle property,
∠ADB = ∠ACD + ∠CAD
∠ADB = $50^0 + 50^0$

$\angle ADB = 100^0$

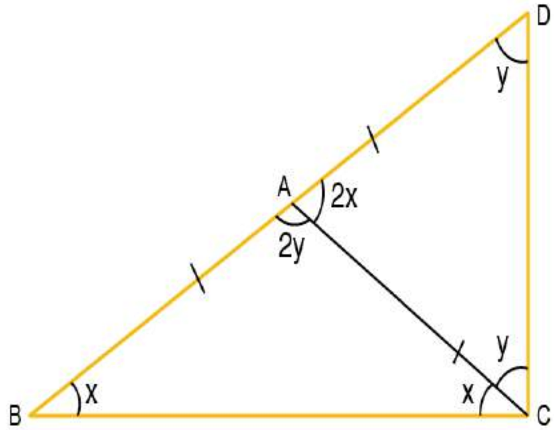
In $\triangle ADB$,
AD = BD …… (given)
$\angle DBA = \angle DAB$ ….. (angles opposite to two equal sides are equal)
Also,
$\angle ADB + \angle DBA + \angle DAB = 180^0$
$100^0 + 2\angle DBA = 180^0$
$2\angle DBA = 180^0 - 100^0$
$2\angle DBA = 80^0$
We get,
$\angle DBA = 40^0$
$\angle DAB = 40^0$

$\angle BAC = \angle DAB + \angle CAD$
$\angle BAC = 40^0 + 50^0$
$\angle BAC = 90^0$
Therefore, the interior angles of $\triangle ABC$ are 50^0, 90^0 and 40^0

5. Side BA of an isosceles triangle ABC is produced so that AB = AD. If AB and AC are the equal sides of the isosceles triangle, prove that ∠BCD is a right angle.



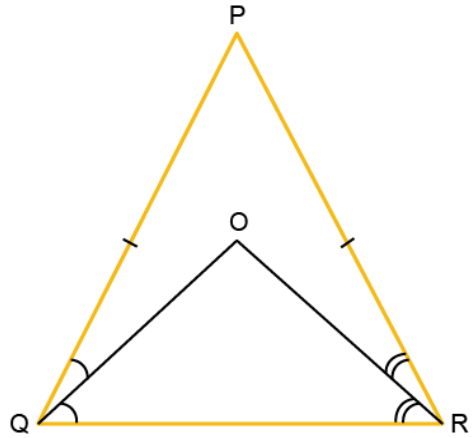
Solution:



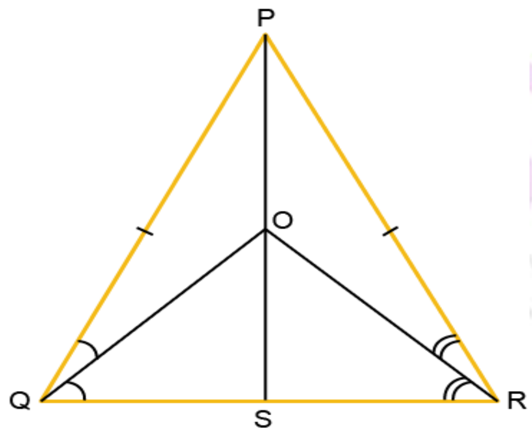
Let ∠ABC = x
Hence,
∠BCA = x (since AB = AC)
In △ABC,
∠ABC + ∠BCA + ∠BAC = 180^0 ……. (1)
But
∠BAC + ∠DAC = 180^0 ……. (2)
From equations (1) and (2)
∠ABC + ∠BCA + ∠BAC = ∠BAC + ∠DAC
∠DAC = ∠ABC + ∠BCA
∠DAC = x + x
We get,
∠DAC = 2x
Let ∠ADC = y,
Hence,
∠DCA = y (since AD = AC)
Now,
In △ADC,
∠ADC + ∠DCA + ∠DAC = 180^0 … (3)
But ∠BAC + ∠DAC = 180^0 ……. (4)
From equations (3) and (4), we get,
∠ADC + ∠DCA + ∠DAC = ∠BAC + ∠DAC
∠BAC = ∠ADC + ∠DCA
∠BAC = y + y
∠BAC = 2y
Now, substituting the value of ∠BAC and ∠DAC in equation (2)
2x + 2y = 180^0
x + y = 90^0
∠BCA + ∠DCA = 90^0

Therefore,
∠BCD is a right angle

6. The bisectors of the equal angles of an isosceles triangle PQR meet at O. If PQ = PR, prove that PO bisects ∠P.



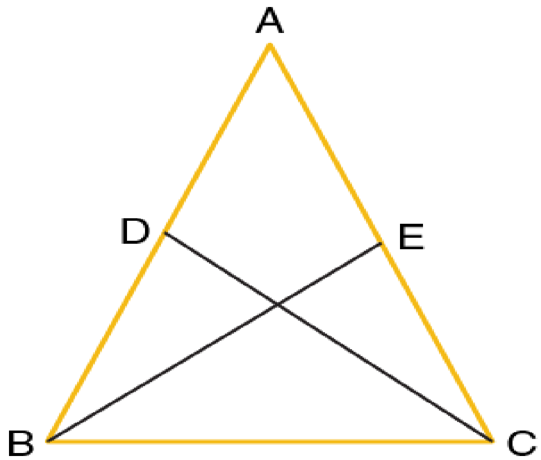
Solution:



Join PO and produce to meet QR at point S
In △PQS and △PRS
PS = PS (common)
PQ = PR (given)
So,
∠Q = ∠R (angles opposite to two equal sides are equal)
Hence,
△PQS ≅ △PRS
Thus,
∠QPS = ∠RPS
Therefore,
PO bisects ∠P

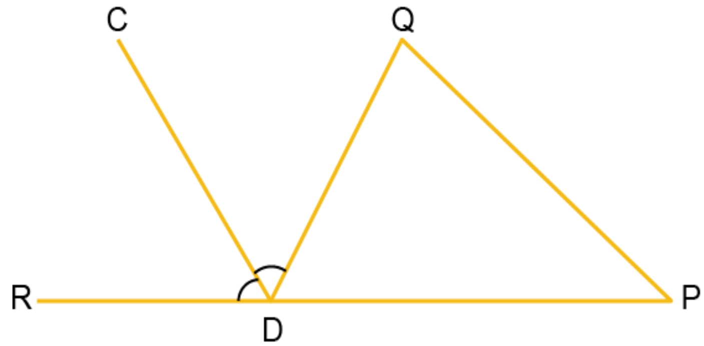
7. Prove that the medians corresponding to equal sides of an isosceles triangle are equal.

Solution:



Let △ABC be an isosceles triangle with AB = AC
Let D and E be the mid points of AB and AC respectively
Now,
Join BE and CD
Then BE and CD become the medians of this isosceles triangle
In △ABE and △ACD
AB = AC (given)
AD = AE (D and E are mid points of AB and AC)
∠A = ∠A (common angle)
Hence,
△ABE ≅ △ACD (SAS criteria)
Therefore,
The medians BE and CD are equal i.e BE = CD

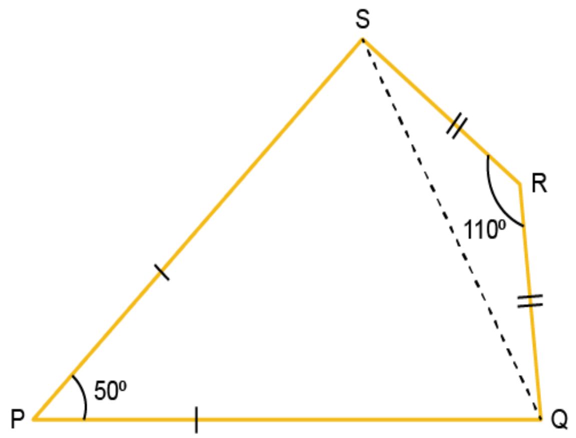
8. DPQ is an isosceles triangle with DP = DQ. A straight line CD bisects the exterior ∠QDR. Prove that DC is parallel to PQ



Solution:

In △QDP,
DP = DQ
Hence,
∠Q = ∠P (angles opposite to two equal sides are equal)
∠QDR = ∠Q + ∠P
2∠QDC = ∠Q + ∠P (DC bisects angle QDR)
2∠QDC = ∠Q + ∠Q
We get,
2∠QDC = 2∠Q
Hence,
∠QDC = ∠Q
But these angles are alternate angles
Therefore,
DC || PQ
Hence, proved

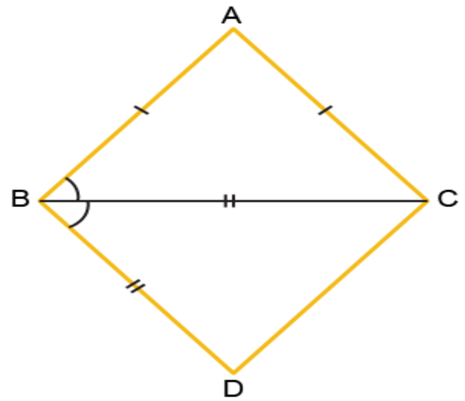
9. In a quadrilateral PQRS, PQ = PS and RQ = RS. If ∠P = 50^0 and ∠R = 110^0, find ∠PSR.



Solution:
In △PQS,
PQ = PS
Therefore,
∠PQS = ∠PSQ (angles opposite to two equal sides are equal)
∠P + ∠PQS + ∠PSQ = 180^0
50^0 + 2∠PQS = 180^0
2∠PQS = 180^0 - 50^0
We get,
2∠PQS = 130^0
∠PQS = 65^0

So,
∠PQS = ∠PSQ = 65^0 …….. (1)
In △SRQ,
SR = RQ
Hence,
∠RQS = ∠RSQ (angles opposite to two equal sides are equal)
∠R + ∠RQS + ∠RSQ = 180^0
110^0 + 2∠RQS = 180^0
2∠RQS = 180^0 - 110^0
We get,
2∠RQS = 70^0
∠RQS = 35^0
So,
∠RQS = ∠RSQ = 35^0 ….. (2)
Adding equations (1) and (2), we get,
∠PSQ + ∠RSQ = 65^0 + 35^0
∠PSR = 100^0

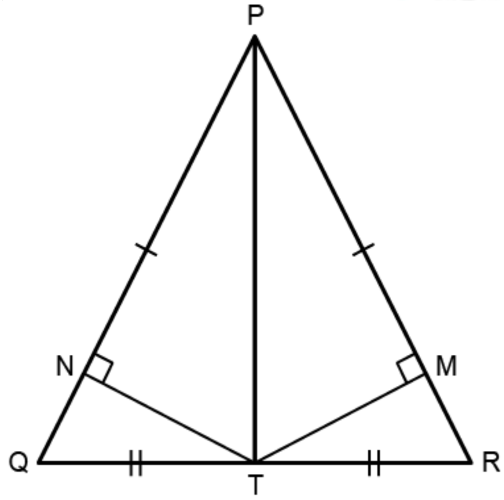
10. △ABC is an isosceles triangle with AB = AC. Another triangle BDC is drawn with base BC = BD in such a way that BC bisects ∠B. If the measure of ∠BDC is 70^0, find the measures of ∠DBC and ∠BAC.



Solution:
In △BDC,
∠BDC = 70^0
BD = BC
Hence,
∠BDC = ∠BCD (angles opposite to two equal sides are equal)
∠BCD = 70^0
Now,

$\angle BCD + \angle BDC + \angle DBC = 180^0$
$70^0 + 70^0 + \angle DBC = 180^0$
$\angle DBC = 180^0 - 140^0$
We get,
$\angle DBC = 40^0$
$\angle DBC = \angle ABC$ (BC is the angle bisector)
Hence,
$\angle ABC = 40^0$
In $\triangle ABC$,
Since AB = AC, $\angle ABC = \angle ACB$
Hence,
$\angle ACB = 40^0$
$\angle ACB + \angle ABC + \angle BAC = 180^0$
$40^0 + 40^0 + \angle BAC = 180^0$
$\angle BAC = 180^0 - 80^0$
$\angle BAC = 100^0$
Therefore, the measure of $\angle BAC = 100^0$ and $\angle DBC = 40^0$

11. $\triangle$PQR is isosceles with PQ = PR. T is the mid-point of QR, and TM and TN are perpendiculars on PR and PQ respectively. Prove that,
(i) TM = TN
(ii) PM = PN and
(iii) PT is the bisector of $\angle$P



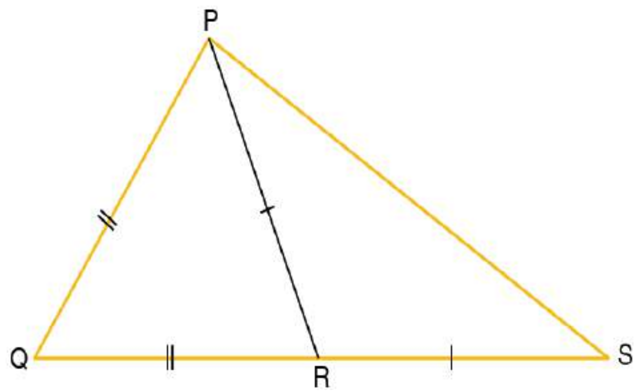
Solution:
(i) In $\triangle$PQR,
PQ = PR
Hence,

$\angle R = \angle Q$ …… (1)
Now,
In $\triangle QNT$ and $\triangle RMT$
$\angle QNT = \angle RMT = 90^0$
$\angle Q = \angle R$ [from equation (1)]
QT = TR (given)
Hence,
$\triangle QNT \cong \triangle RMT$ (AAS criteria)
Therefore,
TM = TN

(ii) Since, $\triangle QNT \cong \triangle RMT$
NQ = MR ……… (2)
But,
PQ = PR ….. (3) [given]
Now, subtracting (2) from (3), we get,
PQ – NQ = PR - MR
PN = PM

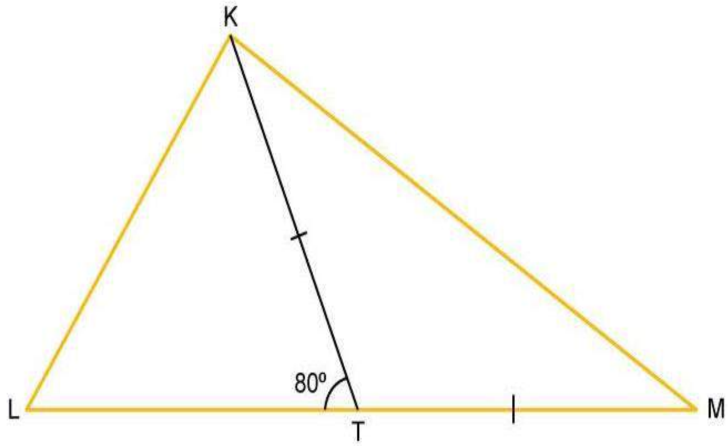
(iii) In $\triangle PNT$ and $\triangle PMT$
TN = TM (proved)
PT = PT (common)
$\angle PNT = \angle PMT = 90^0$
Hence,
$\triangle PNT \cong \triangle PMT$
So,
$\angle NPT = \angle MPT$
Therefore,
PT is the bisector of $\angle P$

12. $\triangle PQR$ is isosceles with PQ = QR. QR is extended to S so that $\triangle PRS$ becomes isosceles with PR = PS. Show that $\angle PSR : \angle QPS = 1:3$
Solution:



In △PQR,
PQ = QR (given)
∠PRQ = ∠QPR …… (1)
In △PRS,
PR = RS (given)
∠PSR = ∠RPS …….. (2)
Now,
Adding equations (1) and (2), we get,
∠QPR + ∠RPS = ∠PRQ + ∠PSR
∠QPS = ∠PRQ + ∠PSR …….. (3)
Now,
In △PRS,
∠PRQ = ∠RPS + ∠PSR
∠PRQ = ∠PSR + ∠PSR [from equation(2)]
∠PRQ = 2∠PSR …… (4)
Now,
∠QPS = 2∠PSR + ∠PSR [from equation (3) and (4)]
∠QPS = 3∠PSR
∠PSR / ∠QPS = 1 / 3
Therefore,
∠PSR: ∠QPS = 1: 3
Hence, proved

13. In △KLM, KT bisects ∠LKM and KT = TM. If ∠LTK is 80^0, find the value of ∠LMK and ∠KLM.



Solution:

In △KTM,

KT = TM (given)

Hence,

∠TKM = ∠TMK ……. (1)

Now,

∠KTL = ∠TKM + ∠TMK

80^0 = ∠TKM + ∠TKM …….. [from (1)]

80^0 = 2∠TKM

We get,

∠TKM = 40^0 = ∠TMK = ∠LMK …… (2)

But,

∠TKM = ∠TKL (KT is the angle bisector)

Hence,

∠TKL = 40^0

In △KTL,

∠TKL + ∠KTL + ∠KLT = 180^0

$40^0 + 80^0$ + ∠KLT = 180^0

∠KLT = $180^0 – 40^0 - 80^0$

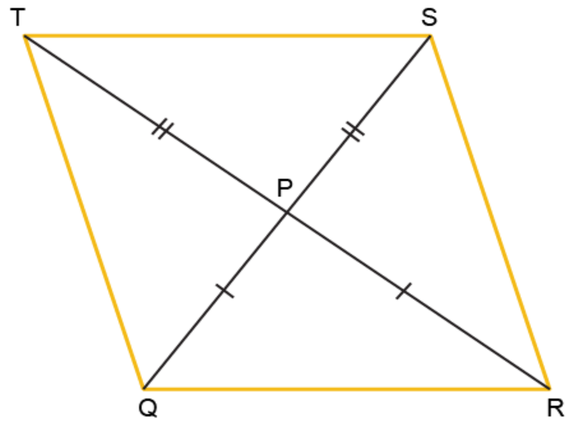
We get,

∠KLT = 60^0 = ∠KLM

Therefore,

∠KLM = 60^0 and ∠LMK = 40^0

14. Equal sides QP and RP of an isosceles △PQR are produced beyond P to S and T such that △PST is an isosceles triangle with PS = PT. Prove that TQ = SR.



Solution:

In △PTQ and △PSR

PQ = PR (given)

PT = PS (given)

∠TPQ = ∠SPR (vertically opposite angles)

Hence,

△PTQ ≅ △PSR

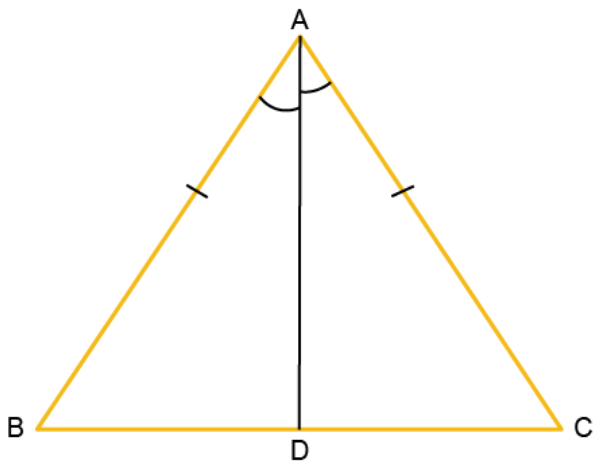
Therefore,

TQ = SR

Hence, proved

15. Prove that the bisector of the vertex angle of an isosceles triangle bisects the base perpendicularly.

Solution:



In △ADB and △ADC

AB = AC (given)

AD = AD (common)

∠BAD = ∠CAD (AD bisects ∠BAC)

Hence,
$\triangle ADB \cong \triangle ADC$
Therefore,
BD = DC and $\angle BDA = \angle CDA$
But,
$\angle BDA + \angle CDA = 180^0$
$\angle BDA = \angle CDA = 90^0$
Therefore,
AD bisects BC perpendicularly
Hence, proved