1. Name the greatest and the smallest sides in the following triangles:
(a) $\triangle ABC$, $\angle A = 56^0$, $\angle B = 64^0$ and $\angle C = 60^0$
(b) $\triangle DEF$, $\angle D = 32^0$, $\angle E = 56^0$ and $\angle F = 92^0$
(c) $\triangle XYZ$, $\angle X = 76^0$, $\angle Y = 84^0$
Solution:
(a) In the given $\triangle ABC$,
The greatest angle is $\angle B$ and the opposite side to the $\angle B$ is AC
Therefore,
The greatest side is AC
The smallest angle in the $\triangle ABC$ is $\angle A$ and the opposite side to the $\angle A$ is BC
Therefore,
The smallest side is BC

(b) In the given $\triangle DEF$,
The greatest angle is $\angle F$ and the opposite side to the $\angle F$ is DE
Therefore,
The greatest side is DE
The smallest angle in the $\triangle DEF$ is $\angle D$ and the opposite side to the $\angle D$ is EF
Therefore,
The smallest side is EF

(c) In $\triangle XYZ$,
$\angle X + \angle Y + \angle Z = 180^0$
$76^0 + 84^0 + \angle Z = 180^0$
$160^0 + \angle Z = 180^0$
$\angle Z = 180^0 - 160^0$
We get,
$\angle Z = 20^0$
Therefore,
$\angle X = 76^0$, $\angle Y = 84^0$, $\angle Z = 20^0$
In the given $\triangle XYZ$,
The greatest angle is $\angle Y$ and the opposite side to the $\angle Y$ is XZ
Therefore,
The greatest side is XZ
The smallest angle in the $\triangle XYZ$ is $\angle Z$ and the opposite side to the $\angle Z$ is XY
Therefore,
The smallest side is XY

2. Arrange the sides of the following triangles in an ascending order:

(a) $\triangle ABC$, $\angle A = 45^0$, $\angle B = 65^0$
(b) $\triangle DEF$, $\angle D = 38^0$, $\angle E = 58^0$
Solution:
(a) In $\triangle ABC$,
$\angle A + \angle B + \angle C = 180^0$
$45^0 + 65^0 + \angle C = 180^0$
$110^0 + \angle C = 180^0$
$\angle C = 180^0 - 110^0$
We get,
$\angle C = 70^0$
Hence,
$\angle A = 45^0$, $\angle B = 65^0$, $\angle C = 70^0$
$45^0 < 65^0 < 70^0$
Hence,
Ascending order of the angles in the given triangle is,
$\angle A < \angle B < \angle C$
Therefore,
Ascending order of sides in triangle is,
BC, AC, AB

(b) In $\triangle DEF$,
$\angle D + \angle E + \angle F = 180^0$
$38^0 + 58^0 + \angle F = 180^0$
$96^0 + \angle F = 180^0$
$\angle F = 180^0 - 96^0$
We get,
$\angle F = 84^0$
Hence,
$\angle D = 38^0$, $\angle E = 58^0$, $\angle F = 84^0$
$38^0 < 58^0 < 84^0$
Hence,
Ascending order of the angles in the given triangle is,
$\angle D < \angle E < \angle F$
Therefore,
Ascending order of sides in triangle is,
EF, DF, DE

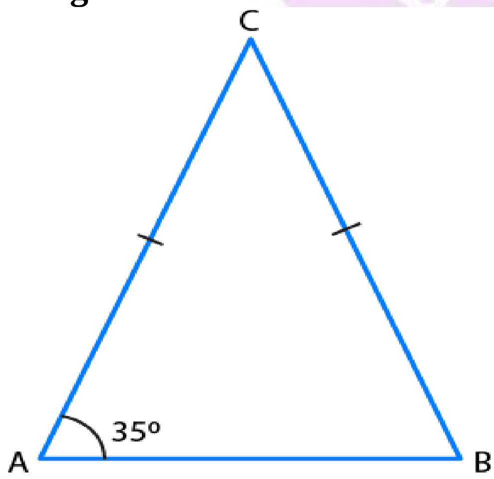
3. Name the smallest angle in each of these triangles:
(i) In $\triangle ABC$, AB = 6.2 cm, BC = 5.6 cm and AC = 4.2 cm

(ii) In △PQR, PQ = 8.3 cm, QR = 5.4 cm and PR = 7.2 cm
(iii) In △XYZ, XY = 6.2 cm, XY = 6.8 cm and YZ = 5 cm
Solution:
(i) We know that,
In a triangle, the angle opposite to the smallest side is the smallest
In △ABC,
AC = 4.2 cm is the smallest side
Hence,
∠B is the smallest angle

(ii) We know that,
In a triangle, the angle opposite to the smallest side is the smallest
In △PQR,
QR = 5.4 cm is the smallest side
Hence,
∠P is the smallest angle

(iii) We know that,
In a triangle, the angle opposite to the smallest side is the smallest
In △XYZ,
YZ = 5 cm is the smallest side
Therefore,
∠X is the smallest angle

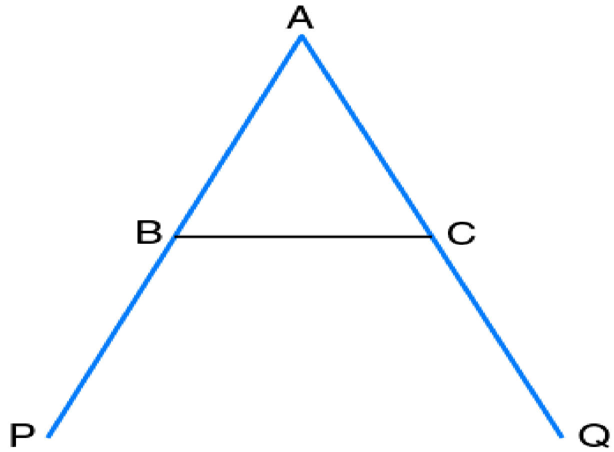
4. In a triangle ABC, BC = AC and ∠A = 35^0. Which is the smallest side of the triangle?



Solution:
In △ABC,

BC = AC (given)
$\angle A = \angle B = 35^0$ [angles opposite to the two equal sides are equal]
Let $\angle C = x^0$
In $\triangle ABC$,
$\angle A + \angle B + \angle C = 180^0$
$35^0 + 35^0 + x^0 = 180^0$
$70^0 + x^0 = 180^0$
$x^0 = 180^0 - 70^0$
We get,
$x^0 = 110^0$
Hence,
$\angle C = x^0 = 110^0$
Therefore,
Angles in a triangle are $\angle A = \angle B = 35^0$ and $\angle C = 110^0$
In $\triangle ABC$,
The greatest angle is $\angle C$
Here, the smallest angles are $\angle A$ and $\angle B$
Hence,
Smallest sides are BC and AC

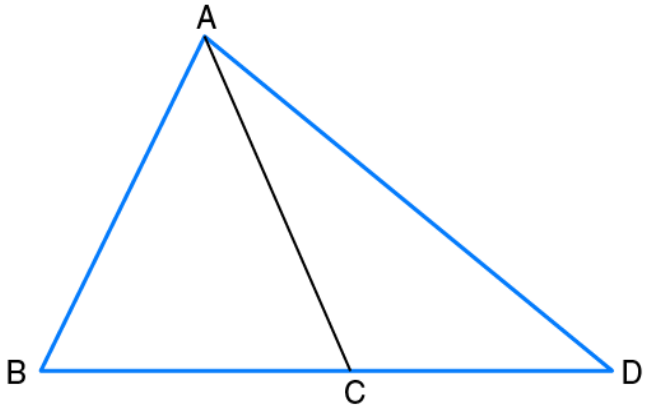
5. In $\triangle ABC$, the exterior $\angle PBC$ > exterior $\angle QCB$. Prove that AB> AC.



Solution:
In the given triangle,
Given that, $\angle PBC > \angle QCB$ ……. (1)
$\angle PBC + \angle ABC = 180^0$ (linear pair angles)
$\angle PBC = 180^0 - \angle ABC$ …. (2)
Similarly,
$\angle QCB = 180^0 - \angle ACB$ ……(3)

Now,
From (2) and (3), we get,
$180^0 - \angle ABC > 180^0 - \angle ACB$
$- \angle ABC > - \angle ACB$
Therefore,
$\angle ABC < \angle ACB$ or $\angle ACB > \angle ABC$
We know that,
In a triangle, the greater angle has the longer side opposite to it
Therefore,
AB > AC
Hence proved

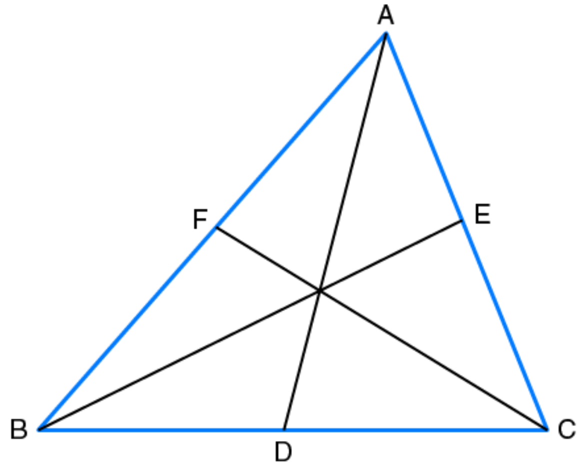
6. △ABC is isosceles with AB = AC. If BC is extended at D, then prove that AD > AB.



Solution:
In △ACD,
We have,
$\angle ACB = \angle CDA + \angle CAD$ [Using exterior angle property]
$\angle ACB > \angle CDA$ ….. (1)
Now,
AB = AC …. (given)
$\angle ACB = \angle ABC$ …. (2)
From equations (1) and (2), we get,
$\angle ABC > \angle CDA$
We know that,
In a triangle, the greater angle has the longer side opposite to it.
Now,
In △ABD,
We have,

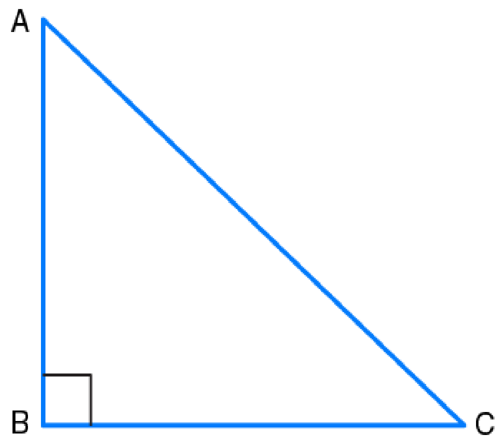
∠ABC > ∠CDA
Therefore,
AD > AB
Hence, proved

7. Prove that the perimeter of a triangle is greater than the sum of its three medians.
Solution:



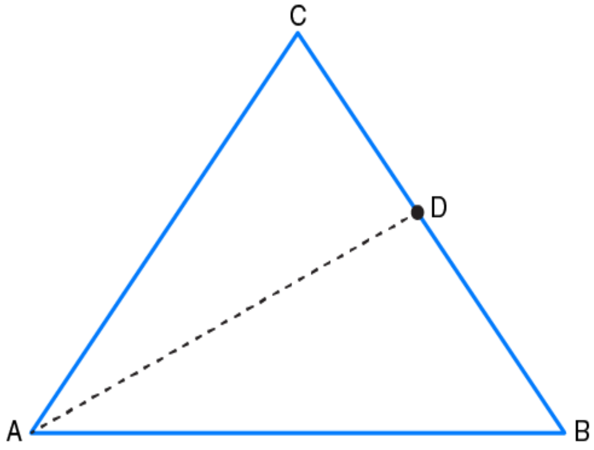
Given
In △ABC,
AD, BE and CF are its medians
We know that,
The sum of any two sides of a triangle is greater than twice the median bisecting the third side.
Hence,
AD is the median bisecting BC
So,
AB + AC > 2 AD ……..(1)
BE is the median bisecting AC
AB + BC > 2BE ……..(2)
CF is the median bisecting AB
BC + AC > 2CF ……… (3)
Now,
Adding equations (1), (2) and (3), we get,
(AB + AC) + (AB + BC) + (BC + AC) > 2AD + 2BE + 2CF
2 (AB + BC + AC) > 2 (AD + BE + CF)
We get,
AB + BC + AC > AD + BE + CF
Hence, proved

8. Prove that the hypotenuse is the longest side in a right angled triangle.
Solution:



Let △ABC be a right angled triangle in which the right angle is at B
By angle sum property of a triangle,
$\angle A + \angle B + \angle C = 180^0$
$\angle A + 90^0 + \angle C = 180^0$
We get,
$\angle A + \angle C = 90^0$
Hence,
The other two angles must be acute i.e less than 90^0
Therefore,
∠B is the largest angle in △ABC
∠B > ∠A and
∠B > ∠C
AC > BC and
AC > AB
∵ In any triangle, the side opposite to the greater angle is longer
Hence,
AC is the largest side in △ABC
But, AC is the hypotenuse of △ABC
Therefore,
Hypotenuse is the longest side in a right angled triangle
Hence, proved

9. D is a point on the side of the BC of △ABC. Prove that the perimeter of △ABC is greater than twice of AD.
Solution:



Construction: Join AD

In $\triangle ACD$,

$AC + CD > AD$ ….. (1)

$\because$ Sum of two sides of a triangle is greater than the third side

Similarly,

In $\triangle ADB$,

$AB + BD > AD$ ……... (2)

Adding equations (1) and (2), we get,

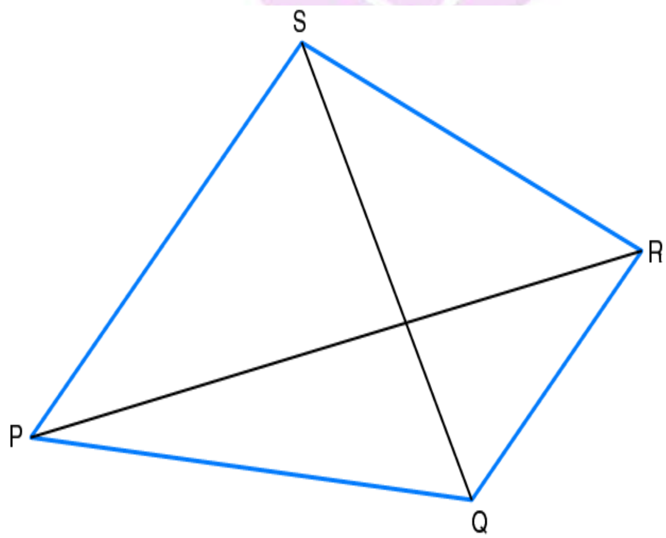
$AC + CD + AB + BD > 2\ AD$

$AB + BC + AC > 2AD$ (since, $CD + BD = BC$)

Hence, proved

10. For any quadrilateral, prove that its perimeter is greater than the sum of its diagonals.

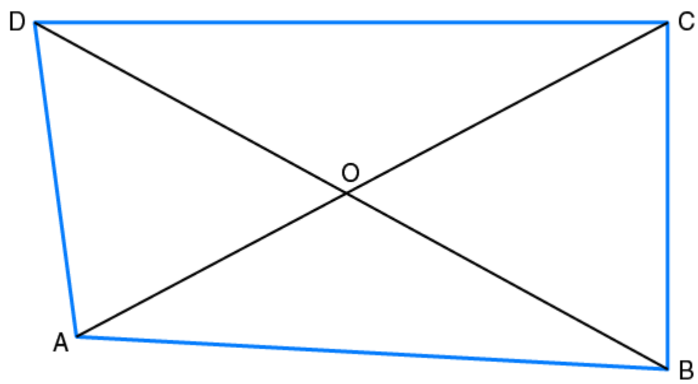
Solution:



Given

PQRS is a quadrilateral
PR and QS are the diagonals of quadrilateral
To prove: PQ + QR + SR + PS > PR + QS
Proof: In △PQR,
PQ + QR > PR (Sum of two sides of a triangle is greater than the third side)
Similarly,
In △PSR,
PS + SR > PR
In △PQS,
PS + PQ > QS and
In △QRS,
QR + SR > QS
Now,
We have,
PQ + QR > PR
PS + SR > PR
PS + PQ > QS
QR + SR > QS
After adding all the above inequalities, we get,
PQ + QR + PS + SR + PS + PQ + QR + SR > PR + PR + QS + QS
2PQ + 2QR + 2PS + 2SR > 2PR + 2QS
2 (PQ + QR + PS + SR) > 2 (PR + QS)
We get,
PQ + QR + PS + SR > PR + QS
Hence, proved

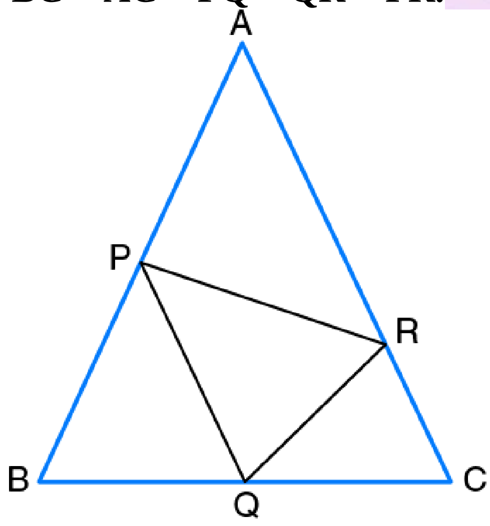
11. ABCD is a quadrilateral in which the diagonals AC and BD intersect at O. Prove that AB + BC + CD + AD < 2 (AC + BD)



Solution:
We know that,

Sum of two sides of a triangle is greater than the third side
Hence,
In $\triangle AOB$,
$OA + OB > AB$ ……… (1)
In $\triangle BOC$,
$OB + OC > BC$ ……… (2)
In $\triangle COD$,
$OC + OD > CD$ ……… (3)
In $\triangle AOD$,
$OA + OD > AD$ ………. (4)
Now,
Adding equations (1) (2) and (3) and (4), we get,
$2 (OA + OB + OC + OD) > AB + BC + CD + AD$
$2[(OA + OC) + (OB + OD)] > AB + BC + CD + AD$
We get,
$2 (AC + BD) > AB + BC + CD + AD$
$\because OA + OC = AC$ and $OB + OD = BD$
Therefore,
$AB + BC + CD + AD < 2 (AC + BD)$
Hence, proved

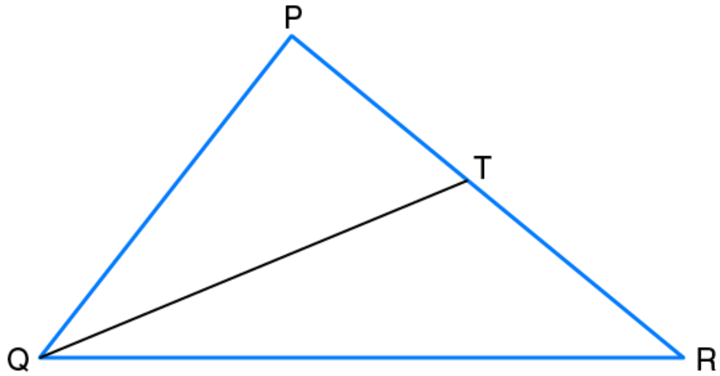
12. In $\triangle ABC$, P, Q and R are points on AB, BC and AC respectively. Prove that AB + BC + AC > PQ + QR + PR.



Solution:
We know that,
Sum of two sides of a triangle is greater than the third side
Hence,

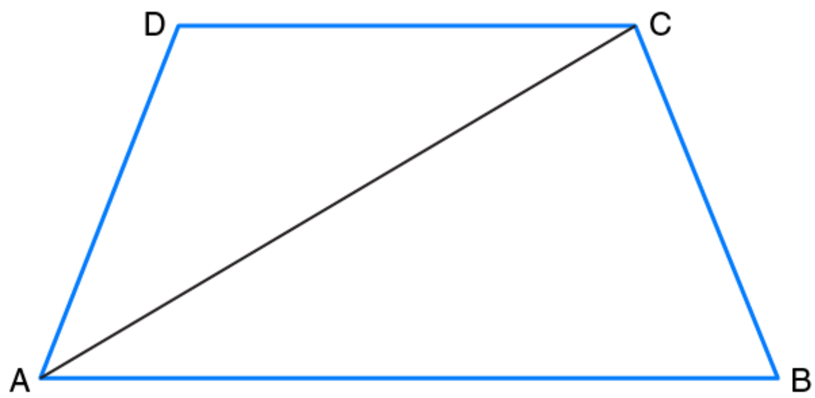
In $\triangle APR$,
$AP + AR > PR$ …….. (1)
In $\triangle BPQ$,
$BQ + PB > PQ$ …….. (2)
In $\triangle QCR$,
$QC + CR > QR$ ……... (3)
Now,
Adding equations (1), (2) and (3), we get,
$AP + AR + BQ + PB + QC + CR > PR + PQ + QR$
$(AP + PB) + (BQ + QC) + (CR + AR) > PR + PQ + QR$
We get,
$AB + BC + AC > PQ + QR + PR$
Hence, proved

13. In $\triangle PQR$, PR > PQ and T is a point on PR such that PT = PQ. Prove that QR > TR.



Solution:
In $\triangle PQT$,
We have
$PT = PQ$ ……. (1) (given)
In $\triangle PQR$,
$PQ + QR > PR$
$PQ + QR > PT + TR$ ($\because PR = PT + TR$)
$PQ + QR > PQ + TR$ [Using equation (1)]
We get,
$QR > TR$
Hence, proved

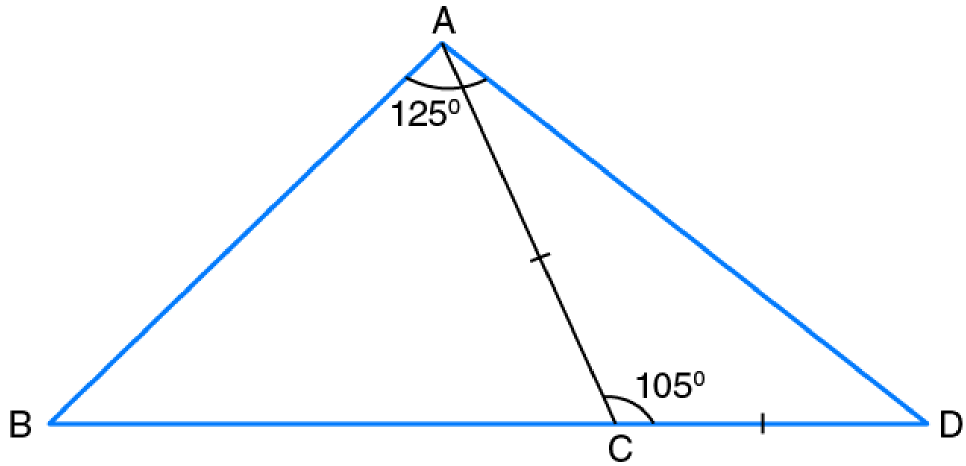
14. ABCD is a trapezium. Prove that:



(i) CD + DA + AB + BC > 2AC
(ii) CD + DA + AB > BC
Solution:
(i) In △ABC,
We have,
AB + BC > AC …… (1)
In △ACD,
We have,
AD + CD > AC ………. (2)
Adding equations (1) and (2), we get,
AB + BC + AD + CD > 2AC
Hence, proved

(ii) In △ACD,
We have,
CD + DA > CA
CD + DA + AB > CA + AB
CD + DA + AB > BC (since, AB + AC > BC)
Hence, proved

15. In △ABC, BC produced to D, such that, AC = CD; ∠BAD = 125^0 and ∠ACD = 105^0. Show that BC > CD.



Solution:

In $\triangle ACD$,

$AC = CD$ … (given)

$\angle CDA = \angle DAC$ …….. ($\triangle ACD$ is an isosceles triangle)

Let $\angle CDA = \angle DAC = x^0$

$\angle CDA + \angle DAC + \angle ACD = 180^0$

$x^0 + x^0 + 105^0 = 180^0$

$2x^0 + 105^0 = 180^0$

$2x^0 = 180^0 - 105^0$

We get,

$2x^0 = 75^0$

$x = (75^0 / 2)$

We get,

$x = 37.5^0$

$\angle CDA = \angle DAC = x^0 = 37.5^0$ ………… (1)

$\angle DAB = \angle DAC + \angle BAC$

$125^0 = 37.5^0 + \angle BAC$ {from equation (1)}

$125^0 – 37.5^0 = \angle BAC$

$87.5^0 = \angle BAC$

Also,

$\angle BCA + \angle ACD = 180^0$

$\angle BCA + 105^0 = 180^0$

We get,

$\angle BCA = 75^0$

Now,

In $\triangle BAC$,

$\angle ACB + \angle BAC + \angle ABC = 180^0$

$75^0 + 87.5^0 + \angle ABC = 180^0$

$\angle ABC = 180^0 - 75^0 - 87.5^0$
We get,
$\angle ABC = 17.5^0$
Since, $87.5^0 > 17.5^0$
Hence,
$\angle BAC > \angle ABC$
BC > AC
Therefore,
BC > CD …. (Since AC = CD)
Hence, proved