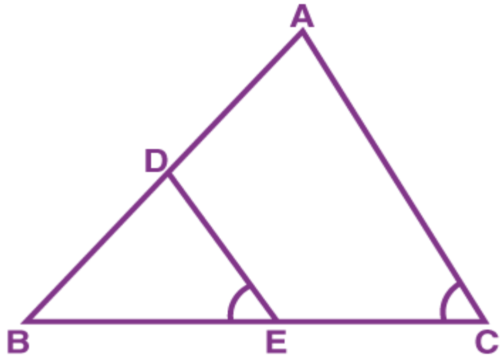
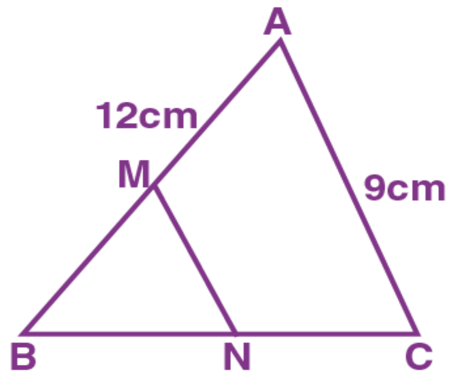
1. In △ABC, D is the mid-point of AB and E is the mid-point of BC



Calculate:
(i) DE, if AC = 8.6 cm
(ii) ∠DEB, if ∠ACB = 72^0
Solution:
In △ABC,
D and E are the mid-points of AB and BC respectively
Hence, by mid-point theorem DE || AC and DE = (1/2) AC
(i) DE = (1/2) AC = (1/2) x 8.6
We get,
= 4.3 cm
(ii) ∠DEB = ∠C = 72^0 (Corresponding angles are equal, since DE || AC)

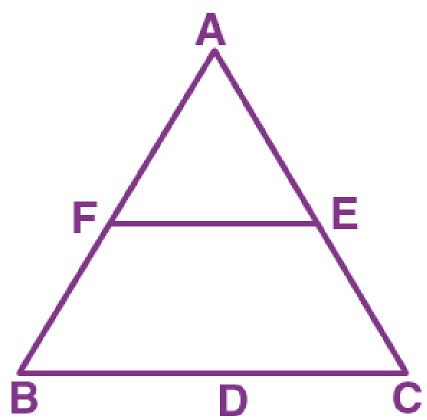
2. In △ABC, AB = 12 cm and AC = 9 cm. If M is the mid-point of AB and a straight line through M parallel to AC cuts BC in N, what is the length of MN?
Solution:



MN || AC and M is the mid-point of AB
Hence, N is the mid-point of BC
Therefore, MN = (1/2) AC
= (9/2) cm
We get,

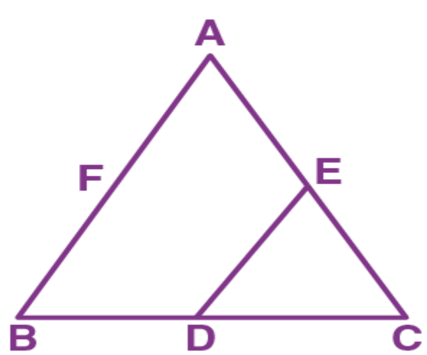
= 4.5 cm

3. (a) In △ABC, D, E, F are the mid-points of BC, CA and AB respectively. Find FE, if BC = 14 cm
(b) In △ABC, D, E, F are the mid-points of BC, CA and AB respectively. Find DE, if AB = 8 cm
(c) In △ABC, D, E, F are the mid-points of BC, CA and AB respectively. Fine ∠FDB if ∠ACB = 115^0
Solution:
(a)



F is the mid-point of AB and E is the mid-point of AC
Hence,
FE = (1/2) BC ………… (Mid-point Theorem)
= (1/2) x 14
We get,
= 7 cm

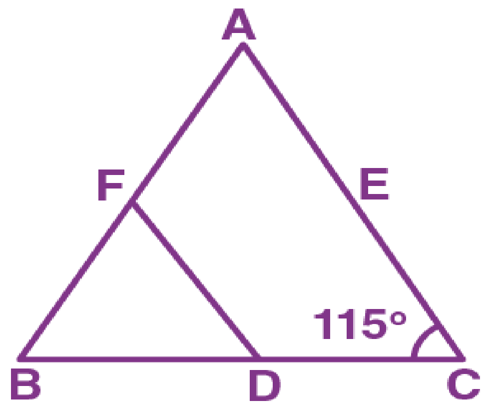
(b)



In △ABC,
D is the mid-point of BC and E is the mid-point of AC

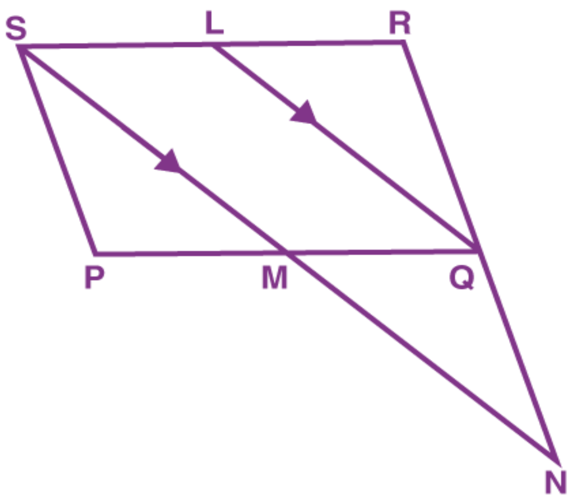
Hence,
DE = (1/2) AB .... (Mid-point Theorem)
= (1/2) x 8
We get,
= 4 cm

(c)



In △ABC,
FD || AC
Hence,
∠FDB = ∠ACB = 115^0 ....(Corresponding angles are equal)

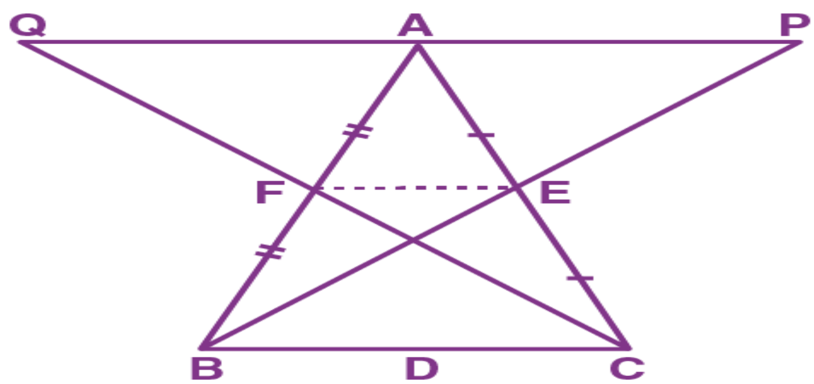
4. In parallelogram PQRS, L is mid-point of side SR and SN is drawn parallel to LQ which meets RQ produced at N and cuts side PQ at M. Prove that M is the mid-point of PQ



Solution:

In △NSR,
MQ = (1/2) SR
But L is the mid-point of SR and SR = PQ (Sides of parallelogram)
So, it can be written as,
MQ = (1/2) PQ
MQ = PM = LS= LR
Hence, M is the mid-point of PQ

5. In △ABC, BE and CF are medians. P is a point on BE produced such that BE = EP and Q is a point on CF produced such that CF = FQ. Prove that:
(a) QAP is a straight line
(b) A is the mid-point of PQ
Solution:

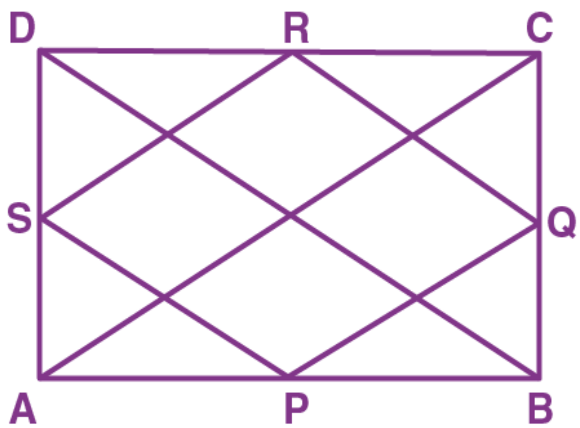


Since, BE and CF are the medians,
F is the mid-point of AB and E is the mid-point of AC
We know that the line joining the mid-points of any two sides is parallel and half of the third side
We have,
In △ACQ,
EF|| AQ and EF = (1/2) AQ ………(1)
In △ABP,
EF || AP and EF = (1/2) AP ……….(2)

(a) From (1) and (2)
We get,
AP || AQ (both are parallel to EF)
As AP and AQ are parallel and have a common point A
This is possible only if QAP is a straight line
Thus, proved

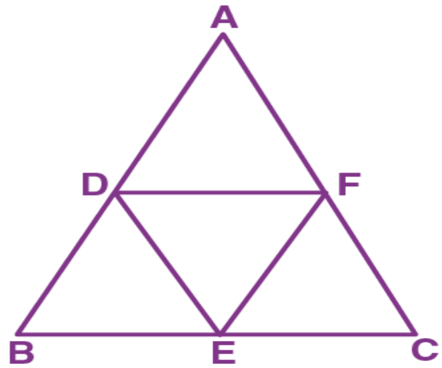
(b) From (1) and (2),
EF = (1/2) AQ and EF = (1/2) AP
⇒ (1/2) AQ = (1/2) AP
⇒ AQ = AP
Therefore, A is the mid-point of QP

6. Prove that the figure obtained by joining the mid-points of the adjacent sides of a rectangle is a rhombus
Solution:



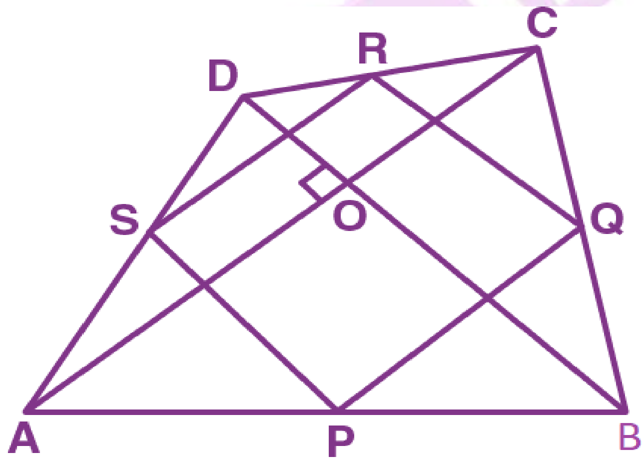
In the given rectangle ABCD,
Join AC and BD
In △ABC, P and Q are the mid-points of AB and BC respectively
PQ = (1/2) AC …….. (1) and PQ ||AC
In △BDC, R and Q are the mid-points of CD and BC respectively
QR = (1/2) BD ……..(2) and QR || BD
But AC and BD are diagonals of the rectangle
From equations (1) and (2)
PQ = QR
Similarly,
QR = RS and RS = SP
And,
RS || AC and SP || BD
Therefore, PQ = QR = RS = SP
Hence, PQRS is a rhombus

7. D, E and F are the mid-points of the sides AB, BC and CA of an isosceles △ABC in which AB = BC. Prove that △DEF is also isosceles.
Solution:



E and F are mid-points of BC and AC
Hence, EF = (1/2) AB …….(1)
D and F are the mid-points of AB and AC
Hence, DF = (1/2) BC ……..(2)
But AB = BC
From (1) and (2)
EF = DF
Thus, △DEF is an isosceles triangle

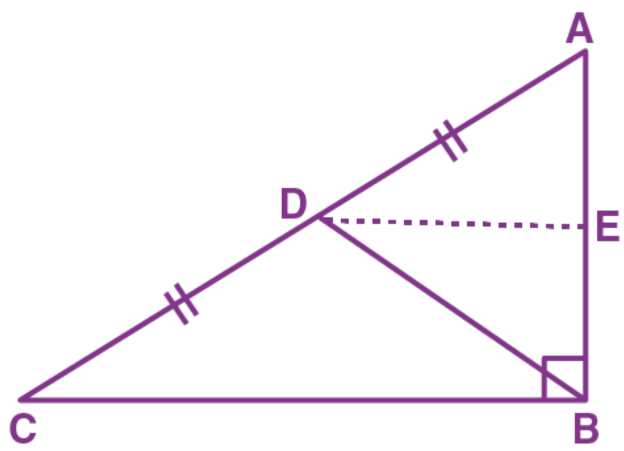
8. The diagonals of a quadrilateral intersect each other at right angle. Prove that the figure obtained by joining the mid-points of the adjacent sides of the quadrilateral is a rectangle.
Solution:



P and Q are the mid-points of AB and BC
Hence,
PQ || AC and PQ = (1/2) AC ……. (i)
S and R are the mid-points of AD and DC
Hence,
SR || AC and SR = (1/2) AC …….(ii)
From (i) and (ii),

PQ || SR and PQ = SR
Therefore, PQRS is a parallelogram
Further AC and BC intersect at right angles
∴ SP || BD and BD ⊥ AC
∴ SP ⊥ AC
⇒ SP ⊥ SR
⇒ ∠RSP = 90^0
∴ ∠RSP = ∠SRQ = ∠RQP = ∠SPQ = 90^0
Hence, PQRS is a rectangle

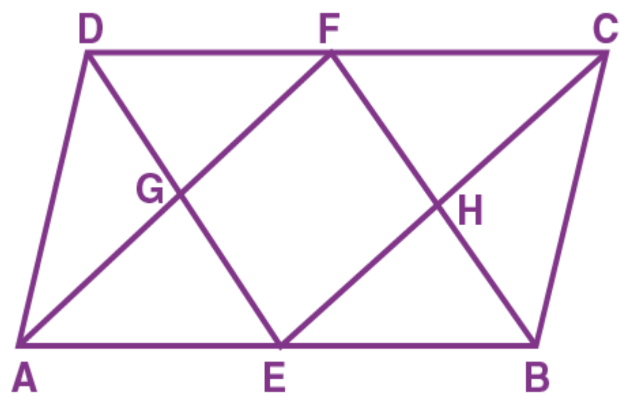
9. In a right angled triangle ABC. ∠ABC = 90^0 and D is the mid-point of AC. Prove that BD = (1/2) AC
Solution:



Draw line segment DE ||CB, which meets AB at point E
Now,
DE || CB and AB is the transversal,
∴∠AED = ∠ABC …. (corresponding angles)
∠ABC = 90^0 (given)
⇒ ∠AED = 90^0
Also,
Since D is the mid-point of AC and DE || CB,
DE bisects side AB,
Hence,
AE = BE …..(i)
In △AED and △BED,
∠AED = ∠BED …..(Each 90^0)
AE = BE ……[From (i)]
DE = DE ….(Common)

Therefore, △AED ≅ △BED ….(By SAS)
⇒ AD = BD …..(C.P.C.T.C)
⇒ BD = (1/2) AC
Hence, proved

10. In a parallelogram ABCD, E and F are the mid-points of the sides AB and CD respectively. The line segments AF and BF meet the line segments DE and CE at points G and H respectively. Prove that:
(a) △GEA ≅ △GFD
(b) △HEB ≅ △HFC
(c) EGFH is a parallelogram
Solution:



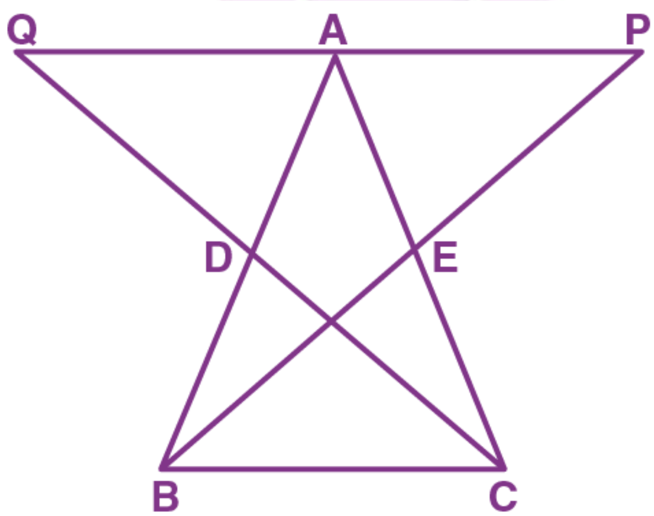
Since ABCD is a parallelogram,
AB = CD and AD = BC
Now,
E and F are the mid-points of AB and CD respectively,
Hence,
AE = EB = DF = FC ……….(1)
(a) In △GEA and △GFD,
AE = DF …… [From (1)]
∠AGE = ∠DGF ……… (vertically opposite angles)
∠GAE = ∠GFD …….. (Alternate interior angles)
Therefore,
△GEA ≅ △GFD

(b) In △HEB and △HFC,
BE = FC …..[From (1)]
∠EHB = ∠FHC (vertically opposite angles)

∠HBE = ∠HFC (Alternate interior angles)
Therefore,
△HEB ≅ △HFC

(c) In quadrilateral AECF,
AE = CF …..[From (1)]
AE || CF …...(since AB || DC)
Hence,
AECF is a parallelogram
⇒ EC || AF or EH || GF …… (i)
In quadrilateral BFDE,
BE = DF ….[From (1)]
BE || DF ….(since AB || DC)
⇒ BEDF is a parallelogram
⇒ BF || ED or HF || EG ……(ii)
From equations (i) and (ii),
We get,
EGFH is a parallelogram

11. In △ABC, the medians BE and CD are produced to the points P and Q respectively such that BE = EP and CD = DQ. Prove that:
(a) Q, A and P are collinear
(b) A is the mid-point of PQ
Solution:



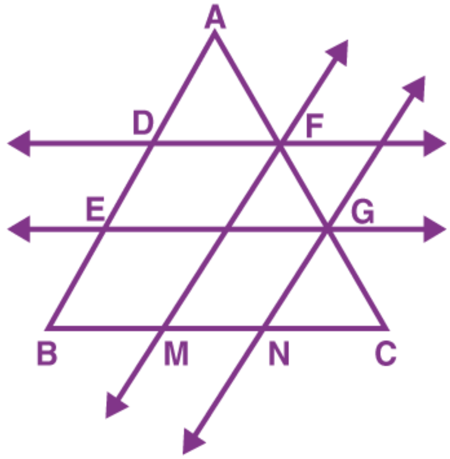
In △BDC and △ADQ,
CD = DQ … (given)
∠BDC = ∠ADQ ……(vertically opposite angles)

BD = AD ……(D is the mid-point of AB)
Therefore,
$\triangle BDC \cong \triangle ADQ$
$\Rightarrow \angle DBC = \angle DAQ$ (cpct)…….(i)
And, BC = AQ (cpct) …….(ii)
Similarly,
We can prove
$\triangle CEB \cong \triangle AEP$
$\Rightarrow \angle ECB = \angle EAP$ (cpct)……..(iii)
And, BC = AP (cpct)……..(iv)

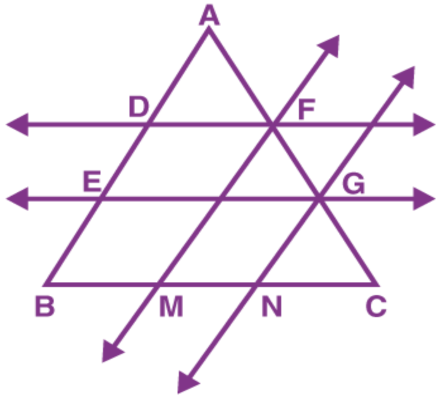
(a) In $\triangle ABC$,
$\angle ABC + \angle ACB + \angle BAC = 180^0$
$\Rightarrow \angle DBC + \angle ECB + \angle BAC = 180^0$
$\Rightarrow \angle DAQ + \angle EAP + \angle BAC = 180^0$ [From (i) and (iii)]
$\Rightarrow$ Q, A, P are collinear

(b) From (ii) and (iv),
AQ = AP
Therefore, A is the mid-point of PQ

12. In $\triangle ABC$, D and E are two points on the side AB such that AD = DE = EB. Through D and E, lines are drawn parallel to BC which meet the side AC at points F and G respectively. Through F and G, lines are drawn parallel to AB which meet the side BC at points M and N respectively. Prove that BM = MN = NC

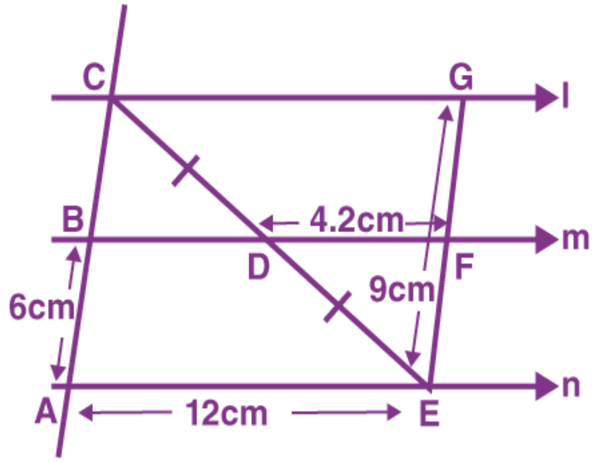


Solution:



In △AEG,
D is the mid-point of AE and DF || EG
Hence,
F is the mid-point of AG
AF = FG ……….. (1)
In △ABC,
DF || EG|| BC
DE = BE
Hence,
GF = GC …………. (2)
From (1) and (2) we get,
AF = FG = GC
Similarly, since GN|| FM|| AB
Thus, BM = MN = NC (proved)

13. In the given figure, the lines l, m and n are parallel to each other. D is the mid-point of CE. Find
(a) BC
(b) EF
(c) CG
(d) BD



Solution:

According to equal intercept theorem,

Since CD = DE

AB = BC ………(i)

EF = GF ……….(ii)

(a) BC = AB = 6 cm ….. [From (i)]

(b) EG = EF + FG

EG = 2EF …... [From (ii)]

9 = 2EF

EF = (9/2)

EF = 4.5 cm

(c) CG = 2DF

CG = 2 x 4.2

CG = 8.4 cm

(d) AE = 2BD

BD = (1/2) AE

BD = (1/2) x 12

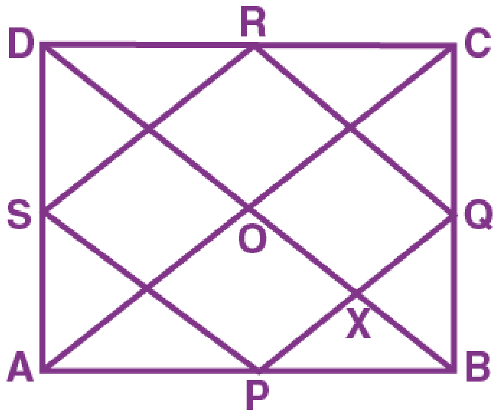
We get,

BD = 6 cm

14. The diagonals AC and BD of a quadrilateral ABCD intersect at right angles. Prove that the quadrilateral formed by joining the mid-points of quadrilateral ABCD is a rectangle.

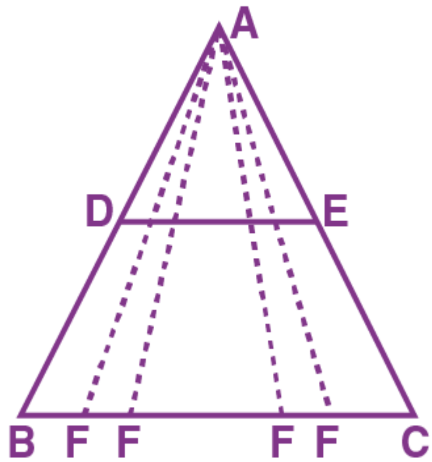
Solution:

The figure is as shown below



Let ABCD be a quadrilateral where P, Q, R, S are the mid-points of sides AB, BC, CD, DA. Diagonals AC and BD intersect at point 'O'.
We need to prove that PQRS is a rectangle
Proof:
In $\triangle ABC$ and $\triangle ADC$,
$2PQ = AC$ and $PQ \parallel AC$ …….. (1)
$2RS = AC$ and $RS \parallel AC$ …….. (2)
From (1) and (2)
We get,
$PQ = RS$ and $PQ \parallel RS$
Similarly, we can show that
$PS = RQ$ and $PS \parallel RQ$
Hence,
PQRS is a parallelogram
$PQ \parallel AC$
Therefore, $\angle AOD = \angle PXO = 90^0$ …. [Corresponding angles]
Again $BD \parallel RQ$
Therefore, $\angle PXO = \angle RQX = 90^0$ …[Corresponding angles]
Similarly,
$\angle QRS = \angle RSP = \angle SPQ = 90^0$
Hence,
PQRS is a rectangle

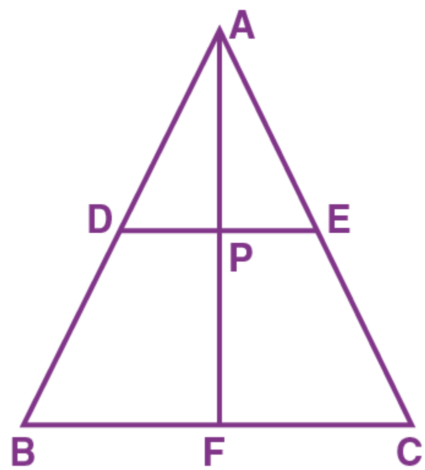
15. In $\triangle ABC$, D and E are the midpoints of the sides AB and AC respectively. F is any point on the side BC. If DE intersects AF at P show that DP = PE.
Solution:
Note: The given question is incomplete
According to the question given, F could be any point on BC as shown below



So, this makes it impossible to prove DP = DE

Since P too would shift as F shift because P too would be any point on DE as F is

Note: If we are given F to be the mid-point of BC, the result can be proved



Here,

D and E are the mid-points of AB and AC respectively

DE || BC and DE = (1/2) BC

But F is the mid-point of BC

BF = FC = (1/2) BC = DE

Since D is the mid-point of AB, and DP || BF

Since P is the mid-point of AF and E is the mid-point of AC,

PE = (1/2) FC

Also,

D and P are the mid-points of AB and AF respectively

⇒ DP = (1/2) BF = (1/2) FC = PE ……. (since BF = FC)

⇒ DP = PE

Hence, proved