1. Without using tables, evaluate the following:
(i) $\sin 60^0 \sin 30^0 + \cos 30^0 \cos 60^0$
(ii) $\sec 30^0 \operatorname{cosec} 60^0 + \cos 60^0 \sin 30^0$
(iii) $\sec 45^0 \sin 45^0 – \sin 30^0 \sec 60^0$
(iv) $\sin^2 30^0 \sin^2 45^0 + \sin^2 60^0 \sin^2 90^0$
(v) $\tan^2 30^0 + \tan^2 60^0 + \tan^2 45^0$
(vi) $\sin^2 30^0 \cos^2 45^0 + 4 \tan^2 30^0 + \sin^2 90^0 + \cos^2 0^0$
(vii) $\operatorname{cosec}^2 45^0 \sec^2 30^0 – \sin^2 30^0 – 4 \cot^2 45^0 + \sec^2 60^0$
(viii) $\operatorname{cosec}^3 30^0 \cos 60^0 \tan^3 45^0 \sin^2 90^0 \sec^2 45^0 \cot 30^0$
(ix) $(\sin 90^0 + \sin 45^0 + \sin 30^0) (\sin 90^0 – \cos 45^0 + \cos 60^0)$
(x) $4(\sin^4 30^0 + \cos^4 60^0) – 3 (\cos^2 45^0 – \sin^2 90^0)$
Solution:
(i) $\sin 60^0 \sin 30^0 + \cos 30^0 \cos 60^0$
$\sin 60^0 = (\sqrt{3} / 2)$
$\sin 30^0 = (1 / 2)$
$\cos 30^0 = (\sqrt{3} / 2)$
$\cos 60^0 = (1 / 2)$
On substituting, we get,
$\sin 60^0 \sin 30^0 + \cos 30^0 \cos 60^0 = (\sqrt{3} / 2) \times (1 / 2) + (\sqrt{3} / 2) \times (1/ 2)$
$= (\sqrt{3} / 4) + (\sqrt{3} / 4)$
We get,
$= (\sqrt{3} / 2)$

(ii) $\sec 30^0 \operatorname{cosec} 60^0 + \cos 60^0 \sin 30^0$
$\cos 30^0 = (\sqrt{3} / 2) \Rightarrow \sec 30^0 = (2 / \sqrt{3})$
$\sin 60^0 = (\sqrt{3} / 2) \Rightarrow \operatorname{cosec} 60^0 = (2 /\sqrt{3})$
$\cos 60^0 = (1 / 2)$
$\sin 30^0 = (1 / 2)$
On substituting, we get,
$\sec 30^0 \operatorname{cosec} 60^0 + \cos 60^0 \sin 30^0 = (2 / \sqrt{3}) \times (2 / \sqrt{3}) + (1 / 2) \times (1 / 2)$
$= (4 / 3) + (1 / 4)$
$= (16 + 3) / 12$
We get,
$= 19 / 12$

(iii) $\sec 45^0 \sin 45^0 – \sin 30^0 \sec 60^0$
$\cos 45^0 = (1 / \sqrt{2}) \Rightarrow \sec 45^0 = \sqrt{2}$
$\sin 45^0 = (1 / \sqrt{2})$
$\sin 30^0 = (1 / 2)$

$\cos 60^0 = (1 / 2) \Rightarrow \sec 60^0 = 2$
$\sec 45^0 \sin 45^0 – \sin 30^0 \sec 60^0$
On substituting, we get,
$= (\sqrt{2}) \times (1 / \sqrt{2}) – (1 / 2) \times 2$
$= 1 – 1$
We get,
$= 0$

(iv) $\sin^2 30^0 \sin^2 45^0 + \sin^2 60^0 \sin^2 90^0$
$\sin 30^0 = (1 / 2)$
$\sin 45^0 = (1 / \sqrt{2})$
$\sin 60^0 = (\sqrt{3} / 2)$
$\sin 90^0 = 1$
On substituting, we get,
$\sin^2 30^0 \sin^2 45^0 + \sin^2 60^0 \sin^2 90^0 = (1 / 2)^2 (1 / \sqrt{2})^2 + (\sqrt{3} / 2)^2 (1)^2$
$= (1 / 4) \times (1 / 2) + (3 / 4) (1)$
$= (1 / 8) + (3 / 4)$
$= (1 + 6) / 8$
We get,
$= 7 / 8$

(v) $\tan^2 30^0 + \tan^2 60^0 + \tan^2 45^0$
$\tan 30^0 = (1 / \sqrt{3})$
$\tan 60^0 = \sqrt{3}$
$\tan 45^0 = 1$
On substituting, we get,
$\tan^2 30^0 + \tan^2 60^0 + \tan^2 45^0 = (1 / \sqrt{3})^2 + (\sqrt{3})^2 + 1$
$= (1 / 3) + 3 + 1$
$= (1 + 9 + 3) / 3$
We get,
$= 13 / 3$

(vi) $\sin^2 30^0 \cos^2 45^0 + 4 \tan^2 30^0 + \sin^2 90^0 + \cos^2 0^0$
$\sin 30^0 = (1 / 2)$
$\cos 45^0 = (1 / \sqrt{2})$
$\tan 30^0 = (1 / \sqrt{3})$
$\sin 90^0 = 1$
$\cos 0^0 = 1$
On substituting, we get,

$\sin^2 30^0 \cos^2 45^0 + 4 \tan^2 30^0 + \sin^2 90^0 + \cos^2 0^0$
$= (1 / 2)^2 (1 / \sqrt{2})^2 + 4 (1 / \sqrt{3})^2 + 1 + 1$
$= (1 / 4) (1 / 2) + (4 / 3) + 2$
$= (1 / 8) + (4 / 3) + 2$
$= (3 + 32 + 48) / 24$
We get,
$= 83 / 24$

(vii) $\text{cosec}^2 45^0 \sec^2 30^0 – \sin^2 30^0 – 4 \cot^2 45^0 + \sec^2 60^0$
$\sin 45^0 = (1 / \sqrt{2})$
$\text{cosec } 45^0 = (\sqrt{2} / 1)$
$\sin 30^0 = \cos 60^0 = (1 / 2)$
$\sec 60^0 = 2$
$\cos 30^0 = (\sqrt{3} / 2)$
$\sec 30^0 = (2 / \sqrt{3})$
$\tan 45^0 = 1$
$\cot 45^0 = 1$
On substituting, we get,
$\text{cosec}^2 45^0 \sec^2 30^0 – \sin^2 30^0 – 4 \cot^2 45^0 + \sec^2 60^0 = (\sqrt{2} / 1)^2 (2 / \sqrt{3})^2 – (1 / 2)^2 – 4 (1)^2 + (2)^2$
$= 2 \times (4 / 3) – (1 / 4) – 4 + 4$
$= (8 / 3) – (1 / 4)$
$= (32 – 3) / 12$
We get,
$= 29 / 12$

(viii) $\text{cosec}^3 30^0 \cos 60^0 \tan^3 45^0 \sin^2 90^0 \sec^2 45^0 \cot 30^0$
$\sin 30^0 = (1 / 2)$
$\text{cosec } 30^0 = 2$
$\cos 60^0 = (1 / 2)$
$\sec 60^0 = 2$
$\cos 45^0 = (1 / \sqrt{2})$
$\sec 45^0 = \sqrt{2}$
$\tan 45^0 = 1$
$\sin 90^0 = 1$
$\tan 30^0 = (1 / \sqrt{3})$
$\cot 30^0 = \sqrt{3}$
On substituting, we get,
$\text{cosec}^3 30^0 \cos 60^0 \tan^3 45^0 \sin^2 90^0 \sec^2 45^0 \cot 30^0$

$= (2)^3 (1 / 2) (1)^3 (1)^2 (\sqrt{2})^2 (\sqrt{3})$
$= 8 \times (1 / 2) \times 2 \times \sqrt{3}$
We get,
$= 8\sqrt{3}$

(ix) $(\sin 90^0 + \sin 45^0 + \sin 30^0) (\sin 90^0 - \cos 45^0 + \cos 60^0)$
$\sin 30^0 = (1 / 2)$
$\sin 45^0 = (1 / \sqrt{2})$
$\sin 90^0 = 1$
$\cos 45^0 = (1 / \sqrt{2})$
$\cos 60^0 = (1 / 2)$
$(\sin 90^0 + \sin 45^0 + \sin 30^0) (\sin 90^0 - \cos 45^0 + \cos 60^0)$
On substituting, we get,
$= \{(1) + (1/ \sqrt{2}) + (1 / 2)\} \{(1) - (1 / \sqrt{2}) + (1 / 2)\}$
$= \{(3 / 2) + (1 / \sqrt{2})\} \{(3 / 2) - (1 / \sqrt{2})\}$
$= (3 / 2)^2 - (1 / \sqrt{2})^2$
$= (9 / 4) - (1 / 2)$
We get,
$= (9 - 2) / 4$
$= 7 / 4$

(x) $4 (\sin^4 30^0 + \cos^4 60^0) - 3 (\cos^2 45^0 - \sin^2 90^0)$
$\sin 30^0 = (1 / 2)$
$\sin 90^0 = 1$
$\cos 45^0 = (1 / \sqrt{2})$
$\cos 60^0 = (1 / 2)$
On substituting, we get,
$4 (\sin^4 30^0 + \cos^4 60^0) - 3 (\cos^2 45^0 - \sin^2 90^0)$
$= 4 \{(1 / 2)^4 + (1 / 2)^4\} - 3 \{(1 / \sqrt{2})^2 - (1)^2\}$
$= 4 \{(1 / 16) + (1 / 16)\} - 3 \{(1 / 2) - 1\}$
On calculating further, we get,
$= 4 \times (2 / 16) + 3 \times (1 / 2)$
$= (1 / 2) + (3 / 2)$
$= 4 / 2$
We get,
$= 2$

2. Without using tables, find the value of the following:
(i) $(\sin 30^0 - \sin 90^0 + 2 \cos 0^0) / \tan 30^0 \tan 60^0$

(ii) (sin 30^0 / sin 45^0) + (tan 45^0 / sec 60^0) – (sin 60^0 / cot 45^0) – (cos 30^0 / sin 90^0)
(iii) (tan 45^0 / cosec 30^0) + (sec 60^0 / cot 45^0) – (5 sin 90^0 / 2 cos 0^0)
(iv) ($\tan^2 60^0$ + 4 $\cos^2 45^0$ + 3 $\sec^2 30^0$ + 5 cos 90^0) / (cosec 30^0 + sec 60^0 – $\cot^2 30^0$)
(v) (4 / $\cot^2 30^0$) + (1 / $\sin^2 60^0$) – $\cos^2 45^0$

Solution:

(i) (sin 30^0 – sin 90^0 + 2 cos 0^0) / tan 30^0 tan 60^0
= {(1 / 2) – (1) + 2 x 1} / (1 / √3) x √3
On further calculation, we get,
= {(1 / 2) – 1 + 2} / 1
= (1 / 2) – 1 + 2
= (1 / 2) + 1
We get,
= 3 / 2

(ii) (sin 30^0 / sin 45^0) + (tan 45^0 / sec 60^0) – (sin 60^0 / cot 45^0) – (cos 30^0 / sin 90^0)
= {(1 / 2) / (1 / √2)} + (1 / 2) – {(√3 / 2) / 1} – {(√3 / 2) / 1}
On calculating further, we get,
= (√2 / 2) + (1 / 2) – (√3 / 2) – (√3 / 2)
= (√2 + 1 - 2√3) / 2

(iii) (tan 45^0 / cosec 30^0) + (sec 60^0 / cot 45^0) – (5 sin 90^0 / 2 cos 0^0)
= (1 / 2) + (2 / 1) – (5 x 1) / (2 x 1)
On further calculation, we get,
= (1 / 2) + (2 / 1) – (5 / 2)
= (1 + 4 – 5) / 2
We get,
= 0

(iv) ($\tan^2 60^0$ + 4 $\cos^2 45^0$ + 3 $\sec^2 30^0$ + 5 cos 90^0) / (cosec 30^0 + sec 60^0 – $\cot^2 30^0$)
= {$(\sqrt{3})^2$ + 4 x $(1 / \sqrt{2})^2$ + 3 x $(2 / \sqrt{3})^2$ + 5 x 0} / (2) + (2) – $(\sqrt{3})^2$
On further calculation, we get,
= {3 + 4 x (1 / 2) + 3 x (4 / 3) + 0} / (2 + 2 – 3)
= (3 + 2 + 4) / (4 – 3)
We get,
= 9

(v) (4 / $\cot^2 30^0$) + (1 / $\sin^2 60^0$) – $\cos^2 45^0$
= {4 / $(\sqrt{3})^2$} + {1 / $(\sqrt{3} / 2)^2$} – $(1 / \sqrt{2})^2$
On further calculation, we get,

= (4 / 3) + {1 / (3 / 4)} – (1 / 2)
= (4 / 3) + (4 / 3) – (1 / 2)
= (8 + 8 – 3) / 6
We get,
= 13 / 6

3. Prove that:
(a) $\sin 60^0 . \cos 30^0 - \sin 60^0 . \sin 30^0 = (1 / 2)$
(b) $\cos 60^0 . \text{Cos } 30^0 - \sin 60^0 . \sin 30^0 = 0$
(c) $\sec^2 45^0 - \tan^2 45^0 = 1$
(d) $\{(\cot 30^0 + 1) / (\cot 30^0 - 1)\}^2 = (\sec 30^0 + 1) / (\sec 30^0 - 1)$
Solution:
(a) Consider L.H.S.
$\sin 60^0 . \cos 30^0 - \cos 60^0 . \sin 30^0$
= (√3 / 2) x (√3 / 2) – (1 / 2) x (1 / 2)
On simplification, we get,
= (3 / 4) – (1 / 4)
= (3 – 1) / 4
= 2 / 4
We get,
= 1 / 2
= R.H. S.
Hence, proved

(b) Consider L.H.S.
$\cos 60^0 . \cos 30^0 - \sin 60^0 . \sin 30^0$
= (1 / 2) x (√3 / 2) – (√3 / 2) x (1 / 2)
On calculating further, we get,
= (√3 / 4) – (√3 / 4)
We get,
= 0
= R.H.S.
Hence, proved

(c) Consider L.H.S.
$\sec^2 45^0 - \tan^2 45^0$
$= (\sqrt{2})^2 - (1)^2$
= 2 – 1
= 1

= R.H.S.
Hence, proved

(d) Consider L.H.S.
$\{(\cot 30^0 + 1) / (\cot 30^0 - 1)\}^2$
$= \{(\sqrt{3}) + 1 / (\sqrt{3}) - 1\}^2$
$= \{(\sqrt{3} + 1) / (\sqrt{3} - 1) \times (\sqrt{3} + 1) / (\sqrt{3} + 1)\}^2$
On further calculation, we get,
$= \{(\sqrt{3})^2 + (1)^2 + 2\sqrt{3}\} / \{(\sqrt{3})^2 + (1)^2 - 2\sqrt{3}\}$
$= (3 + 1 + 2\sqrt{3}) / (3 + 1 - 2\sqrt{3})$
$= (4 + 2\sqrt{3}) / (4 - 2\sqrt{3})$
Taking 2 as common, we get,
$= 2 (2 + \sqrt{3}) / 2 (2 - \sqrt{3})$
$= (2 + \sqrt{3}) / (2 - \sqrt{3})$
$= \{(2 / \sqrt{3}) + 1\} / \{(2 / \sqrt{3}) - 1\}$
$= (\sec 30^0 + 1) / (\sec 30^0 - 1)$
= R.H.S.
Hence, proved

4. Find the value of ‘A’, if
(a) 2 cos A = 1
(b) 2 sin 2A = 1
(c) cosec 3A = (2 / √3)
(d) 2 cos 3A = 1
(e) √3 cot A = 1
(f) cot 3A = 1
Solution:
(a) 2 cos A = 1
cos A = (1 / 2)
$\cos A = \cos 60^0$
$A = 60^0$
Therefore, the value of ‘A’ is 60^0

(b) 2 sin 2A = 1
sin 2A = (1 / 2)
$\sin 2A = \sin 30^0$
$2A = 30^0$
We get,
$A = 15^0$

Therefore, the value of ‘A’ is 15^0

(c) cosec 3A = (2 / √3)
cosec 3A = cosec 60^0
3A = 60^0
We get,
A = 20^0
Therefore, the value of ‘A’ is 20^0

(d) 2 cos 3A = 1
cos 3A = (1 / 2)
cos 3A = cos 60^0
3A = 60^0
We get,
A = 20^0
Therefore, the value of ‘A’ is 20^0

(e) √3 cot A = 1
cot A = (1 / √3)
cot A = cot 60^0
A = 60^0
Therefore, the value of ‘A’ is 60^0

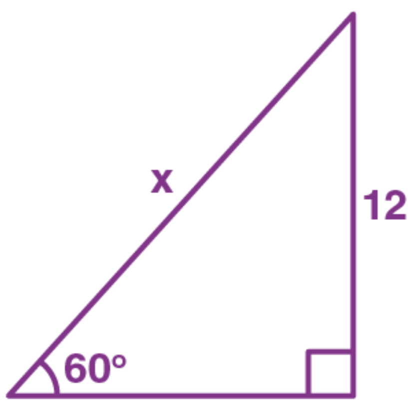
(f) cot 3A = 1
cot 3A = cot 45^0
3A = 45^0
We get,
A = 15^0
Therefore, the value of ‘A’ is 15^0

5. Find the value of ‘A’, if
(a) (1 – cosec A) (2 – sec A) = 0
(b) (2 – cosec 2A) cos 3A = 0
Solution:
(a) (1 – cosec A) (2 – sec A) = 0
Here,
1 – cosec A = 0 and 2 – sec A = 0
On calculating further, we get,
cosec A = 1 and sec A = 2

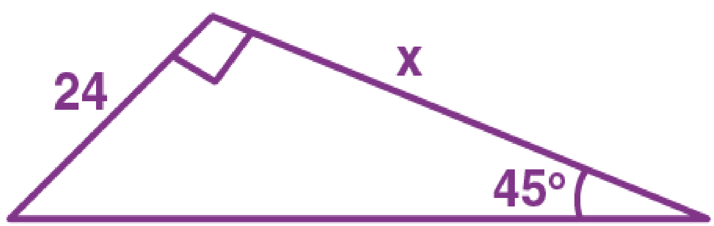
cosec A = cosec 90^0 and sec A = sec 60^0
A = 90^0 and A = 60^0

(b) (2 – cosec 2A) cos 3A = 0
Here,
2 – cosec 2A = 0 and cos 3A = 0
On further calculation, we get,
cosec 2A = 2 and cos 3A = 0
cosec 2A = cosec 30^0 and cos 3A = cos 90^0
We get,
2A = 30^0 and 3A = 90^0
A = 15^0 and A = 30^0

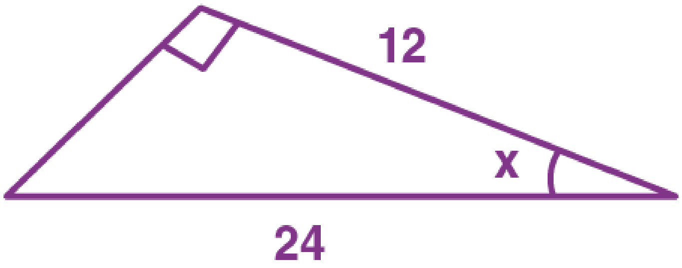
6. Find the value of 'x' in each of the following:
(a)



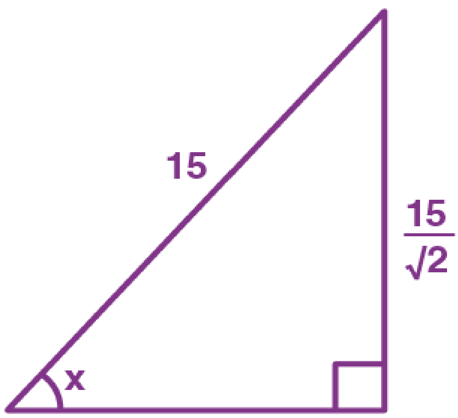
(b)



(c)

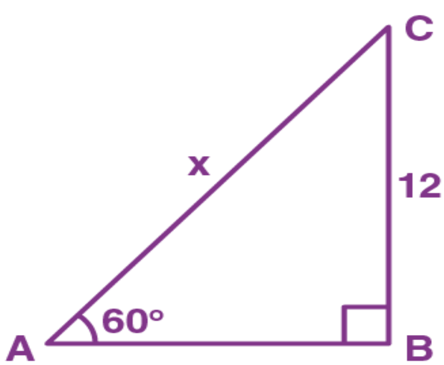


(d)



Solution:

(a)



From the figure,

We have,

sin 60^0 = BC / AC

√3 / 2 = 12 / x

On simplification, we get,

x = (2 x 12) / √3

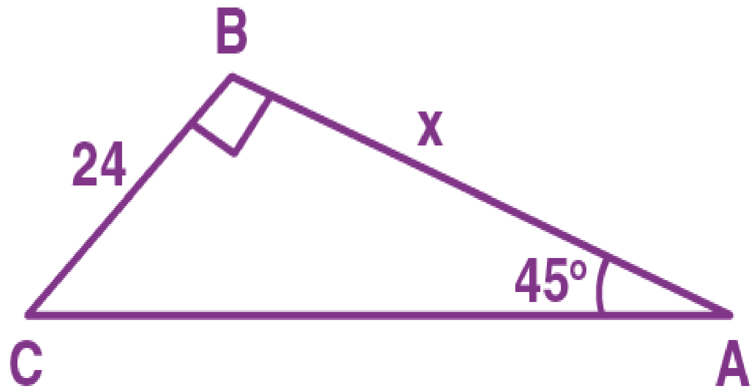
x = 24 / √3

x = (8 x 3) / √3

We get,

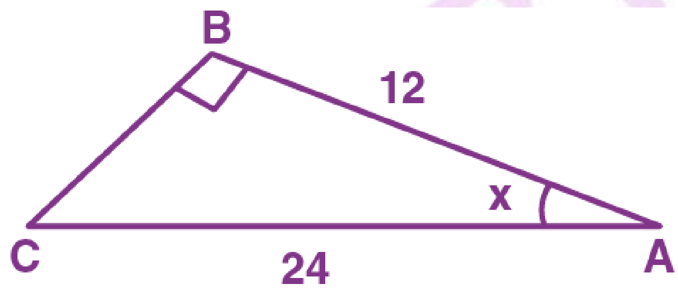
x = 8√3

(b)



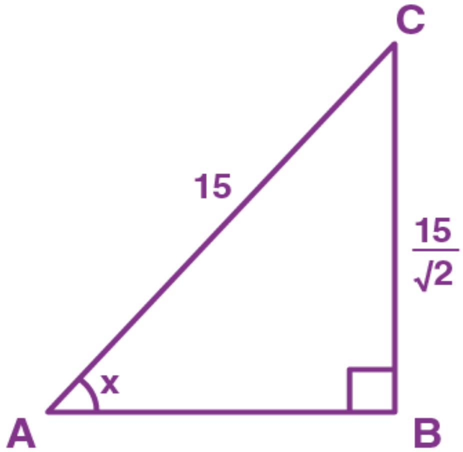
From the figure,
We have
tan 45^0 = BC / AB
1 = 24 / x
We get,
x = 24

(c)



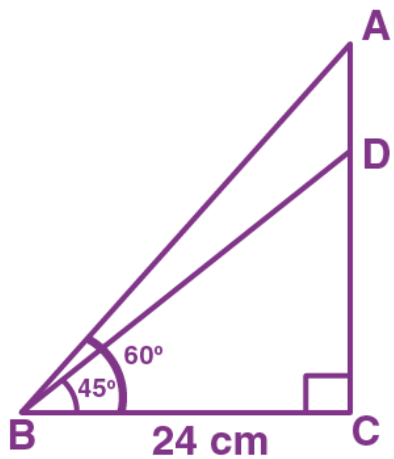
From the figure,
We have,
cos x = AB / AC
cos x = 12 / 24
cos x = 1 / 2
cos x = cos 60^0
We get,
x = 60^0

(d)

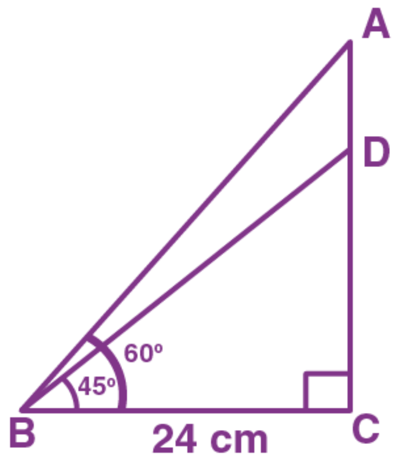


From the figure,
We have,
sin x = BC / AC
sin x = (15 / √2) / 15
sin x = (1 / √2)
sin x = sin 45^0
We get,
x = 45^0

7. Find the length of AD.
Given: ∠ABC = 60^0, ∠DBC = 45^0 and BC = 24 cm.



Solution:

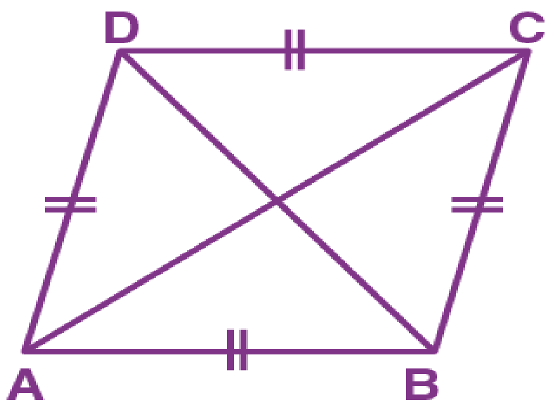


In △ABC,
tan 60^0 = AC / BC
√3 = AC / 24
We get,
AC = 24√3 cm

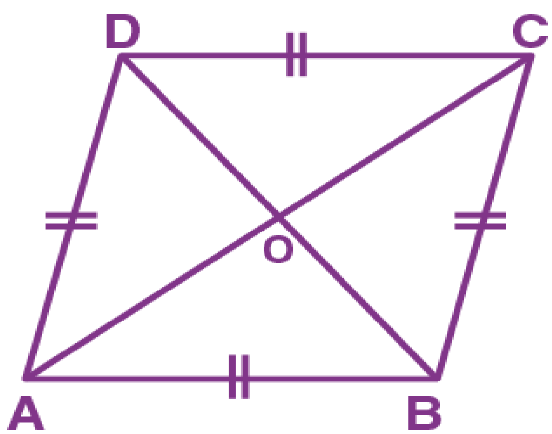
In △DBC,
tan 45^0 = DC / BC
1 = DC / 24
We get,
DC = 24 cm

Now,
AC = AD + DC
AD = AC – DC
Substituting the values of AC and DC, we get,
AD = 24√3 – 24
AD = 24 (√3 – 1) cm
Therefore, the length of AD is 24(√3 – 1) cm

8. Find lengths of diagonals AC and BD. Given AB = 24 cm and ∠BAD = 60^0



Solution:



Since all sides are equal,

∴ The given figure is a rhombus

We know that,

Diagonals of a rhombus bisect each other at right angles and also bisect the angle of vertex

Let the diagonals AC and BD intersect each other at point O

Hence,

OA = OC = (1 / 2) AC

OB = OD = (1 / 2) BD

$\angle AOB = 90^0$

Given

$\angle BAD = 60^0$

$\Rightarrow \angle OAB = (1/ 2) \angle BAD$

$\Rightarrow \angle OAB = 30^0$

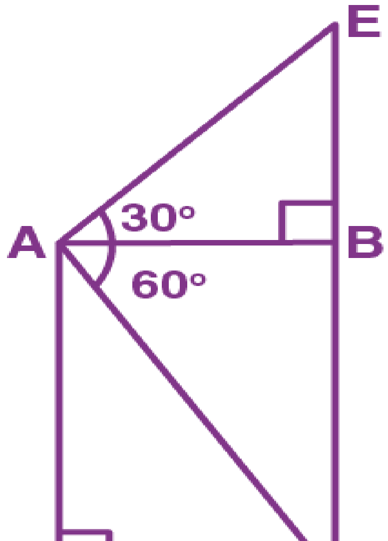
In right-angled △AOB,

$\sin 30^0 = OB / AB$

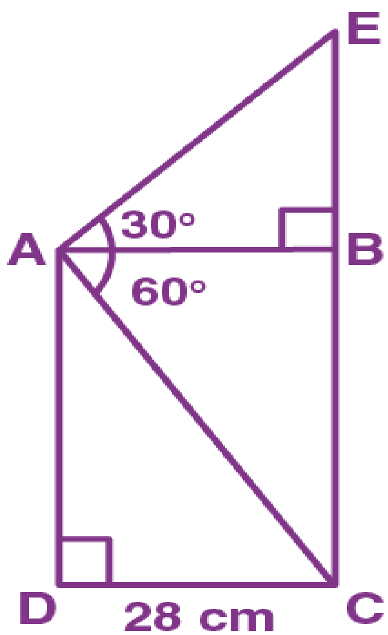
= (1 / 2)
Given AB = 24,
⇒OB / 24 = (1 / 2)
OB = 24 / 2
We get,
OB = 12 cm

cos 30^0 = OA / AB
= √3 / 2
⇒OA / 24 = √3 / 2
OA = (24√3) / 2
We get,
OA = 12√3 cm
Therefore,
Length of diagonal AC = 2 x OA = 2 x 12√3 = 24√3 cm and
Length of diagonal BD = 2 x OB = 2 x 12 = 24 cm

9. Find the length of EC.



Solution:



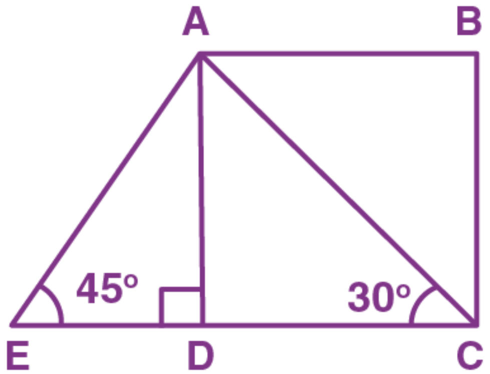
CD = 28 cm
⇒ AB = 28 cm

In right △ABE,
tan 30^0 = BE / AB
1 / √3 = BE / 28
We get,
BE = (28 / √3)

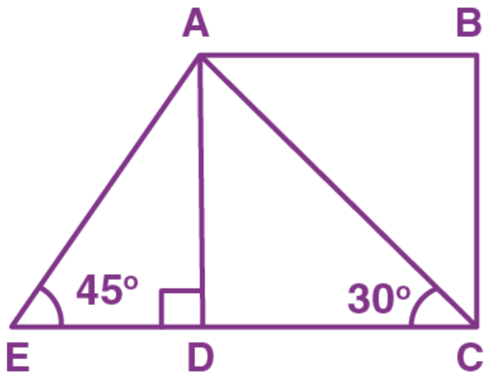
In right △ABC,
tan 60^0 = CB / AB
√3 = CB / 28
We get,
CB = 28√3
Hence,
Length of EC = CB + BE
= 28√3 + (28 / √3)
On further calculation, we get,
= (84 + 28) / √3
= (112 / √3)
Hence, the length of EC is (112 / √3) cm

10. In the given figure, AB and EC are parallel to each other. Sides AD and BC are 1.5 cm each and are perpendicular to AB. Given that ∠AED = 45^0 and ∠ACD = 30^0.

Find:
(a) AB
(b) AC
(c) AE



Solution:



(a) In right △ADC,
tan 30^0 = AD / DC
(1 / √3) = 1.5 / DC (given AD = 1.5 cm)
DC = 1.5√3
Here,
AB || DC and AD ⊥ EC, ABCD is a parallelogram
Therefore, opposite sides are equal
⇒ AB = DC = 1.5√3 cm

(b) In right △ADC,
sin 30^0 = AD / AC
(1 / 2) = 1.5 / AC (given AD = 1.5 cm)
AC = 2 x 1.5
We get,
AC = 3 cm

(c) In right △ADE,
sin 45^0 = AD / AE
(1 / √2) = 1.5 / AE (given AD = 1.5 cm)
We get,
AE = 1.5√2 cm

11. Evaluate the following:
(a) sin 62^0 / cos 28^0
(b) sec 34^0 / cosec 56^0
(c) tan 12^0 / cot 78^0
(d) sin 25^0 cos 43^0 / sin 47^0 cos 65^0
(e) sec 32^0 cot 26^0 / tan 64^0 cosec 58^0
(f) cos 34^0 cos 33^0 / sin 57^0 sin 56^0
Solution:
(a) sin 62^0 / cos 28^0
This can be written as,
= sin (90^0 – 28^0) / cos 28^0
= cos 28^0 / cos 28^0
We get,
= 1

(b) sec 34^0 / cosec 56^0
This can be written as,
= sec (90^0 – 56^0) / cosec 56^0
= cosec 56^0 / cosec 56^0
We get,
= 1

(c) tan 12^0 / cot 78^0
This can be written as,
= tan (90^0 – 78^0) / cot 78^0
= cot 78^0 / cot 78^0
We get,
= 1

(d) sin 25^0 cos 43^0 / sin 47^0 cos 65^0
This can be written as,
= sin (90^0 – 65^0) cos (90^0 – 47^0) / sin 47^0 cos 65^0

= cos 65^0 sin 47^0 / sin 47^0 cos 65^0
We get,
= 1

(e) sec 32^0 cot 26^0 / tan 64^0 cosec 58^0
This can be written as,
= sec (90^0 – 58^0) cot (90^0 – 64^0) / tan 64^0 cosec 58^0
= cosec 58^0 tan 64^0 / tan 64^0 cosec 58^0
We get,
= 1

(f) cos 34^0 cos 33^0 / sin 57^0 sin 56^0
This can be written as,
= cos (90^0 – 56^0) cos (90^0 – 57^0) / sin 57^0 sin 56^0
= sin 56^0 sin 57^0 / sin 57^0 sin 56^0
We get,
= 1

12. Evaluate the following:
(a) sin 31^0 – cos 59^0
(b) cot 27^0 – tan 63^0
(c) cosec 54^0 – sec 36^0
(d) sin 28^0 sec 62^0 + tan 49^0 tan 41^0
(e) sec 16^0 tan 28^0 – cot 62^0 cosec 74^0
(f) sin 22^0 cos 44^0 – sin 46^0 cos 68^0
Solution:
(a) sin 31^0 – cos 59^0
= sin (90^0 – 59^0) – cos 59^0
= cos 59^0 – cos 59^0
We get,
= 0

(b) cot 27^0 – tan 63^0
= cot (90^0 – 63^0) – tan 63^0
= tan 63^0 – tan 63^0
We get,
= 0

(c) cosec 54^0 – sec 36^0

= cosec (90^0 – 36^0) – sec 36^0
= sec 36^0 – sec 36^0
We get,
= 0

(d) sin 28^0 sec 62^0 + tan 49^0 tan 41^0
= sin 28^0 sec (90^0 – 28^0) + tan 49^0 tan (90^0 – 49^0)
= sin 28^0 cosec 28^0 + tan 49^0 cot 49^0
= sin 28^0 x (1 / sin 28^0) + tan 49^0 x (1 / tan 49^0)
= 1 + 1
We get,
= 2

(e) sec 16^0 tan 28^0 – cot 62^0 cosec 74^0
= sec (90^0 – 74^0) tan (90^0 – 62^0) – cot 62^0 cosec 74^0
= cosec 74^0 cot 62^0 – cot 62^0 cosec 74^0
We get,
= 0

(f) sin 22^0 cos 44^0 – sin 46^0 cos 68^0
= sin (90^0 – 68^0) cos (90^0 – 46^0) – sin 46^0 cos 68^0
= cos 68^0 sin 46^0 – sin 46^0 cos 68^0
We get,
= 0

13. Evaluate the following:
(a) (sin 36^0 / cos 54^0) + (sec 31^0 / cosec 59^0)
(b) (tan 42^0 / cot 48^0) – (cos 33^0 / sin 57^0)
(c) (2 sin 28^0 / cos 62^0) + (3 cot 49^0 / tan 41^0)
(d) (5 sec 68^0 / cosec 22^0) + (3 sin 52^0 sec 38^0) / (cot 51^0 cot 39^0)
Solution:
(a) (sin 36^0 / cos 54^0) + (sec 31^0 / cosec 59^0)
This can be written as,
= {sin (90^0 – 54^0) / cos 54^0} + {sec (90^0 – 59^0) / cosec 59^0}
= (cos 54^0 / cos 54^0) + (cosec 59^0 / cosec 59^0)
= 1 + 1
We get,
= 2

(b) $(\tan 42^0 / \cot 48^0) - (\cos 33^0 / \sin 57^0)$
This can be written as,
$= \{\tan (90^0 - 48^0) / \cot 48^0\} - \{\cos (90^0 - 57^0) / \sin 57^0\}$
$= (\cot 48^0 / \cot 48^0) - (\sin 57^0 / \sin 57^0)$
$= 1 - 1$
We get,
$= 0$

(c) $(2 \sin 28^0 / \cos 62^0) + (3 \cot 49^0 / \tan 41^0)$
This can be written as,
$= \{2 \sin (90^0 - 62^0) / \cos 62^0\} + \{3 \cot (90^0 - 41^0) / \tan 41^0\}$
$= (2 \cos 62^0 / \cos 62^0) + (3 \tan 41^0 / \tan 41^0)$
$= 2 + 3$
We get,
$= 5$

(d) $(5 \sec 68^0 / \text{cosec } 22^0) + (3 \sin 52^0 \sec 38^0) / (\cot 51^0 \cot 39^0)$
This can be written as,
$= \{5 \sec (90^0 - 22^0) / \text{cosec } 22^0\} + \{3 \sin 52^0 \sec (90^0 - 52^0) / \cot 51^0 \cot (90^0 - 51^0)\}$
$= (5 \text{ cosec } 22^0 / \text{cosec } 22^0) + (3 \sin 52^0 \text{ cosec } 52^0 / \cot 51^0 \tan 51^0)$
$= 5 + \{3 \sin 52^0 \text{ x } (1 / \sin 52^0) / \cot 51^0 \text{ x } (1 / \cot 51^0)\}$
$= 5 + 3 / 1$
$= 5 + 3$
We get,
$= 8$

14. Express each of the following in terms of trigonometric ratios of angles between 0^0 and 45^0
(a) $\sin 65^0 + \cot 59^0$
(b) $\cos 72^0 - \cos 88^0$
(c) $\text{cosec } 64^0 + \sec 70^0$
(d) $\tan 77^0 - \cot 63^0 + \sin 57^0$
(e) $\sin 53^0 + \sec 66^0 - \sin 50^0$
(f) $\cos 84^0 + \text{cosec } 69^0 - \cot 68^0$
Solution:
(a) $\sin 65^0 + \cot 59^0$
This can be written as,
$= \sin (90^0 - 25^0) + \cot (90^0 - 31^0)$
We get,

= cos 25^0 + tan 31^0

(b) cos 72^0 – cos 88^0
This can be written as,
= cos (90^0 – 18^0) – cos (90^0 – 2^0)
We get,
= sin 18^0 – sin 2^0

(c) cosec 64^0 + sec 70^0
This can be written as,
= cosec (90^0 – 26^0) + sec (90^0 – 20^0)
We get,
= sec 26^0 + cosec 20^0

(d) tan 77^0 – cot 63^0 + sin 57^0
This can be written as,
= tan (90^0 – 13^0) – cot (90^0 – 27^0) + sin (90^0 – 33^0)
We get,
= cot 13^0 – tan 27^0 + cos 33^0

(e) sin 53^0 + sec 66^0 – sin 50^0
This can be written as,
= sin (90^0 – 37^0) + sec (90^0 – 24^0) – sin (90^0 – 40^0)
We get,
= cos 37^0 + cosec 24^0 – cos 40^0

(f) cos 84^0 + cosec 69^0 – cot 68^0
This can be written as,
= cos (90^0 – 6^0) + cosec (90^0 – 21^0) – cot (90^0 – 22^0)
We get,
= sin 6^0 + sec 21^0 – tan 22^0

15. Evaluate the following:
(a) sin 35^0 sin 45^0 sec 55^0 sec 45^0
(b) cot 20^0 cot 40^0 cot 45^0 cot 50^0 cot 70^0
(c) cos 39^0 cos 48^0 cos 60^0 cosec 42^0 cosec 51^0
(d) sin (35^0 + θ) – cos (55^0 – θ) – tan (42^0 + θ) + cot (48^0 – θ)
(e) tan (78^0 + θ) + cosec (42^0 + θ) – cot (12^0 – θ) – sec (48^0 – θ)
(f) (3 sin 37^0 / cos 53^0) – (5 cosec 39^0 / sec 51^0) + {(4 tan 23^0 tan 37^0 tan 67^0 tan 53^0) /

(cos 17^0 cos 67^0 cosec 73^0 cosec 23^0)}

(g) (sin 0^0 sin 35^0 sin 55^0 sin 75^0) / (cos 22^0 cos 64^0 cos 68^0 cos 90^0)

(h) {(2 sin 25^0 sin 35^0 sec 55^0 sec 65^0) / (5 tan 29^0 tan 45^0 tan 61^0)} + {(3 cos 20^0 cos 50^0 cot 70^0 cot 40^0) / (5 tan 20^0 tan 50^0 sin 70^0 sin 40^0)}

(i) {(3 $\sin^2 40^0$) / (4 $\cos^2 50^0$)} – {($\text{cosec}^2 28^0$) / (4 $\sec^2 62^0$)} + {(cos 10^0 cos 25^0 cos 45^0 cosec 80^0) / (2 sin 15^0 sin 45^0 sin 65^0 sec 75^0)}

(j) {(5 cot 5^0 cot 15^0 cot 25^0 cot 35^0 cot 45^0) / (7 tan 45^0 tan 55^0 tan 65^0 tan 75^0 tan 85^0)} + {(2 cosec 12^0 cosec 24^0 cos 78^0 cos 66^0) / (7 sin 14^0 sin 23^0 sec 76^0 sec 67^0)}

Solution:

(a) sin 35^0 sin 45^0 sec 55^0 sec 45^0

This can be written as,

= sin (90^0 – 55^0) x (1 / √2) x (1 / cos 55^0) x (√2)

= cos 55^0 x (1 / cos 55^0) x (1 / √2) x (√2)

We get,

= 1

(b) cot 20^0 cot 40^0 cot 45^0 cot 50^0 cot 70^0

This can be written as,

= cot (90^0 – 70^0) x cot (90^0 – 50^0) x 1 x cot 50^0 cot 70^0

= tan 70^0 x tan 50^0 x cot 50^0 x cot 70^0

= tan 70^0 x cot 70^0 x tan 50^0 x cot 50^0

= tan 70^0 x (1 / tan 70^0) x tan 50^0 x (1 / tan 50^0)

We get,

= 1

(c) cos 39^0 cos 48^0 cos 60^0 cosec 42^0 cosec 51^0

This can be written as,

= cos (90^0 – 51^0) x cos (90^0 – 42^0) x (1 / 2) x (1 / sin 42^0) x (1 / sin 51^0)

= sin 51^0 x sin 42^0 x (1 / 2) x (1 / sin 42^0) x (1 / sin 51^0)

We get,

= 1 / 2

(d) sin (35^0 + θ) – cos (55^0 – θ) – tan (42^0 + θ) + cot (48^0 – θ)

This can be written as,

= sin {90^0 – (55^0 – θ)} – cos (55^0 – θ) – tan {90^0 – (48^0 – θ)} + cot (48^0 – θ)

= cos (55^0 – θ) – cos (55^0 – θ) – cot (48^0 – θ) + cot (48^0 – θ)

We get,

= 0

(e) tan $(78^0 + \theta)$ + cosec $(42^0 + \theta)$ – cot $(12^0 - \theta)$ – sec $(48^0 - \theta)$
This can be written as,
= tan $\{90^0 - (12^0 - \theta)\}$ + cosec $\{90^0 - (48^0 - \theta)\}$ – cot $(12^0 - \theta)$ – sec $(48^0 - \theta)$
= cot $(12^0 - \theta)$ + sec $(48^0 - \theta)$ – cot $(12^0 - \theta)$ – sec $(48^0 - \theta)$
We get,
= 0

(f) (3 sin 37^0 / cos 53^0) – (5 cosec 39^0 / sec 51^0) + {(4 tan 23^0 tan 37^0 tan 67^0 tan 53^0) / (cos 17^0 cos 67^0 cosec 73^0 cosec 23^0)}
This can be written as,
= {3 sin $(90^0 - 53^0)$ / cos 53^0} – {5 cosec $(90^0 - 51^0)$ / sec 51^0} + [{4 tan $(90^0 - 67^0)$ tan $(90^0 - 53^0)$ x (1 / cot 67^0) x (1 / cot 53^0)}] / {cos $(90^0 - 73^0)$ cos $(90^0 - 23^0)$ x (1 / sin 73^0) x (1 / sin 23^0)}
= (3 cos 53^0 / cos 53^0) – (5 sec 51^0 / sec 51^0) + [4 cot 67^0 cot 53^0 x (1 / cot 67^0) x (1 / cot 53^0] / {sin 73^0 sin 23^0 x (1 / sin 73^0) x (1 / sin 23^0)}
On calculating further, we get,
= 3 – 5 + 4
= 2

(g) (sin 0^0 sin 35^0 sin 55^0 sin 75^0) / (cos 22^0 cos 64^0 cos 68^0 cos 90^0)
= 0 x sin 35^0 sin 55^0 sin 75^0) / (cos 22^0 cos 64^0 cos 68^0 x 0)
(∵ sin 0^0 = 0 and cos 90^0 = 0)
We get,
= 0

(h) {(2 sin 25^0 sin 35^0 sec 55^0 sec 65^0) / (5 tan 29^0 tan 45^0 tan 61^0)} + {(3 cos 20^0 cos 50^0 cot 70^0 cot 40^0) / (5 tan 20^0 tan 50^0 sin 70^0 sin 40^0)}
This can be written as,
= {2 sin $(90^0 - 65^0)$ sin $(90^0 - 55^0)$ sec 55^0 sec 65^0} / {5 tan $(90^0 - 61^0)$ x 1 x tan 61^0} + {3 cos $(90^0 - 70^0)$ cos $(90^0 - 40^0)$ cot $(90^0 - 20^0)$ cot $(90^0 - 50^0)$} / (5 tan 20^0 tan 50^0 sin 70^0 sin 40^0)
= {2 cos 65^0 cos 55^0 x (1 / cos 55^0) x (1 / cos 65^0)} / {5 cot 61^0 x 1 x (1 / cot 61^0)} + {3 sin 70^0 sin 40^0 tan 20^0 tan 50^0} / (5 tan 20^0 tan 50^0 sin 70^0 sin 40^0)
On calculating further, we get,
= (2 / 5) + (3 / 5)
= (2 + 3) / 5
= 5 / 5
= 1

(i) $\{(3 \sin^2 40^0) / (4 \cos^2 50^0)\} – \{(\text{cosec}^2 28^0) / (4 \sec^2 62^0)\} + \{(\cos 10^0 \cos 25^0 \cos 45^0 \text{ cosec } 80^0) / (2 \sin 15^0 \sin 45^0 \sin 65^0 \sec 75^0)\}$

$= \{3 \sin^2 (90^0 – 50^0) / 4 \cos^2 50^0\} – \{\text{cosec}^2 (90^0 – 62^0) / 4 \sec^2 62^0\} + \{\cos (90^0 – 80^0) \cos 25^0 \text{ x } (1 / \sqrt{2}) \text{ x } (1 / \sin 80^0\} / \{2 \sin (90^0 – 75^0) \text{ x } (1 / \sqrt{2}) \text{ x } \sin (90^0 - 25^0) \text{ x } (1 / \cos 75^0)$

$= 3 \cos^2 50^0 / 4 \cos^2 50^0) – (\sec^2 62^0 / 4 \sec^2 62^0) + (\sin 80^0 \text{ x } \cos 25^0 \text{ x } (1 / \sin 80^0) / \{2 \cos 75^0 \text{ x } \cos 25^0 \text{ x } (1 / \cos 75^0)\}$

On further calculation, we get,

$= (3 / 4) – (1 / 4) + (1 / 2)$

$= (1 / 2) + (1 / 2)$

$= 1$

(j) $\{(5 \cot 5^0 \cot 15^0 \cot 25^0 \cot 35^0 \cot 45^0) / (7 \tan 45^0 \tan 55^0 \tan 65^0 \tan 75^0 \tan 85^0)\} + \{(2 \text{ cosec } 12^0 \text{ cosec } 24^0 \cos 78^0 \cos 66^0) / (7 \sin 14^0 \sin 23^0 \sec 76^0 \sec 67^0)\}$

$= \{5 \cot (90^0 – 85^0) \cot (90^0 – 75^0) \cot (90^0 – 65^0) \cot (90^0 – 55^0) \text{ x } 1\} / (7 \text{ x } 1 \text{ x } \tan 55^0 \tan 65^0 \tan 75^0 \tan 85^0) + \{2 \text{ cosec } (90^0 – 78^0) \text{ cosec } (90^0 – 66^0) \cos 78^0 \cos 66^0\} / \{7 \sin (90^0 – 76^0) \sin (90^0 – 67^0) \sec 76^0 \sec 67^0\}$

$= (5 \tan 85^0 \tan 75^0 \tan 65^0 \tan 55^0) / (7 \text{ x } \tan 55^0 \tan 65^0 \tan 75^0 \tan 85^0) + \{(2 \sec 78^0 \sec 66^0 \text{ x } (1 / \sec 78^0) \text{ x } (1 / \sec 66^0)\} / \{7 \cos 76^0 \cos 67^0 \text{ x } (1 / \cos 76^0) \text{ x } (1 / \cos 67^0)\}$

On further calculation, we get,

$= (5 / 7) + (2 / 7)$

$= (5 + 2) / 7$

$= 7 / 7$

$= 1$