EXERCISE 13.1 PAGE NO: 122

Write the correct answer in each of the following:

1. The radius of a sphere is 2*r*, then its volume will be

(A) $\frac{4}{3}\pi r^3$ (B) $4\pi r^3$ (C) $\frac{8\pi r^3}{3}$ (D) $\frac{32}{3}\pi r^3$

Solution:

(D) $(32\pi r^3)/3$

Explanation:

Given, the radius of a sphere is 2r.

The volume of a sphere = $(4/3) \times (\text{radius})^3$

Thus, the volume of the given sphere = $(4/3) \times \pi \times (2r)^3$

$= (4/3) \times \pi \times 8 \times r^3$

$= (32\pi r^3/3)$

Hence, option D is the correct answer.

2. The total surface area of a cube is 96 cm2. The volume of the cube is:

(A) 8 cm^3 (B) 512 cm^3 (C) 64 cm^3 (D) 27 cm^3

Solution:

(C) 64 cm^3

Explanation:

The surface area of a cube is 96 cm^2

Let the length of the cube is l cm.

Thus,

$6 \times l^2 = 96$

According to formula

$l^2 = 96/6$

$l^2 = 16$

$l = 4$

Thus the length of the cube is 4 cm.

Volume of a cube $= l^3$

$= 4^3$

$= 64$

Thus volume is 64 cm^3 which is option c.

Hence, option C is the correct answer.

3. A cone is 8.4 cm high and the radius of its base is 2.1 cm. It is melted and recast into a sphere. The radius of the sphere is:

(A) 4.2 cm (B) 2.1 cm (C) 2.4 cm (D) 1.6 cm

Solution:

(B) 2.1 cm

Explanation:

Height of cone, h = 8.4cm

Radius of base, r = 2.1cm

Thus volume of a cone = (1/3) × π × h × r^2

=(1/3) × π × 8.4 × $(2.1)^2$

Now when it is melted to form a sphere, say of radius r1 cm, the volumes of both are going to be equal.

Volume of sphere = (4/3) × π × r_1^3

$$\therefore \frac{4}{3}\pi \times r_1^3 = \frac{1}{3}\pi \times 2.1^2 \times 8.4$$

$$= 2.1^2 \times 8.4$$

$$r_1^3 = 2.1^2 \times 2.1$$

$$r_1^3 = 2.1^3$$

$$\therefore r_1 = 2.1\text{cm}$$

Thus the radius of the sphere is 2.1cm which is option b.

Hence, option B is the correct answer.

4. In a cylinder, radius is doubled and height is halved, curved surface area will be
(A) halved (B) doubled (C) same (D) four times

Solution:

(C) same

Explanation:

Let the radius of a cylinder = r unit

Let the height of a cylinder = h unit.

According to question,

Radius is doubled = 2r

Height is halved = h/2

Then,

Curved surface area of a cylinder = 2 × π × r × h

And according to the above condition,

Curved surface area = 2 × π × 2 × r × h/2

= 2 × π × r × h

This is same as the curved surface area of the cylinder with radius r and height h.

Hence, option C is the correct answer.

5. The total surface area of a cone whose radius is r/2 and slant height 2l is

(A) $2\pi r(l + r)$ (B) $\pi r\left(l + \frac{r}{4}\right)$ (C) $\pi r(l + r)$ (D) $2\pi rl$

Solution:

(B) πr(l+(r/4))

Explanation:

Given, radius of a cone is r/2 and slant height is 2L.

Total surface area = π × radius × slant height + π × radius^2

$$= \pi \times \frac{r}{2} \times 2 \times l + \pi \times \left(\frac{r}{2}\right)^2$$

$$= \pi r l + \frac{\pi r^2}{4}$$

$$= \pi r \left(l + \frac{r}{4}\right)$$

Hence, option B is the correct answer

EXERCISE 13.2 PAGE NO: 124

Write True or False and justify your answer in each of the following:

1. The volume of a sphere is equal to two-third of the volume of a cylinder whose height and diameter are equal to the diameter of the sphere.

Solution:

True

Justification:

Let the radius of the sphere = r.

According to the question,

height and diameter of cylinder = diameter of sphere.

So, the radius of the cylinder = r

And, the height of the cylinder = 2r

We know that,

Volume of sphere = 2/3 volume of cylinder

$$\Rightarrow \frac{4}{3}\pi r^3 = \frac{2}{3}(\pi r^2 \times 2r) = \frac{4}{3}\pi r^3$$

Hence, the given statement "the volume of a sphere is equal to two-third of the volume of a cylinder whose height and diameter are equal to the diameter of the sphere" is true.

2. If the radius of a right circular cone is halved and height is doubled, the volume will remain unchanged.

Solution:

False

Justification:

Let the original radius of the cone = r

Let height of the cone = h.

The volume of cone = 1/3 $\pi r^2 h$

Now, when radius of a height circular cone is halved and height is doubled, then

$$V = \frac{1}{3}\pi\left(\frac{r}{2}\right)^2 \times 2h = \frac{1}{3}\pi \times \frac{r^2}{4} \times 2h = \frac{1}{2}\left(\frac{1}{3}\pi r^2 h\right)$$

We can observe that the new volume = half of the original volume.

Hence, the given statement "if the radius of a right circular cone is halved and height is doubled, the volume will remain unchanged" is false.

3. In a right circular cone, height, radius and slant height do not always be sides of a right triangle.

Solution:

Consider a right circular cone, with

Height = h

Radius = r

And slant height = l

We know, right triangle = one angle 90°

Using Pythagoras theorem in
⇒ $l^2 = h^2 + r^2$
This justifies that height, radius and slant height of cone can always be the sides of a right triangle.
Hence, the given statement "in a right circular cone, height, radius and slant height do not always be sides of a right triangle" is true.

4. If the radius of a cylinder is doubled and its curved surface area is not changed, the height must be halved.
Solution:
True
Justification:
Let radius of the cylinder = r
Height of the cylinder = h
Then, curved surface area of the cylinder, CSA = 2πrh
According to the question,
Radius is doubled and curved surface area is not changed.
New radius of the cylinder, R = 2r
New curved surface area of the cylinder, CSA' = 2πrh …(i)
Alternate case:
When R = 2r and CSA' = 2πrh
But curved surface area of cylinder in this case, CSA'= 2πRh = 2π(2r)h = 4πrh …(ii)
Comparing equations (i) and (ii),
We get,
Since, 2πrh ≠ 4πrh
equation (i) ≠ equation (ii)
Thus, if h = h/2 (height is halved)
Then,
CSA' = 2π(2r)(h/2) = 2πrh
Hence, the given statement "If the radius of a cylinder is doubled and its curved surface area is not changed, the height must be halved" is true.

5. The volume of the largest right circular cone that can be fitted in a cube whose edge is 2r equals to the volume of a hemisphere of radius r.
Solution:
According to the question,
Edge of cube, l = 2r
Then, diameter of the cone = 2r
⇒ radius of the cone = 2r/2
= r
Height of the cone, h = height of the cube
= 2r
Volume of the cone is given by,
Volume of cone = $1/3\ \pi r^2 h$
= $1/3\ \pi r^2(2r)$
= $2/3\ \pi r^3$

= Volume of hemisphere of radius r

Hence, the given statement "the volume of the largest right circular cone that can be fitted in a cube whose edge is 2r equals to the volume of a hemisphere of radius r" is true.

EXERCISE 13.3 PAGE NO: 126

1. Metal spheres, each of radius 2 cm, are packed into a rectangular box of internal dimensions 16 cm × 8 cm × 8 cm. When 16 spheres are packed the box is filled with preservative liquid. Find the volume of this liquid. Give your answer to the nearest integer. [Use π =3.14]

Solution:

According to the question,
Radius of each sphere, r = 2 cm
Volume of a sphere is given by
Volume of 1 sphere = $4/3\ \pi r^3$
Since there are 16 spheres in our question,
Volume of 16 spheres = $16 \times 4/3\ \pi r^3$
$= 16 \times 4/3 \times 3.14 \times 2^3$
$= 535.89\ cm^3$
We know that,
Dimensions of rectangular box = 16 cm × 8 cm × 8 cm
Volume of rectangular box = $16 \times 8 \times 8 = 1024\ cm^3$
In order to find volume of the liquid that is filled in rectangular box,
We need to find the space left in the rectangular box after the space occupied by the spheres.
So, we can say that,
Volume of the liquid = (Volume of the rectangular box) – (Volume of the 16 spheres)
⇒ Volume of the liquid = 1024 – 535.89
$= 488.11\ cm^3$
Thus, volume of this liquid is $488.11\ cm^3$.

2. A storage tank is in the form of a cube. When it is full of water, the volume of water is 15.625 m^3.If the present depth of water is 1.3 m, find the volume of water already used from the tank.

Solution:

When the cubical tank is full:
Volume of water = Volume of cube = $15.625\ m^3$
So, we know that,
Volume of cube = $(\text{length of edge of cube})^3$
⇒ $(\text{length of edge of cube})^3 = 15.625$
⇒ length of edge of cube = $\sqrt[3]{15.625}$
$= 2.5$ m
We know that,
Length of the edge of the cube = 2.5 m.
When the present depth of the water is 1.3 m:
Length of tank = 2.5 m
Breadth of tank = 2.5 m
So, volume of water upto 1.3 m depth = length × breadth × depth
$= 2.5 \times 2.5 \times 1.3$
$= 8.125\ m^3$
Then, volume of water already used from the tank = (Volume of tank when it was full of water) – (Volume of water when depth is 1.3 m)

= 15.625 – 8.125
= 7.5 m^3

Thus, volume of water already used from the tank is 7.5 m^3.

3. Find the amount of water displaced by a solid spherical ball of diameter 4.2 cm, when it is completely immersed in water.

Solution:

Water displaced when a solid spherical ball is immersed completely in water equals to its own volume.
According to the question,
Diameter of spherical ball = 4.2 cm
⇒ radius of spherical ball = 4.2/2 = 2.1 cm
So, volume of a sphere = $4/3\ \pi r^3$
$(4/3)\ (22/7)(2.1)^3 = 38.81$
Thus, volume of water displaced is 38.81 cm^3.

4. How many square metres of canvas is required for a conical tent whose height is 3.5 m and the radius of the base is 12 m?

Solution:

According to the question,
Dimensions of conical tent are:
Height = 3.5 m
Radius = 12 m
Curved surface area of cone = $\pi r\sqrt{(r^2 + h^2)}$
$= 22/7 \times 12\ \sqrt{(12^2 + 3.5^2)}$
$= 22/7 \times 12 \times \sqrt{156.25}$
$= 22/7 \times 12 \times 12.5$
= 471.43 m^2
Since, area of canvas = curved surface area of conical tent
Therefore, area of canvas required is 471.43 m^2.

EXERCISE 13.4 PAGE NO: 127

1. A cylindrical tube opened at both the ends is made of iron sheet which is 2 cm thick. If the outer diameter is 16 cm and its length is 100 cm, find how many cubic centimeters of iron has been used in making the tube?

Solution:

According to the question,
Outer diameter d = 16cm
Then,
Outer radius r – 16/2 = 8cm
Height = length = 100cm
Thickness of iron sheet = 2cm
Volume of cylinder = $\pi r^2 h$, where r= outer radius and $\pi = 3.14$
Thus, Volume of cylinder = $\pi r^2 h$
$= 3.14 \times (8)^2 \times 100$
$= 20{,}096 \text{ cm}^3$
Now, inner diameter = outer diameter – 2× thickness of iron sheet
Inner diameter = 16-(2× 2) = 12cm
Inner radius R = 12/2 = 6cm
Thus, Volume of hollow space = $\pi R^2 h$, where R= inner radius and $\pi = 3.14$
$= \pi R^2 h$
$= 3.14 \times (6)^2 \times 100$
$= 11{,}304 \text{ cm}^3$
Thus,
Volume of iron used = Volume of cylinder – Volume of hollow space
$= (20{,}096 – 11{,}304) \text{ cm}^3$
$= 8800 \text{ cm}^3$

2. A semi-circular sheet of metal of diameter 28cm is bent to form an open conical cup. Find the capacity of the cup.

Solution:

According to the question,
Diameter of semi circular sheet = 28cm
Radius of semi circular sheet (r) = 28/2
= 14cm
Semi Circular sheet is bent to form an open conical cup
Thus, slant height of a conical cup (l)=radius of semicircular sheet(r) = 14cm
We know that,
Circumference of base of a cone = $2\pi R$,
(where, R = radius of a cone and circumference of semi circle = πr)
Thus,
Circumference of base of a cone = Circumference of a Semi circle
$\Rightarrow 2\pi R = \pi r$

$$\Rightarrow R = \frac{\pi r}{2\pi} = \frac{r}{2} = \frac{14}{2} = 7\text{cm}$$

To find height,
$R^2 + h^2 = l^2$
Where,
R=radius of a cone
h=height of a cone
l=slant height of a cone
$\Rightarrow (7)^2 + h^2 = (14)^2$
$\Rightarrow 49 + h^2 = 196$
$\Rightarrow h^2 = 196 - 49 = 147$
$\Rightarrow h = \sqrt{147} = 7\sqrt{3}$ cm
Thus, volume of a cone = 1/3 $\pi R^2 h$

$$\Rightarrow \text{Volume of a cone} = \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 7\sqrt{3} = \frac{1078\sqrt{3}}{3}\ \text{cm}^3 = \frac{1078}{\sqrt{3}}\ \text{cm}^3$$

$$\Rightarrow \text{Volume of a cone} = \frac{1078}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\ \text{cm}^3 = \frac{1078\sqrt{3}}{3}\ \text{cm}^3$$

$$\Rightarrow \text{Volume of a cone} = \frac{1078\sqrt{3}}{3}\ \text{cm}^3 = 622.38\ \text{cm}^3$$

3. A cloth having an area of 165 m^2 is shaped into the form of a conical tent of radius 5 m
(i) How many students can sit in the tent if a student, on an average, occupies 5/7 m^2 on the ground?
(ii) Find the volume of the cone.
Solution:

According to the question,
Area of cloth = 165m^2
Radius of conical tent = 5m
Area covered by 1 student = 5/7 m^2
Curved surface area of cone = πrl
Thus, curved surface area of a conical; tent = πrl

$$\Rightarrow 165 = \frac{22}{7} \times 5 \times l$$

$$\Rightarrow l = \frac{165 \times 7}{22 \times 5} = \frac{21}{2} = 10.5\ \text{m}$$

(i)

$$\text{No. of students} = \frac{\text{Area of circular base of a cone}}{\text{Area covered by 1 student}}$$

$$\Rightarrow \text{No. of student} = \frac{\pi r^2}{\frac{5}{7}} = \frac{\left(\frac{22}{7} \times 5^2\right)}{\frac{5}{7}} = 22 \times 5 = 110$$

$$\text{No. of student occupies} \frac{5}{7}\ \text{m}^2 \text{ of area} = 110$$

(ii) Height of a cone,
$r^2 + h^2 = l^2$
Where,
r=radius of a cone
h=height of a cone
l=slant height of a cone
$\Rightarrow (5)^2 + h^2 = (10.5)^2$
$\Rightarrow 25 + h^2 = 110.25$
$\Rightarrow h^2 = 110.25 - 25 = 85.25$
$\Rightarrow h = \sqrt{85.25} = 9.23$ m
Volume of a come = (1/3) $\pi r^2 h$

$$\text{Volume of a cone} = \frac{1}{3} \times \frac{22}{7} \times 5^2 \times 9.23 = \frac{5076.5}{21} = 241.73 \text{ m}^3$$

4. The water for a factory is stored in a hemispherical tank whose internal diameter is 14 m. The tank contains 50 kilolitres of water. Water is pumped into the tank to fill to its capacity. Calculate the volume of water pumped into the tank.

Solution:

According to the question,
Internal diameter of hemispherical tank = 14 m
Internal radius of hemispherical tank = 14/2 m = 7m
Tank contains 50kl = $50m^3$ of water (since, 1kl = $1m^3$)
Volume of a hemispherical tank = 2/3 πr^3

$$\text{Volume of a hemispherical tank} = \frac{2}{3} \times \frac{22}{7} \times (7)^3 = \frac{2156}{3} = 718.66 \text{ m}^3$$

Volume of water pumped into the tank = Volume of hemispherical tank – $50m^3$
= 718.66 – 50
= 668.66 m^3