EXERCISE 8.1 PAGE NO: 73

Write the correct answer in each of the following:

1. Three angles of a quadrilateral are 75º, 90º and 75º. The fourth angle is

(A) 90º
(B) 95º
(C) 105º
(D) 120º

Solution:

(D) 120º

Explanation:

According to the question,
Three angles of quadrilateral are 75°, 90° and 75°
Consider the fourth angle to be x.
We know that,
Sum of all angles of a quadrilateral = 360°
$\Rightarrow 75° + 90° + 75° + x = 360°$
$\Rightarrow 240° + x = 360°$
$\Rightarrow x = 360° – 240°$
$\Rightarrow x = 120°$
Hence, the fourth angle is 120°.
Therefore, option (D) is the correct answer.

2. A diagonal of a rectangle is inclined to one side of the rectangle at 25º. The acute angle between the diagonals is

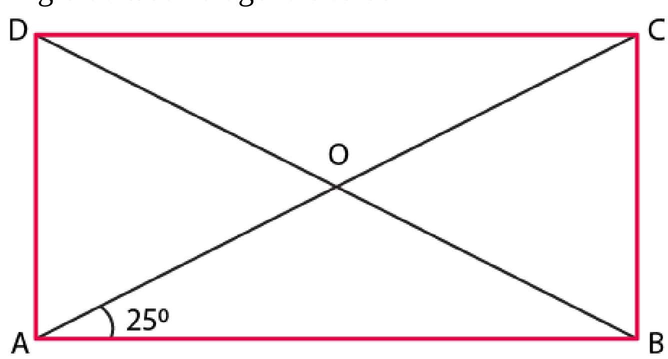
(A) 55º
(B) 50º
(C) 40º
(D) 25º

Solution:

(B) 50º

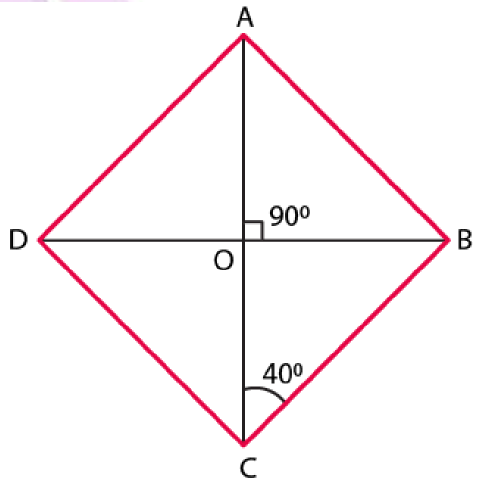
Explanation:

According to the question,
A diagonal of a rectangle is inclined to one side of the rectangle at 25º
i.e., Angle between a side of rectangle and its diagonal = 25°
Consider the acute angle between diagonals to be = x



We know that diagonals of a rectangle are equal in length i.e.,
AC = BD
Dividing RHS and LHS by 2,
⇒ ½ AC = ½ BD
Since, O is mid-point of AC and BD
⇒ OD = OC
Since, angles opposite to equal sides are equal
⇒ $\angle y = 25°$
We also know that,
Exterior angle is equal to the sum of two opposite interior angles.
So, $\angle BOC = \angle ODC + \angle OCD$
⇒ $\angle x = \angle y + 25°$
⇒ $\angle x = 25° + 25°$
⇒ $\angle x = 50°$
Hence, the acute angle between diagonals is 50°.
Therefore, option (B) is the correct answer.

3. ABCD is a rhombus such that ∠ACB = 40º. Then ∠ADB is
(A) 40º
(B) 45º
(C) 50º
(D) 60º
Solution:
(C) 50º
Explanation:
According to the question,
ABCD is a rhombus
$\angle ACB = 40°$



∵ $\angle ACB = 40°$
⇒ $\angle OCB = 40°$
∵ AD ∥ BC
⇒ $\angle DAC = \angle BCA = 40°$ [Alternate interior angles]
⇒ $\angle DAO = 40°$

Since, diagonals of a rhombus are perpendicular to each other
We have,
$\angle AOD = 90°$
We know that,
Sum of all angles of a triangle = 180°
$\Rightarrow \angle AOD + \angle ADO + \angle DAO = 180°$
$\Rightarrow 90° + \angle ADO + 40° = 180°$
$\Rightarrow 130° + \angle ADO = 180°$
$\Rightarrow \angle ADO = 180° - 130°$
$\Rightarrow \angle ADO = 50°$
$\Rightarrow \angle ADB = 50°$
Hence, $\angle ADB = 50°$
Therefore, option (C) is the correct answer.

4. The quadrilateral formed by joining the mid-points of the sides of a quadrilateral PQRS, taken in order, is a rectangle, if
(A) PQRS is a rectangle
(B) PQRS is a parallelogram
(C) diagonals of PQRS are perpendicular
(D) diagonals of PQRS are equal.
Solution:

(C) diagonals of PQRS are perpendicular
Explanation:
Let the rectangle be ABCD,
We know that,
Diagonals of rectangle are equal
∴ AC = BD
⇒ PQ = QR
∴ PQRS is a rhombus
Diagonals of a rhombus are perpendicular.
Hence, diagonals of PQRS are perpendicular
Therefore, option (C) is the correct answer.

5. The quadrilateral formed by joining the mid-points of the sides of a quadrilateral PQRS, taken in order, is a rhombus, if
(A) PQRS is a rhombus
(B) PQRS is a parallelogram
(C) diagonals of PQRS are perpendicular
(D) diagonals of PQRS are equal.
Solution:

(D) diagonals of PQRS are equal.
Explanation:
Since, ABCD is a rhombus
We have,
AB = BC = CD = DA
Now,

Since, D and C are midpoints of PQ and PS
By midpoint theorem,
We have,
DC = ½ QS
Also,
Since, B and C are midpoints of SR and PS
By midpoint theorem
We have,
BC = ½ PR
Now, again, ABCD is a rhombus
∴ BC = CD
⇒ ½ QS = ½ PR
⇒ QS = PR
Hence, diagonals of PQRS are equal
Therefore, option (D) is the correct answer.

6. If angles A, B, C and D of the quadrilateral ABCD, taken in order, are in the ratio 3:7:6:4, then ABCD is a
(A) rhombus
(B) parallelogram
(C) trapezium
(D) kite
Solution:

(C) trapezium

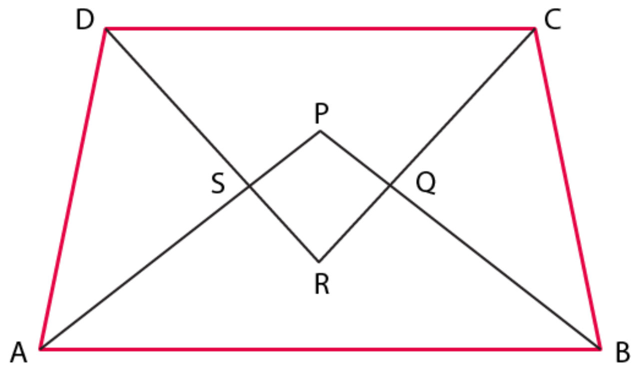
Explanation:
As angle A, B, C and D of the quadrilateral ABCD, taken in order, are in the ratio 3: 7: 6: 4,
We have the angles A, B, C and D = 3x, 7x, 6x and 4x.
Now, sum of the angle of a quadrilateral = 360^o.
$3x + 7x + 6x + 4x = 360^o$
$\Rightarrow 20x = 360^o$
$\Rightarrow x = 360 \div 20 = 18^o$
So, the angles A, B, C and D of quadrilateral ABCD are,
$\angle A = 3\times18^o = 54^o$,
$\angle B = 7\times18^o = 126^o$
$\angle C = 6\times18^o = 108^o$
$\angle D = 4\times18^o = 72^o$
AD and BC are two lines cut by a transversal CD
Now, sum of angles ∠C and ∠D on the same side of transversal,
$\angle C + \angle D = 108^o + 72^o = 180$
Hence, AD|| BC
So, ABCD is a quadrilateral in which one pair of opposite sides are parallel.
Hence, ABCD is a trapezium.
Therefore, option (C) is the correct answer.

7. If bisectors of ∠A and ∠B of a quadrilateral ABCD intersect each other at P, of ∠B and ∠C at Q, of ∠C and ∠D at R and of ∠D and ∠A at S, then PQRS is a

(A) rectangle
(B) rhombus
(C) parallelogram
(D) quadrilateral whose opposite angles are supplementary
Solution:

(D) quadrilateral whose opposite angles are supplementary

Explanation:



We know that,
Sum of all angles of a quadrilateral = 360°
⇒ ∠A + ∠B + ∠C + ∠D = 360°
Dividing LHS and RHS by 2,
⇒ ½ (∠A + ∠B + ∠C + ∠D) = ½ × 360° = 180°
Since, AP, PB, RC and RD are bisectors of ∠A, ∠B, ∠C and ∠D
⇒ ∠PAB + ∠ABP + ∠RCD + ∠RDC = 180° … (1)
We also know that,
Sum of all angles of a triangle = 180°
∠PAB + ∠APB + ∠ABP = 180°
⇒ ∠PAB + ∠ABP = 180° – ∠APB …(2)
Similarly,
∴ ∠RDC + ∠RCD + ∠CRD = 180°
⇒ ∠RDC + ∠RCD = 180° – ∠CRD …(3)
Substituting the value of equations (2) and (3) in equation (1),
180° – ∠APB + 180° – ∠CRD = 180°
⇒ 360° – ∠APB – ∠CRD = 180°
⇒ ∠APB + ∠CRD = 360° – 180°
⇒ ∠APB + ∠CRD = 180° …(4)
Now,
∠SPQ = ∠APB [vertically opposite angles]
∠SRQ = ∠DRC [vertically opposite angles]
Substituting in equation (4),
⇒ ∠SPQ + ∠SRQ = 180°
Hence, PQRS is a quadrilateral whose opposite angles are supplementary.
Therefore, option (D) is the correct answer.

EXERCISE 8.2 PAGE NO: 75

1. Diagonals AC and BD of a parallelogram ABCD intersect each other at O. If OA = 3 cm and OD = 2 cm, determine the lengths of AC and BD.

Solution:

According to the question,
OA = 3 cm
OD = 2 cm
We know that,
Diagonals of parallelogram bisect each other.
Then,
AC = 2AO
AC = 2 × 3 cm
AC = 6 cm
And,
BD = 2OD
BD = 2 × 2 cm
BD = 4 cm
Hence, AC = 6 cm and BD = 4cm

2. Diagonals of a parallelogram are perpendicular to each other. Is this statement true? Give reason for your answer.

Solution:

The statement "diagonals of a parallelogram are perpendicular to each other" is false.
Justification:
Diagonals of a parallelogram bisect each other but not at 90°.
So, they are not perpendicular to each other.
Hence, this statement is false.

3. Can the angles 110º, 80º, 70º and 95º be the angles of a quadrilateral? Why or why not?

Solution:

The angles 110º, 80º, 70º and 95º cannot be the angles of a quadrilateral.
Justification:
We know that,
Sum of all angles of a quadrilateral = 360°
Sum of given angles,
110° + 80° + 70° + 95° = 355° ≠ 360°
Hence, 110°, 80°, 70° and 95° cannot be the angles of a quadrilateral.

4. In quadrilateral ABCD, ∠A + ∠D = 180º. What special name can be given to this quadrilateral?

Solution:

According to the question,
In quadrilateral ABCD, ∠A + ∠D = 180º
We know that,
In a trapezium,

Sum of co-interior angles = 180°
Hence, the given quadrilateral is a trapezium.

5. All the angles of a quadrilateral are equal. What special name is given to this quadrilateral?
Solution:

According to the question,
All the angles of a quadrilateral are equal.
Suppose all the angles of the quadrilateral = x
We know that,
Sum of all angles of a quadrilateral = 360°
$\Rightarrow x + x + x + x = 360°$
$\Rightarrow 4x = 360°$
$\Rightarrow x = 360°/4$
$\Rightarrow x = 90°$
Hence, the quadrilateral is a rectangle.

6. Diagonals of a rectangle are equal and perpendicular. Is this statement true? Give reason for your answer.
Solution:

The statement "diagonals of a rectangle are equal and perpendicular" is false.
We know that,
Diagonals of a rectangle bisect each other.
Therefore, they are equal but they are not perpendicular.
Hence, the statement is not true.

7. Can all the four angles of a quadrilateral be obtuse angles? Give reason for your answer.
Solution:

All the four angles of a quadrilateral cannot be obtuse angles.
Justification:
We know that,
Sum of all angles of a quadrilateral = 360°
So, at least one angle should be acute angle.
Hence, all the four angles of a quadrilateral cannot be obtuse angles.

EXERCISE 8.3

1. One angle of a quadrilateral is of 108º and the remaining three angles are equal. Find each of the three equal angles.

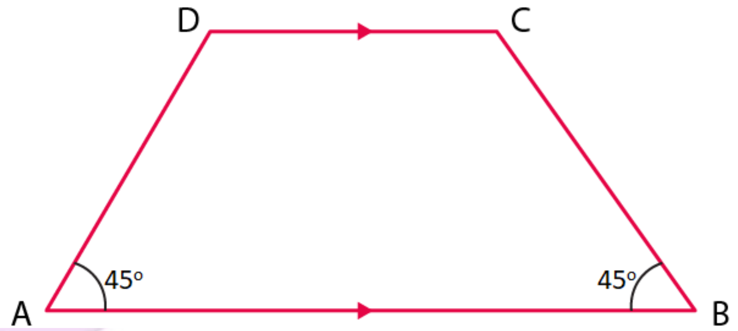
Solution:

Let the remaining three equal angles be x.
We know,
Sum of all interior angles of a quadrilateral is = 360^o
$108^o + x + x + x = 360^o$
$108^o + 3x = 360^o$
$3x = 360^o – 108^o$
$3x = 252^o$
$x = 252/3$
$x = 84^o$
Each of three equal angles, $x = 84^o$.

2. ABCD is a trapezium in which AB || DC and ∠A = ∠B = 45º. Find angles C and D of the trapezium.

Solution:

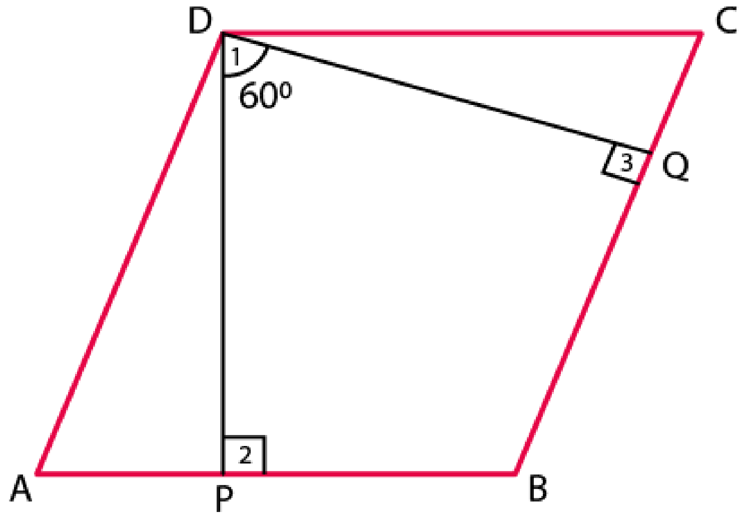
According to the question,
ABCD is a trapezium
$\angle A = \angle B = 45^o$



We know that,
Angles opposite to each other in quadrilateral are supplementary.
Then, we have,
$\angle A + \angle C = 180^o$
$45^o + \angle C = 180^o$
$\angle C = 180^o – 45^o$
$\angle C = 135^o$
Similarly,
We have,
$\angle B + \angle D = 180^o$
$45^o + \angle D = 180^o$
$\angle D = 180^o – 45^o$
$\angle D = 135^o$

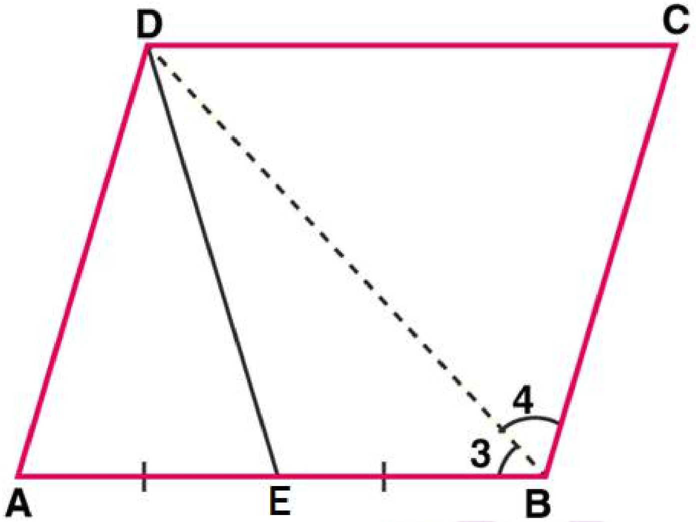
3. The angle between two altitudes of a parallelogram through the vertex of an obtuse angle of the parallelogram is 60º. Find the angles of the parallelogram.

Solution:



According to the question,
ABCD is parallelogram,
DP ⊥ AB
DQ ⊥ BC.
$\angle PDQ = 60^o$
In quad. DPBQ,
Using angle sum property of a quadrilateral,
We have,
$\angle PDQ + \angle Q + \angle P + \angle B = 360^o$
$60^o + 90^o + 90^o + \angle B = 360^o$
$240^o + \angle B = 360^o$
$\angle B = 360^o - 240^o$
$\angle B = 120^o$
Since, opposite angles in parallelogram are equal,
We have,
$\angle B = \angle D = 120^o$
Since, opposite sides are parallel in parallelogram,
We have,
AB||CD
Also, since sum of adjacent interior angles is 180º,
We have,
$\angle B + \angle C = 180^o$
$120^o + \angle C = 180^o$
$\angle C = 180^o - 120^o$
$\angle C = 60^o$
Since, opposite angles in parallelogram are equal,
We have,
$\angle C = \angle A = 60^o$

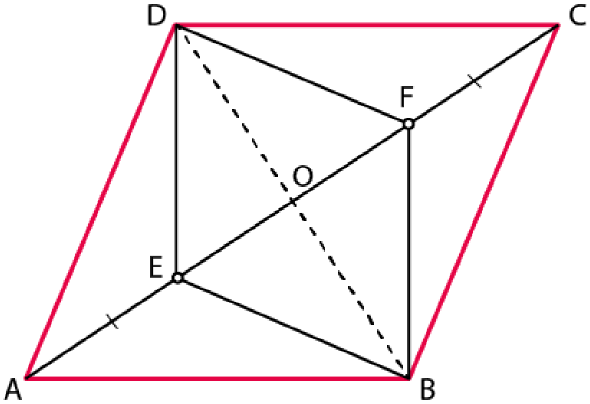
4. ABCD is a rhombus in which altitude from D to side AB bisects AB. Find the angles of the rhombus.
Solution:



According to the question,
We have,
ABCD is a rhombus.
DE is the altitude on AB then AE = EB.
In ΔAED and ΔBED,
We have,
DE = DE (common line)
∠AED = ∠BED (right angle)
AE = EB (DE is an altitude)
∴ ΔAED ≅ ΔBED by SAS property.
∴ AD = BD (by C.P.C.T)
But AD = AB (sides of rhombus are equal)
⇒ AD = AB = BD
∴ ABD is an equilateral triangle.
∴ $\angle A = 60^o$
Since, opposite angles of rhombus are equal, we get,
⇒ $\angle A = \angle C = 60^o$
We also know that,
Sum of adjacent angles of a rhombus = supplementary.
So,
$\angle ABC + \angle BCD = 180^o$
$\angle ABC + 60^o = 180^o$
$\angle ABC = 180^o – 60^o = 120^o$
Since, opposite angles of rhombus are equal, we get,
$\angle ABC = \angle ADC = 120^o$
Hence, Angles of rhombus are:
$\angle A = 60^o$, $\angle C = 60^o$, $\angle B = 120^o$, $\angle D = 120^o$

5. E and F are points on diagonal AC of a parallelogram ABCD such that AE = CF. Show that BFDE is a parallelogram.

Solution:



Construction:

Join BD, meeting AC at O.

According to the question,

Since diagonals of a parallelogram bisect each other,

We get,

OA = OC and OD = OB.

And,

OA = OC and AE = CF,

OA – AE = OC – CF

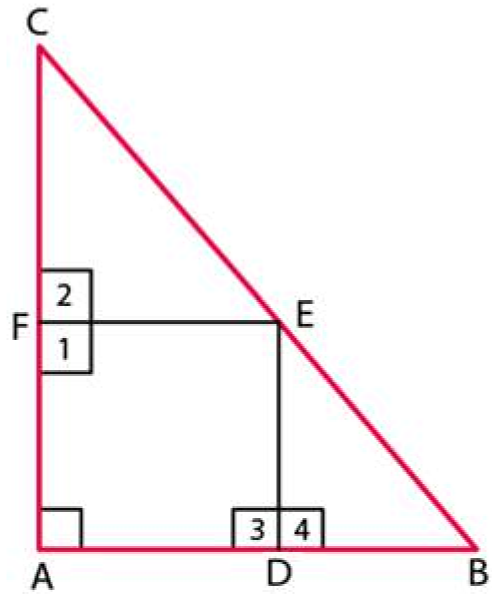
OE = OF

So, BFDE is a quadrilateral whose diagonals bisect each other.

Hence, BFDE is a parallelogram.

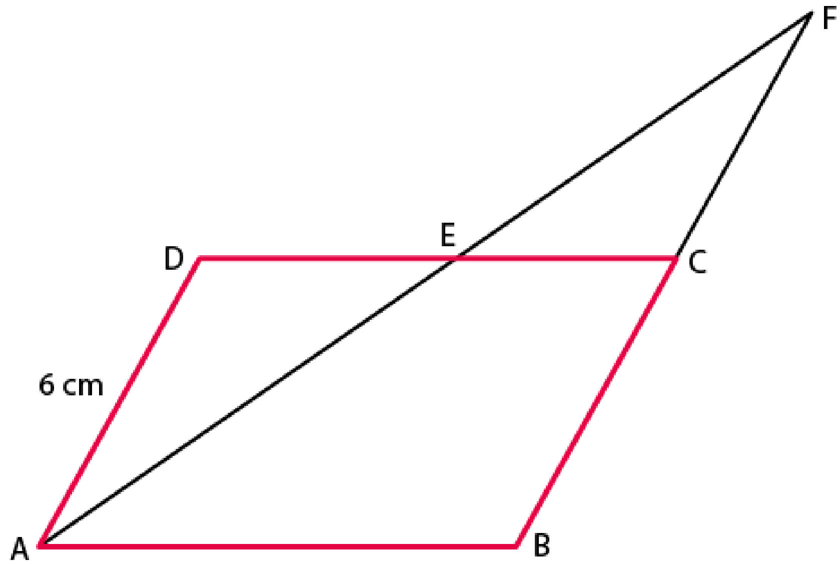
EXERCISE 8.4 PAGE NO: 82

1. A square is inscribed in an isosceles right triangle so that the square and the triangle have one angle common. Show that the vertex of the square opposite the vertex of the common angle bisects the hypotenuse.
Solution:



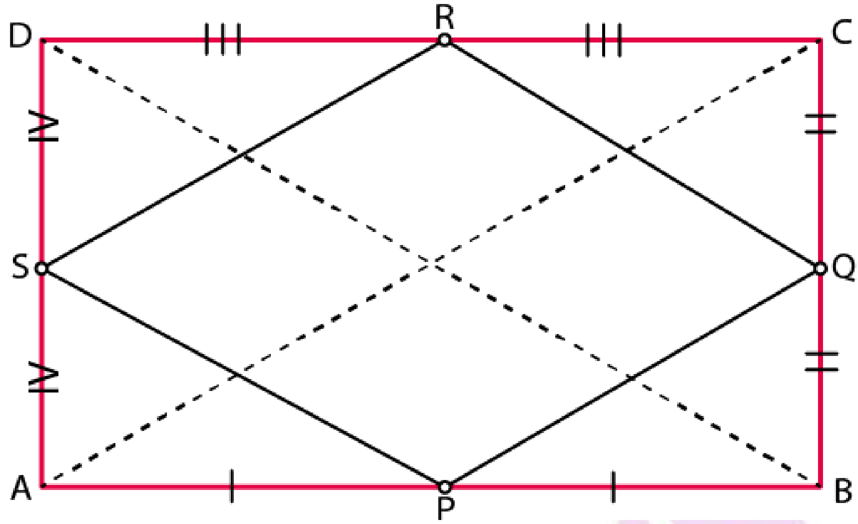
According to the question,
ΔABC with ∠A = 90º and
Since, ABC is an isosceles triangle,
We get,
AB = AC …(i)
Let ADEF be the square inscribed in the isosceles triangle ABC.
Then, we have,
AD = AF = EF = AD …(ii)
Subtracting equation (ii) from (i),
AB – AD = AC – AF
BD = CF
Now,
Considering ΔCFE and ΔEDB,
BD = CF
DE = EF
∠CFE = ∠EDB = 90º (Since, they are the side of a square)
ΔCEF ~ ΔBED (By SAS criteria)
Hence, CE = BE
Therefore, vertex E of the square bisect the hypotenuse BC.

2. In a parallelogram ABCD, AB = 10 cm and AD = 6 cm. The bisector of ∠A meets DC in E. AE and BC produced meet at F. Find the length of CF.
Solution:



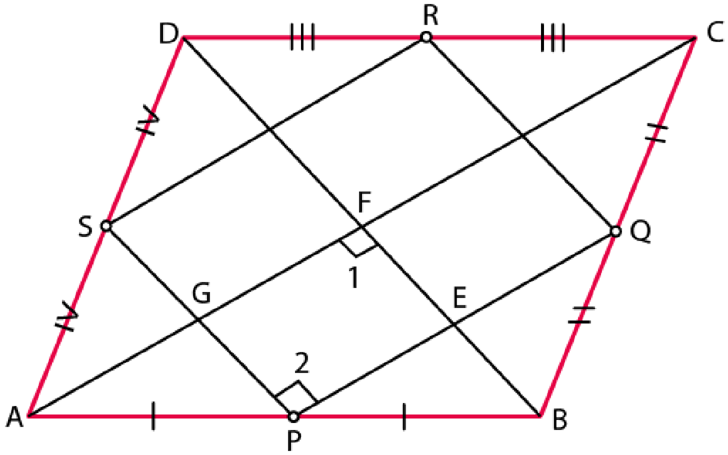
According to the question,
We have,
ABCD is a parallelogram
AB = 10 cm
AD = 6cm.
The bisector of ∠A meets DC at E.
AE and BC produced meet at F.
Since, AF bisects ∠A,
We get,
∠BAE = ∠EAD … (1)
∠EAD = ∠EFB … (2) [Alternate angles]
From equations (1) and (2),
We get,
∠BAE = ∠EFB
Since sides opposite to equal angles are equal,
We get,
BF = AB
Here, AB = 10 cm
So, BF = 10 cm
⇒ BC + CF = 10 cm
6 cm + CF = 10 cm [BC = AD = 6 cm, opposite sides of a parallelogram]
⇒ CF = 10 – 6 cm = 4 cm
⇒ CF = 4 cm

3. P, Q, R and S are respectively the mid-points of the sides AB, BC, CD and DA of a quadrilateral ABCD in which AC = BD. Prove that PQRS is a rhombus.
Solution:



According to the question,
We have,
P is the mid-point of the sides AB
Q is the mid-point of the sides BC
R is the mid-point of the sides CD
S is the mid-point of the sides DA
Also, we know that,
AC = BD.
In ΔADC, by mid-point theorem,
SR = ½ AC
And, SR||AC
In ΔABC, by mid-point theorem,
PQ = ½ AC
And, PQ||AC
Hence, SR = PQ = ½ AC
Similarly,
In ΔBCD, by mid-point theorem,
RQ = ½ BD
And, RQ||BD
In ΔBAD, by mid-point theorem,
SP = ½ BD
And, SP||BD
So, we get,
SP = RQ = ½ BD = ½ AC
Then,
SR = PQ = SP = RQ
Hence, PQRS is a rhombus.

4. P, Q, R and S are respectively the mid-points of the sides AB, BC, CD and DA of a quadrilateral ABCD such that AC ⊥ BD. Prove that PQRS is a rectangle.
Solution:



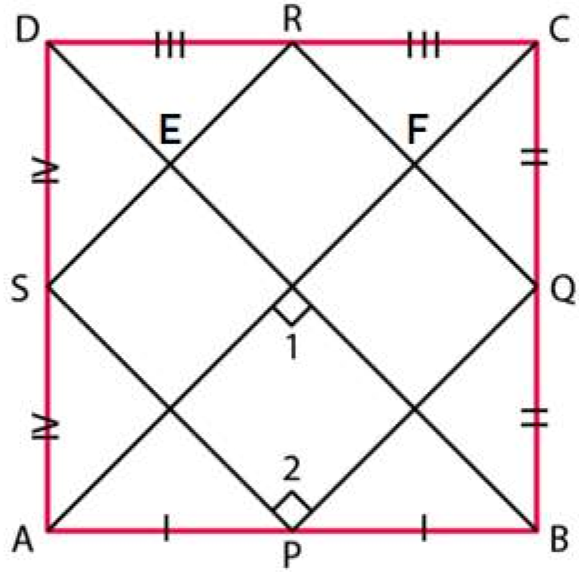
According to the question,
We have,
P is the mid-point of the sides AB
Q is the mid-point of the sides BC
R is the mid-point of the sides CD
S is the mid-point of the sides DA
Also,
AC ⊥ BD
∠COD = ∠AOD = ∠AOB = ∠COB = 90°
In ΔADC, by mid-point theorem,
SR = ½ AC
And, SR||AC
In ΔABC, by mid-point theorem,
PQ = ½ AC
And, PQ||AC
So, we have,
PQ||SR and SR = PQ = ½ AC
Similarly,
SP||RQ and SP = RQ = ½ BD
Now, in quadrilateral EOFR,
OE||FR and OF||ER
So, we get,
∠EOF = ∠ERF = 90°
Hence, PQRS is a rectangle.

5. P, Q, R and S are respectively the mid-points of sides AB, BC, CD and DA of quadrilateral ABCD in which AC = BD and AC ⊥ BD. Prove that PQRS is a square.

Solution:

According to the question,
We have,
P is the mid-point of the sides AB
Q is the mid-point of the sides BC
R is the mid-point of the sides CD

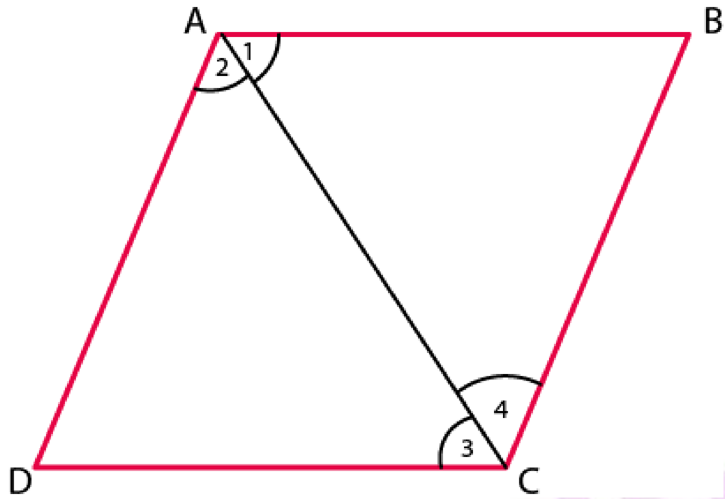
S is the mid-point of the sides DA
Also,
AC ⊥ BD
And AC = BD



In ΔADC, by mid-point theorem,
SR = ½ AC
And, SR||AC
In ΔABC, by mid-point theorem,
PQ = ½ AC
And, PQ||AC
So, we have,
PQ||SR and PQ = SR = ½ AC
Now, in ΔABD, by mid-point theorem,
SP||BD and SP = ½ BD = ½ AC
In ΔBCD, by mid-point theorem,
RQ||BD and RQ = ½ BD = ½ AC
SP = RQ = ½ AC
PQ = SR = SP = RQ
Thus, we get that,
All four sides are equal.
Considering the quadrilateral EOFR,
OE||FR, OF||ER
∠EOF = ∠ERF = 90^o (Opposite angles of parallelogram)
∠QRS = 90^o
Hence, PQRS is a square.

6. A diagonal of a parallelogram bisects one of its angles. Show that it is a rhombus.
Solution:
Let the parallelogram be = ABCD
Diagonal AC bisect ∠A.

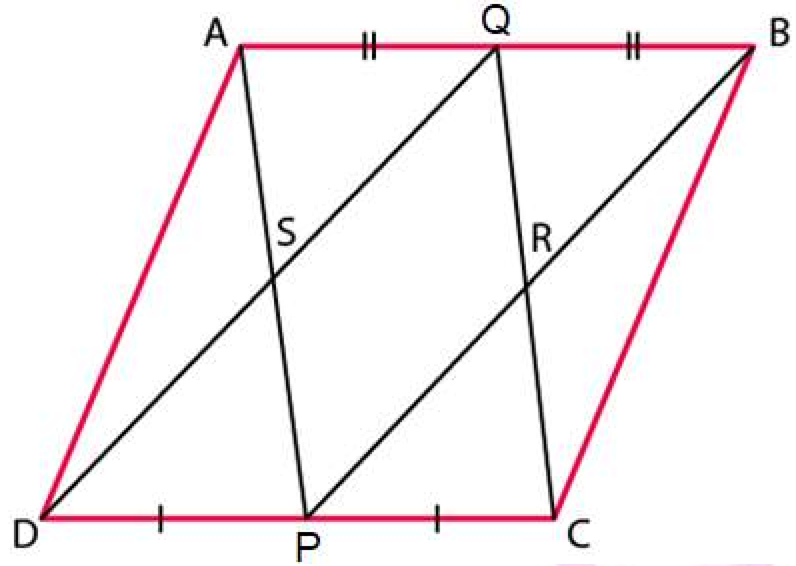
∠CAB = ∠CAD



Now,
AB||CD and AC is a transversal.
∠CAB = ∠ACD
Again, AD||BC and AC is a transversal.
∠DAC = ∠ACB
Now,
∠A = ∠C
½ ∠A = ½ ∠C
∠DAC = ∠DCA
AD = CD
But, AB = CD and AD = BC (Opposite sides of parallelograms)
AB = BC = CD = AD
Thus, ABCD is a rhombus.

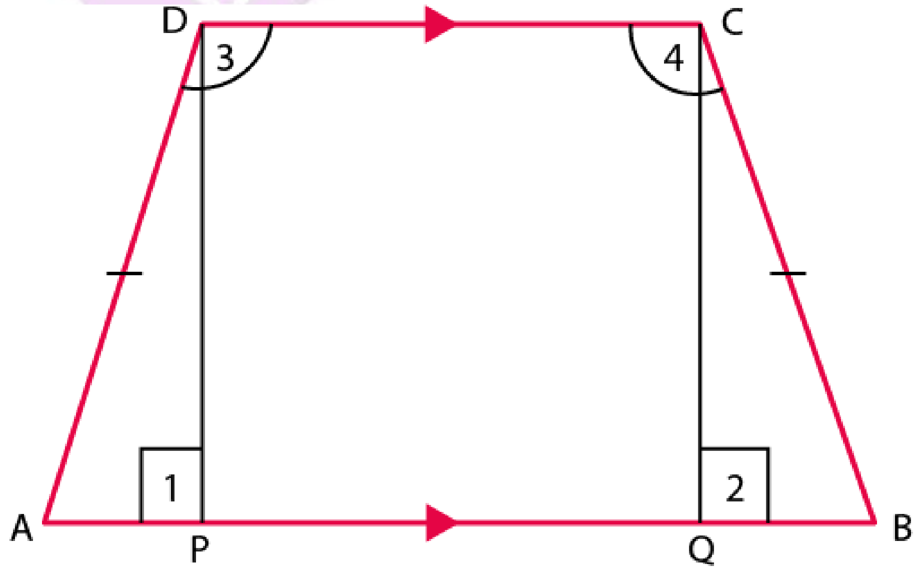
7. P and Q are the mid-points of the opposite sides AB and CD of a parallelogram ABCD. AQ intersects DP at S and BQ intersects CP at R. Show that PRQS is a parallelogram.
Solution:

According to the question,
Q is the midpoint of AB
P is the midpoint of CD
Now,
AB||CD,
Also,
AP||QC
And, AB = DC
½ AB = ½ DC
AP = QC



Now,
AP||QC and AP = QC
APCQ is a parallelogram.
AQ||PC or SQ||PR
Again,
AB||DC means ½ AB = ½ DC
BP = QD
Now, BP||QD and BP = QD
BPDQ is a parallelogram
So, PD||BQ or PS||QR
Thus, SQ||RP and PS||QR
PQRS is a parallelogram.

8. ABCD is a quadrilateral in which AB || DC and AD = BC. Prove that ∠A = ∠B and ∠C = ∠D.
Solution:



According to the question,
We have,
Quadrilateral ABCD
AB||CD and AD = BC.
To prove: ∠A = ∠B and ∠C = ∠D.
Construction: Draw DP ⊥ AB and CQ ⊥ AB.
Proof: In ΔAPD and ΔBQC,
Since ∠1 and ∠2 are equal to 90^o
∠1 = ∠2
Distance between parallel line,
AB = BC [Given]
By RHS criterion of congruence,
We have
ΔAPD ≅ ΔBQC [CPCT]
∠A = ∠B
Now, DC||AB
Since, sum of consecutive interior angles is 180^o
∠A+∠3 =180 …(1)
And,
∠B +∠4 =180 …(2)
From equations (1) and (2),
We get
∠A + ∠3 = ∠B + ∠4
Since, ∠A = ∠B,
We have,
⇒ ∠3 = ∠4
⇒ ∠C = ∠D
Hence, proved.

9. In Fig. 8.11, AB || DE, AB = DE, AC || DF and AC = DF. Prove that BC || EF and BC = EF.

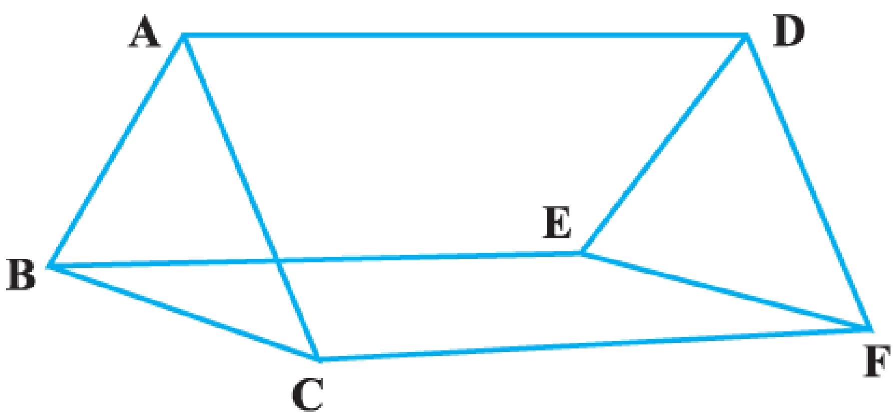
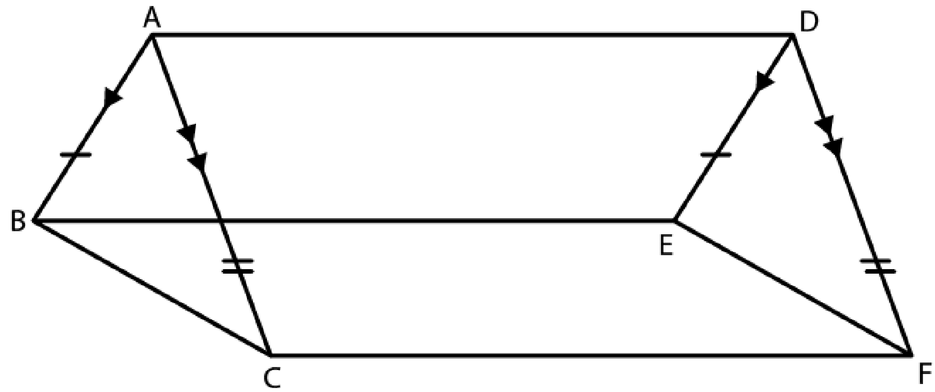


Fig. 8.11

Solution:



According to the question,
In quadrilateral ABED,
We have,
AB||DE and AB = DE
ABED is a parallelogram.
AD||BE and AD = BE
In quadrilateral ACFD,
We have,
AC||FD and AC = FD
ACFD is a parallelogram.
AD||CF and AD = CF
AD = BE = CF and CF||BE
In quadrilateral BCFE,
BE = CF and BE||CF.
BCFE is a parallelogram.
BC = EF and BC||EF
Hence proved.