

**EXERCISE 11.1****PAGE NO: 241**

In each of the following Exercise 1 to 5, find the equation of the circle with

**1. Centre (0, 2) and radius 2****Solution:**

Given:

Centre (0, 2) and radius 2

Let us consider the equation of a circle with centre (h, k) and

Radius r is given as  $(x - h)^2 + (y - k)^2 = r^2$ 

So, centre (h, k) = (0, 2) and radius (r) = 2

The equation of the circle is

$$(x - 0)^2 + (y - 2)^2 = 2^2$$

$$x^2 + y^2 + 4 - 4y = 4$$

$$x^2 + y^2 - 4y = 0$$

 $\therefore$  The equation of the circle is  $x^2 + y^2 - 4y = 0$ **2. Centre (-2, 3) and radius 4****Solution:**

Given:

Centre (-2, 3) and radius 4

Let us consider the equation of a circle with centre (h, k) and

Radius r is given as  $(x - h)^2 + (y - k)^2 = r^2$ 

So, centre (h, k) = (-2, 3) and radius (r) = 4

The equation of the circle is

$$(x + 2)^2 + (y - 3)^2 = (4)^2$$

$$x^2 + 4x + 4 + y^2 - 6y + 9 = 16$$

$$x^2 + y^2 + 4x - 6y - 3 = 0$$

 $\therefore$  The equation of the circle is  $x^2 + y^2 + 4x - 6y - 3 = 0$ **3. Centre (1/2, 1/4) and radius (1/12)****Solution:**

Given:

Centre (1/2, 1/4) and radius 1/12

Let us consider the equation of a circle with centre (h, k) and

Radius r is given as  $(x - h)^2 + (y - k)^2 = r^2$ 

So, centre (h, k) = (1/2, 1/4) and radius (r) = 1/12

The equation of the circle is

$$(x - 1/2)^2 + (y - 1/4)^2 = (1/12)^2$$

$$x^2 - x + 1/4 + y^2 - y/2 + 1/16 = 1/144$$

$$x^2 - x + \frac{1}{4} + y^2 - \frac{y}{2} + \frac{1}{16} = \frac{1}{144}$$

$$144x^2 - 144x + 36 + 144y^2 - 72y + 9 - 1 = 0$$

$$144x^2 - 144x + 144y^2 - 72y + 44 = 0$$

$$36x^2 + 36x + 36y^2 - 18y + 11 = 0$$

$$36x^2 + 36y^2 - 36x - 18y + 11 = 0$$

$$\therefore \text{The equation of the circle is } 36x^2 + 36y^2 - 36x - 18y + 11 = 0$$

#### 4. Centre (1, 1) and radius $\sqrt{2}$

**Solution:**

Given:

Centre (1, 1) and radius  $\sqrt{2}$

Let us consider the equation of a circle with centre (h, k) and

Radius r is given as  $(x - h)^2 + (y - k)^2 = r^2$

So, centre (h, k) = (1, 1) and radius (r) =  $\sqrt{2}$

The equation of the circle is

$$(x-1)^2 + (y-1)^2 = (\sqrt{2})^2$$

$$x^2 - 2x + 1 + y^2 - 2y + 1 = 2$$

$$x^2 + y^2 - 2x - 2y = 0$$

$$\therefore \text{The equation of the circle is } x^2 + y^2 - 2x - 2y = 0$$

#### 5. Centre (-a, -b) and radius $\sqrt{a^2 - b^2}$

**Solution:**

Given:

Centre (-a, -b) and radius  $\sqrt{a^2 - b^2}$

Let us consider the equation of a circle with centre (h, k) and

Radius r is given as  $(x - h)^2 + (y - k)^2 = r^2$

So, centre (h, k) = (-a, -b) and radius (r) =  $\sqrt{a^2 - b^2}$

The equation of the circle is

$$(x + a)^2 + (y + b)^2 = (\sqrt{a^2 - b^2})^2$$

$$x^2 + 2ax + a^2 + y^2 + 2by + b^2 = a^2 - b^2$$

$$x^2 + y^2 + 2ax + 2by + 2b^2 = 0$$

$$\therefore \text{The equation of the circle is } x^2 + y^2 + 2ax + 2by + 2b^2 = 0$$

**In each of the following Exercise 6 to 9, find the centre and radius of the circles.**

#### 6. $(x + 5)^2 + (y - 3)^2 = 36$

**Solution:**

Given:

The equation of the given circle is  $(x + 5)^2 + (y - 3)^2 = 36$

$$(x - (-5))^2 + (y - 3)^2 = 6^2 \text{ [which is of the form } (x - h)^2 + (y - k)^2 = r^2]$$

Where,  $h = -5$ ,  $k = 3$  and  $r = 6$

∴ The centre of the given circle is  $(-5, 3)$  and its radius is 6.

**7.  $x^2 + y^2 - 4x - 8y - 45 = 0$**

**Solution:**

Given:

The equation of the given circle is  $x^2 + y^2 - 4x - 8y - 45 = 0$ .

$$x^2 + y^2 - 4x - 8y - 45 = 0$$

$$(x^2 - 4x) + (y^2 - 8y) = 45$$

$$(x^2 - 2(x)(2) + 2^2) + (y^2 - 2(y)(4) + 4^2) - 4 - 16 = 45$$

$$(x - 2)^2 + (y - 4)^2 = 65$$

$$(x - 2)^2 + (y - 4)^2 = (\sqrt{65})^2 \text{ [which is form } (x-h)^2 + (y-k)^2 = r^2]$$

Where  $h = 2$ ,  $K = 4$  and  $r = \sqrt{65}$

∴ The centre of the given circle is  $(2, 4)$  and its radius is  $\sqrt{65}$ .

**8.  $x^2 + y^2 - 8x + 10y - 12 = 0$**

**Solution:**

Given:

The equation of the given circle is  $x^2 + y^2 - 8x + 10y - 12 = 0$ .

$$x^2 + y^2 - 8x + 10y - 12 = 0$$

$$(x^2 - 8x) + (y^2 + 10y) = 12$$

$$(x^2 - 2(x)(4) + 4^2) + (y^2 - 2(y)(5) + 5^2) - 16 - 25 = 12$$

$$(x - 4)^2 + (y + 5)^2 = 53$$

$$(x - 4)^2 + (y - (-5))^2 = (\sqrt{53})^2 \text{ [which is form } (x-h)^2 + (y-k)^2 = r^2]$$

Where  $h = 4$ ,  $K = -5$  and  $r = \sqrt{53}$

∴ The centre of the given circle is  $(4, -5)$  and its radius is  $\sqrt{53}$ .

**9.  $2x^2 + 2y^2 - x = 0$**

**Solution:**

The equation of the given of the circle is  $2x^2 + 2y^2 - x = 0$ .

$$2x^2 + 2y^2 - x = 0$$

$$(2x^2 + x) + 2y^2 = 0$$

$$(x^2 - 2(x)(1/4) + (1/4)^2) + y^2 - (1/4)^2 = 0$$

$$(x - 1/4)^2 + (y - 0)^2 = (1/4)^2 \text{ [which is form } (x-h)^2 + (y-k)^2 = r^2]$$

Where,  $h = 1/4$ ,  $K = 0$ , and  $r = 1/4$

∴ The center of the given circle is  $(1/4, 0)$  and its radius is  $1/4$ .

**10. Find the equation of the circle passing through the points (4,1) and (6,5) and whose centre is on the line  $4x + y = 16$ .**

**Solution:**

Let us consider the equation of the required circle be  $(x - h)^2 + (y - k)^2 = r^2$

We know that the circle passes through points (4,1) and (6,5)

So,

$$(4 - h)^2 + (1 - k)^2 = r^2 \dots\dots\dots(1)$$

$$(6 - h)^2 + (5 - k)^2 = r^2 \dots\dots\dots(2)$$

Since, the centre (h, k) of the circle lies on line  $4x + y = 16$ ,

$$4h + k = 16 \dots\dots\dots(3)$$

From the equation (1) and (2), we obtain

$$(4 - h)^2 + (1 - k)^2 = (6 - h)^2 + (5 - k)^2$$

$$16 - 8h + h^2 + 1 - 2k + k^2 = 36 - 12h + h^2 + 15 - 10k + k^2$$

$$16 - 8h + 1 - 2k + 12h - 25 - 10k$$

$$4h + 8k = 44$$

$$h + 2k = 11 \dots\dots\dots(4)$$

On solving equations (3) and (4), we obtain  $h=3$  and  $k=4$ .

On substituting the values of h and k in equation (1), we obtain

$$(4 - 3)^2 + (1 - 4)^2 = r^2$$

$$1^2 + (-3)^2 = r^2$$

$$1 + 9 = r^2$$

$$r = \sqrt{10}$$

$$\text{so now, } (x - 3)^2 + (y - 4)^2 = (\sqrt{10})^2$$

$$x^2 - 6x + 9 + y^2 - 8y + 16 = 10$$

$$x^2 + y^2 - 6x - 8y + 15 = 0$$

$\therefore$  The equation of the required circle is  $x^2 + y^2 - 6x - 8y + 15 = 0$

**11. Find the equation of the circle passing through the points (2, 3) and (-1, 1) and whose centre is on the line  $x - 3y - 11 = 0$ .**

**Solution:**

Let us consider the equation of the required circle be  $(x - h)^2 + (y - k)^2 = r^2$

We know that the circle passes through points (2,3) and (-1,1).

$$(2 - h)^2 + (3 - k)^2 = r^2 \dots\dots\dots(1)$$

$$(-1 - h)^2 + (1 - k)^2 = r^2 \dots\dots\dots(2)$$

Since, the centre (h, k) of the circle lies on line  $x - 3y - 11 = 0$ ,

$$h - 3k = 11 \dots\dots\dots(3)$$

From the equation (1) and (2), we obtain

$$(2 - h)^2 + (3 - k)^2 = (-1 - h)^2 + (1 - k)^2$$

$$4 - 4h + h^2 + 9 - 6k + k^2 = 1 + 2h + h^2 + 1 - 2k + k^2$$

$$4 - 4h + 9 - 6k = 1 + 2h + 1 - 2k$$

$$6h + 4k = 11 \dots\dots\dots (4)$$

Now let us multiply equation (3) by 6 and subtract it from equation (4) to get,

$$6h + 4k - 6(h - 3k) = 11 - 66$$

$$6h + 4k - 6h + 18k = 11 - 66$$

$$22k = -55$$

$$K = -5/2$$

Substitute this value of K in equation (4) to get,

$$6h + 4(-5/2) = 11$$

$$6h - 10 = 11$$

$$6h = 21$$

$$h = 21/6$$

$$h = 7/2$$

We obtain  $h = 7/2$  and  $k = -5/2$

On substituting the values of h and k in equation (1), we get

$$(2 - 7/2)^2 + (3 + 5/2)^2 = r^2$$

$$[(4-7)/2]^2 + [(6+5)/2]^2 = r^2$$

$$(-3/2)^2 + (11/2)^2 = r^2$$

$$9/4 + 121/4 = r^2$$

$$130/4 = r^2$$

The equation of the required circle is

$$(x - 7/2)^2 + (y + 5/2)^2 = 130/4$$

$$[(2x-7)/2]^2 + [(2y+5)/2]^2 = 130/4$$

$$4x^2 - 28x + 49 + 4y^2 + 20y + 25 = 130$$

$$4x^2 + 4y^2 - 28x + 20y - 56 = 0$$

$$4(x^2 + y^2 - 7x + 5y - 14) = 0$$

$$x^2 + y^2 - 7x + 5y - 14 = 0$$

∴ The equation of the required circle is  $x^2 + y^2 - 7x + 5y - 14 = 0$

**12. Find the equation of the circle with radius 5 whose centre lies on x-axis and passes through the point (2, 3).**

**Solution:**

Let us consider the equation of the required circle be  $(x - h)^2 + (y - k)^2 = r^2$

We know that the radius of the circle is 5 and its centre lies on the x-axis,  $k = 0$  and  $r = 5$ .

So now, the equation of the circle is  $(x - h)^2 + y^2 = 25$ .

It is given that the circle passes through the point (2, 3) so the point will satisfy the equation of the circle.

$$(2 - h)^2 + 3^2 = 25$$

$$(2 - h)^2 = 25 - 9$$

$$(2 - h)^2 = 16$$

$$2 - h = \pm \sqrt{16} = \pm 4$$

If  $2 - h = 4$ , then  $h = -2$   
 If  $2 - h = -4$ , then  $h = 6$   
 Then, when  $h = -2$ , the equation of the circle becomes

$$(x + 2)^2 + y^2 = 25$$

$$x^2 + 12x + 36 + y^2 = 25$$

$$x^2 + y^2 + 4x - 21 = 0$$

When  $h = 6$ , the equation of the circle becomes

$$(x - 6)^2 + y^2 = 25$$

$$x^2 - 12x + 36 + y^2 = 25$$

$$x^2 + y^2 - 12x + 11 = 0$$

∴ The equation of the required circle is  $x^2 + y^2 + 4x - 21 = 0$  and  $x^2 + y^2 - 12x + 11 = 0$

**13. Find the equation of the circle passing through (0,0) and making intercepts a and b on the coordinate axes.**

**Solution:**

Let us consider the equation of the required circle be  $(x - h)^2 + (y - k)^2 = r^2$

We know that the circle passes through (0, 0),

So,  $(0 - h)^2 + (0 - k)^2 = r^2$

$$h^2 + k^2 = r^2$$

Now, The equation of the circle is  $(x - h)^2 + (y - k)^2 = h^2 + k^2$ .

It is given that the circle intercepts a and b on the coordinate axes.

i.e., the circle passes through points (a, 0) and (0, b).

So,  $(a - h)^2 + (0 - k)^2 = h^2 + k^2$ .....(1)

$(0 - h)^2 + (b - k)^2 = h^2 + k^2$ .....(2)

From equation (1), we obtain

$$a^2 - 2ah + h^2 + k^2 = h^2 + k^2$$

$$a^2 - 2ah = 0$$

$$a(a - 2h) = 0$$

$$a = 0 \text{ or } (a - 2h) = 0$$

However,  $a \neq 0$ ; hence,  $(a - 2h) = 0$

$$h = a/2$$

From equation (2), we obtain

$$h^2 - 2bk + k^2 + b^2 = h^2 + k^2$$

$$b^2 - 2bk = 0$$

$$b(b - 2k) = 0$$

$$b = 0 \text{ or } (b - 2k) = 0$$

However,  $a \neq 0$ ; hence,  $(b - 2k) = 0$

$$k = b/2$$

So, the equation is

$$(x - a/2)^2 + (y - b/2)^2 = (a/2)^2 + (b/2)^2$$

$$[(2x - a)/2]^2 + [(2y - b)/2]^2 = (a^2 + b^2)/4$$

$$4x^2 - 4ax + a^2 + 4y^2 - 4by + b^2 = a^2 + b^2$$

$$4x^2 + 4y^2 - 4ax - 4by = 0$$

$$4(x^2 + y^2 - ax - by) = 0$$

$$x^2 + y^2 - ax - by = 0$$

$\therefore$  The equation of the required circle is  $x^2 + y^2 - ax - by = 0$

**14. Find the equation of a circle with centre (2,2) and passes through the point (4,5).**

**Solution:**

Given:

The centre of the circle is given as  $(h, k) = (2, 2)$

We know that the circle passes through point  $(4, 5)$ , the radius ( $r$ ) of the circle is the distance between the points  $(2, 2)$  and  $(4, 5)$ .

$$r = \sqrt{(2-4)^2 + (2-5)^2}$$

$$= \sqrt{(-2)^2 + (-3)^2}$$

$$= \sqrt{4+9}$$

$$= \sqrt{13}$$

The equation of the circle is given as

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 2)^2 + (y - 2)^2 = (\sqrt{13})^2$$

$$(x - 2)^2 + (y - 2)^2 = 13$$

$$x^2 - 4x + 4 + y^2 - 4y + 4 = 13$$

$$x^2 + y^2 - 4x - 4y = 5$$

$\therefore$  The equation of the required circle is  $x^2 + y^2 - 4x - 4y = 5$

**15. Does the point  $(-2.5, 3.5)$  lie inside, outside or on the circle  $x^2 + y^2 = 25$ ?**

**Solution:**

Given:

The equation of the given circle is  $x^2 + y^2 = 25$ .

$$x^2 + y^2 = 25$$

$$(x - 0)^2 + (y - 0)^2 = 5^2 \text{ [which is of the form } (x - h)^2 + (y - k)^2 = r^2 \text{]}$$

Where,  $h = 0$ ,  $k = 0$  and  $r = 5$ .

So the distance between point  $(-2.5, 3.5)$  and the centre  $(0, 0)$  is

$$\sqrt{[(-2.5 - 0)^2 + (-3.5 - 0)^2]}$$

$$\sqrt{(6.25 + 12.25)}$$

$$\sqrt{18.5}$$

$$4.3 \text{ [which is } < 5 \text{]}$$

Since, the distance between point  $(-2.5, -3.5)$  and the centre  $(0, 0)$  of the circle is less than the radius of the circle, point  $(-2.5, -3.5)$  lies inside the circle.

