EXERCISE 11.2 PAGE NO: 246

In each of the following Exercises 1 to 6, find the coordinates of the focus, axis of the parabola, the equation of the directrix and the length of the latus rectum.

1. $y^2 = 12x$

Solution:

Given:

The equation is $y^2 = 12x$

Here we know that the coefficient of x is positive.

So, the parabola opens towards the right.

On comparing this equation with $y^2 = 4ax$, we get,

$4a = 12$

$a = 3$

Thus, the co-ordinates of the focus = (a, 0) = (3, 0)

Since, the given equation involves y^2, the axis of the parabola is the x-axis.

∴ The equation of directrix, x = -a, then,

$x + 3 = 0$

Length of latus rectum = 4a = 4 × 3 = 12

2. $x^2 = 6y$

Solution:

Given:

The equation is $x^2 = 6y$

Here we know that the coefficient of y is positive.

So, the parabola opens upwards.

On comparing this equation with $x^2 = 4ay$, we get,

$4a = 6$

$a = 6/4$

$= 3/2$

Thus, the co-ordinates of the focus = (0,a) = (0, 3/2)

Since, the given equation involves x^2, the axis of the parabola is the y-axis.

∴ The equation of directrix, y =-a, then,

$y = -3/2$

Length of latus rectum = 4a = 4(3/2) = 6

3. $y^2 = -8x$

Solution:

Given:
The equation is $y^2 = -8x$
Here we know that the coefficient of x is negative.
So, the parabola open towards the left.
On comparing this equation with $y^2 = -4ax$, we get,
$-4a = -8$
$a = -8/-4 = 2$
Thus, co-ordinates of the focus = $(-a,0) = (-2, 0)$
Since, the given equation involves y^2, the axis of the parabola is the x-axis.
∴ Equation of directrix, $x = a$, then,
$x = 2$
Length of latus rectum = $4a = 4(2) = 8$

4. $x^2 = -16y$
Solution:
Given:
The equation is $x^2 = -16y$
Here we know that the coefficient of y is negative.
So, the parabola opens downwards.
On comparing this equation with $x^2 = -4ay$, we get,
$-4a = -16$
$a = -16/-4$
$= 4$
Thus, co-ordinates of the focus = $(0,-a) = (0,-4)$
Since, the given equation involves x^2, the axis of the parabola is the y-axis.
∴ The equation of directrix, $y = a$, then,
$y = 4$
Length of latus rectum = $4a = 4(4) = 16$

5. $y^2 = 10x$
Solution:
Given:
The equation is $y^2 = 10x$
Here we know that the coefficient of x is positive.
So, the parabola open towards the right.
On comparing this equation with $y^2 = 4ax$, we get,
$4a = 10$
$a = 10/4 = 5/2$
Thus, co-ordinates of the focus = $(a,0) = (5/2, 0)$

Since, the given equation involves y^2, the axis of the parabola is the x-axis.
∴ The equation of directrix, x =-a, then,
x = - 5/2
Length of latus rectum = 4a = 4(5/2) = 10

6. $x^2 = -9y$
Solution:
Given:
The equation is $x^2 = -9y$
Here we know that the coefficient of y is negative.
So, the parabola open downwards.
On comparing this equation with $x^2 = -4ay$, we get,
-4a = -9
a = -9/-4 = 9/4
Thus, co-ordinates of the focus = (0,-a) = (0, -9/4)
Since, the given equation involves x^2, the axis of the parabola is the y-axis.
∴ The equation of directrix, y = a, then,
y = 9/4
Length of latus rectum = 4a = 4(9/4) = 9

In each of the Exercises 7 to 12, find the equation of the parabola that satisfies the given conditions:

7. Focus (6,0); directrix x = – 6
Solution:
Given:
Focus (6,0) and directrix x = -6
We know that the focus lies on the x–axis is the axis of the parabola.
So, the equation of the parabola is either of the form $y^2 = 4ax$ or $y^2 = -4ax$.
It is also seen that the directrix, x = -6 is to the left of the y- axis,
While the focus (6, 0) is to the right of the y –axis.
Hence, the parabola is of the form $y^2 = 4ax$.
Here, a = 6
∴ The equation of the parabola is $y^2 = 24x$.

8. Focus (0,–3); directrix y = 3
Solution:
Given:
Focus (0, -3) and directrix y = 3

We know that the focus lies on the y–axis, the y-axis is the axis of the parabola.
So, the equation of the parabola is either of the form $x^2 = 4ay$ or $x^2 = -4ay$.
It is also seen that the directrix, $y = 3$ is above the x- axis,
While the focus (0,-3) is below the x-axis.
Hence, the parabola is of the form $x^2 = -4ay$.
Here, $a = 3$
∴ The equation of the parabola is $x^2 = -12y$.

9. Vertex (0, 0); focus (3, 0)
Solution:
Given:
Vertex (0, 0) and focus (3, 0)
We know that the vertex of the parabola is (0, 0) and the focus lies on the positive x-axis.
[x-axis is the axis of the parabola.]
The equation of the parabola is of the form $y^2 = 4ax$.
Since, the focus is (3, 0), $a = 3$
∴ The equation of the parabola is $y^2 = 4 \times 3 \times x$,
$y^2 = 12x$

10. Vertex (0, 0); focus (–2, 0)
Solution:
Given:
Vertex (0, 0) and focus (-2, 0)
We know that the vertex of the parabola is (0, 0) and the focus lies on the positive x-axis.
[x-axis is the axis of the parabola.]
The equation of the parabola is of the form $y^2=-4ax$.
Since, the focus is (-2, 0), $a = 2$
∴ The equation of the parabola is $y^2 = -4 \times 2 \times x$,
$y^2 = -8x$

11. Vertex (0, 0) passing through (2, 3) and axis is along x-axis.
Solution:
We know that the vertex is (0, 0) and the axis of the parabola is the x-axis
The equation of the parabola is either of the from $y^2 = 4ax$ or $y^2 = -4ax$.
Given that the parabola passes through point (2, 3), which lies in the first quadrant.
So, the equation of the parabola is of the form $y^2 = 4ax$, while point (2, 3) must satisfy the equation $y^2 = 4ax$.
Then,
$3^2 = 4a(2)$

$3^2 = 8a$
$9 = 8a$
$a = 9/8$
Thus, the equation of the parabola is
$y^2 = 4\,(9/8)x$
$= 9x/2$
$2y^2 = 9x$
∴ The equation of the parabola is $2y^2 = 9x$

12. Vertex (0, 0), passing through (5, 2) and symmetric with respect to y-axis.
Solution:
We know that the vertex is (0, 0) and the parabola is symmetric about the y-axis.
The equation of the parabola is either of the from $x^2 = 4ay$ or $x^2 = -4ay$.
Given that the parabola passes through point (5, 2), which lies in the first quadrant.
So, the equation of the parabola is of the form $x^2 = 4ay$, while point (5, 2) must satisfy the equation $x^2 = 4ay$.
Then,
$5^2 = 4a(2)$
$25 = 8a$
$a = 25/8$
Thus, the equation of the parabola is
$x^2 = 4\,(25/8)y$
$x^2 = 25y/2$
$2x^2 = 25y$
∴ The equation of the parabola is $2x^2 = 25y$