

EXERCISE 11.3

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In each of the Exercises 1 to 9, find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse.

1. $x^2/36 + y^2/16 = 1$

Solution:

Given:

The equation is $x^2/36 + y^2/16 = 1$ Here, the denominator of $x^2/36$ is greater than the denominator of $y^2/16$.

So, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get $a = 6$ and $b = 4$.

$$c = \sqrt{a^2 - b^2}$$

$$= \sqrt{36-16}$$

$$= \sqrt{20}$$

$$= 2\sqrt{5}$$

Then,

The coordinates of the foci are $(2\sqrt{5}, 0)$ and $(-2\sqrt{5}, 0)$.The coordinates of the vertices are $(6, 0)$ and $(-6, 0)$ Length of major axis = $2a = 2(6) = 12$ Length of minor axis = $2b = 2(4) = 8$ Eccentricity, $e^{c/a} = 2\sqrt{5}/6 = \sqrt{5}/3$ Length of latus rectum = $2b^2/a = (2 \times 16)/6 = 16/3$

2. $x^2/4 + y^2/25 = 1$

Solution:

Given:

The equation is $x^2/4 + y^2/25 = 1$ Here, the denominator of $y^2/25$ is greater than the denominator of $x^2/4$.

So, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get $a = 5$ and $b = 2$.

$$c = \sqrt{a^2 - b^2}$$

$$= \sqrt{25-4}$$

$$= \sqrt{21}$$

Then,

The coordinates of the foci are $(0, \sqrt{21})$ and $(0, -\sqrt{21})$.

The coordinates of the vertices are (0, 5) and (0, -5)

Length of major axis = $2a = 2(5) = 10$

Length of minor axis = $2b = 2(2) = 4$

Eccentricity, $e^{c/a} = \sqrt{21}/5$

Length of latus rectum = $2b^2/a = (2 \times 2^2)/5 = (2 \times 4)/5 = 8/5$

3. $x^2/16 + y^2/9 = 1$

Solution:

Given:

The equation is $x^2/16 + y^2/9 = 1$ or $x^2/4^2 + y^2/3^2 = 1$

Here, the denominator of $x^2/16$ is greater than the denominator of $y^2/9$.

So, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get

$a = 4$ and $b = 3$.

$c = \sqrt{a^2 - b^2}$

$= \sqrt{16-9}$

$= \sqrt{7}$

Then,

The coordinates of the foci are $(\sqrt{7}, 0)$ and $(-\sqrt{7}, 0)$.

The coordinates of the vertices are (4, 0) and (-4, 0)

Length of major axis = $2a = 2(4) = 8$

Length of minor axis = $2b = 2(3) = 6$

Eccentricity, $e^{c/a} = \sqrt{7}/4$

Length of latus rectum = $2b^2/a = (2 \times 3^2)/4 = (2 \times 9)/4 = 18/4 = 9/2$

4. $x^2/25 + y^2/100 = 1$

Solution:

Given:

The equation is $x^2/25 + y^2/100 = 1$

Here, the denominator of $y^2/100$ is greater than the denominator of $x^2/25$. So, the major axis is along the y-axis, while the minor axis is along the x-axis. On

comparing the given equation with $x^2/b^2 + y^2/a^2 = 1$, we get

$b = 5$ and $a = 10$.

$c = \sqrt{a^2 - b^2}$

$= \sqrt{100-25}$

$= \sqrt{75}$

$= 5\sqrt{3}$

Then,

The coordinates of the foci are $(0, 5\sqrt{3})$ and $(0, -5\sqrt{3})$.

The coordinates of the vertices are $(0, \sqrt{10})$ and $(0, -\sqrt{10})$
 Length of major axis = $2a = 2(10) = 20$
 Length of minor axis = $2b = 2(5) = 10$
 Eccentricity, $e^{c/a} = 5\sqrt{3}/10 = \sqrt{3}/2$
 Length of latus rectum = $2b^2/a = (2 \times 5^2)/10 = (2 \times 25)/10 = 5$

5. $x^2/49 + y^2/36 = 1$

Solution:

Given:

The equation is $x^2/49 + y^2/36 = 1$

Here, the denominator of $x^2/49$ is greater than the denominator of $y^2/36$.

So, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get

$b = 6$ and $a = 7$

$$c = \sqrt{a^2 - b^2}$$

$$= \sqrt{49 - 36}$$

$$= \sqrt{13}$$

Then,

The coordinates of the foci are $(\sqrt{13}, 0)$ and $(-\sqrt{13}, 0)$.

The coordinates of the vertices are $(7, 0)$ and $(-7, 0)$

Length of major axis = $2a = 2(7) = 14$

Length of minor axis = $2b = 2(6) = 12$

Eccentricity, $e^{c/a} = \sqrt{13}/7$

Length of latus rectum = $2b^2/a = (2 \times 6^2)/7 = (2 \times 36)/7 = 72/7$

6. $x^2/100 + y^2/400 = 1$

Solution:

Given:

The equation is $x^2/100 + y^2/400 = 1$

Here, the denominator of $y^2/400$ is greater than the denominator of $x^2/100$. So, the major axis is along the y-axis, while the minor axis is along the x-axis. On

comparing the given equation with $x^2/b^2 + y^2/a^2 = 1$, we get

$b = 10$ and $a = 20$.

$$c = \sqrt{a^2 - b^2}$$

$$= \sqrt{400 - 100}$$

$$= \sqrt{300}$$

$$= 10\sqrt{3}$$

Then,

The coordinates of the foci are $(0, 10\sqrt{3})$ and $(0, -10\sqrt{3})$.

The coordinates of the vertices are (0, 20) and (0, -20)

Length of major axis = $2a = 2(20) = 40$

Length of minor axis = $2b = 2(10) = 20$

Eccentricity, $e^{c/a} = 10\sqrt{3}/20 = \sqrt{3}/2$

Length of latus rectum = $2b^2/a = (2 \times 10^2)/20 = (2 \times 100)/20 = 10$

7. $36x^2 + 4y^2 = 144$

Solution:

Given:

The equation is $36x^2 + 4y^2 = 144$ or $x^2/4 + y^2/36 = 1$ or $x^2/2^2 + y^2/6^2 = 1$

Here, the denominator of $y^2/6^2$ is greater than the denominator of $x^2/2^2$.

So, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing the given equation with $x^2/b^2 + y^2/a^2 = 1$, we get

$b = 2$ and $a = 6$.

$c = \sqrt{a^2 - b^2}$

$= \sqrt{36-4}$

$= \sqrt{32}$

$= 4\sqrt{2}$

Then,

The coordinates of the foci are (0, $4\sqrt{2}$) and (0, $-4\sqrt{2}$).

The coordinates of the vertices are (0, 6) and (0, -6)

Length of major axis = $2a = 2(6) = 12$

Length of minor axis = $2b = 2(2) = 4$

Eccentricity, $e^{c/a} = 4\sqrt{2}/6 = 2\sqrt{2}/3$

Length of latus rectum = $2b^2/a = (2 \times 2^2)/6 = (2 \times 4)/6 = 4/3$

8. $16x^2 + y^2 = 16$

Solution:

Given:

The equation is $16x^2 + y^2 = 16$ or $x^2/1 + y^2/16 = 1$ or $x^2/1^2 + y^2/4^2 = 1$

Here, the denominator of $y^2/4^2$ is greater than the denominator of $x^2/1^2$.

So, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing the given equation with $x^2/b^2 + y^2/a^2 = 1$, we get

$b = 1$ and $a = 4$.

$c = \sqrt{a^2 - b^2}$

$= \sqrt{16-1}$

$= \sqrt{15}$

Then,

The coordinates of the foci are (0, $\sqrt{15}$) and (0, $-\sqrt{15}$).

The coordinates of the vertices are (0, 4) and (0, -4)

Length of major axis = $2a = 2(4) = 8$

Length of minor axis = $2b = 2(1) = 2$

Eccentricity, $e^{c/a} = \sqrt{15}/4$

Length of latus rectum = $2b^2/a = (2 \times 1^2)/4 = 2/4 = 1/2$

9. $4x^2 + 9y^2 = 36$

Solution:

Given:

The equation is $4x^2 + 9y^2 = 36$ or $x^2/9 + y^2/4 = 1$ or $x^2/3^2 + y^2/2^2 = 1$

Here, the denominator of $x^2/3^2$ is greater than the denominator of $y^2/2^2$.

So, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get

$a = 3$ and $b = 2$.

$c = \sqrt{a^2 - b^2}$

$= \sqrt{9-4}$

$= \sqrt{5}$

Then,

The coordinates of the foci are $(\sqrt{5}, 0)$ and $(-\sqrt{5}, 0)$.

The coordinates of the vertices are $(3, 0)$ and $(-3, 0)$

Length of major axis = $2a = 2(3) = 6$

Length of minor axis = $2b = 2(2) = 4$

Eccentricity, $e^{c/a} = \sqrt{5}/3$

Length of latus rectum = $2b^2/a = (2 \times 2^2)/3 = (2 \times 4)/3 = 8/3$

In each of the following Exercises 10 to 20, find the equation for the ellipse that satisfies the given conditions:

10. Vertices $(\pm 5, 0)$, foci $(\pm 4, 0)$

Solution:

Given:

Vertices $(\pm 5, 0)$ and foci $(\pm 4, 0)$

Here, the vertices are on the x-axis.

So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.

Then, $a = 5$ and $c = 4$.

It is known that $a^2 = b^2 + c^2$.

So, $5^2 = b^2 + 4^2$

$25 = b^2 + 16$

$$b^2 = 25 - 16$$

$$b = \sqrt{9}$$

$$= 3$$

∴ The equation of the ellipse is $x^2/5^2 + y^2/3^2 = 1$ or $x^2/25 + y^2/9 = 1$

11. Vertices (0, ± 13), foci (0, ± 5)

Solution:

Given:

Vertices (0, ± 13) and foci (0, ± 5)

Here, the vertices are on the y-axis.

So, the equation of the ellipse will be of the form $x^2/b^2 + y^2/a^2 = 1$, where 'a' is the semi-major axis.

Then, $a = 13$ and $c = 5$.

It is known that $a^2 = b^2 + c^2$.

$$13^2 = b^2 + 5^2$$

$$169 = b^2 + 25$$

$$b^2 = 169 - 25$$

$$b = \sqrt{144}$$

$$= 12$$

∴ The equation of the ellipse is $x^2/12^2 + y^2/13^2 = 1$ or $x^2/144 + y^2/169 = 1$

12. Vertices (± 6, 0), foci (± 4, 0)

Solution:

Given:

Vertices (± 6, 0) and foci (± 4, 0)

Here, the vertices are on the x-axis.

So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.

Then, $a = 6$ and $c = 4$.

It is known that $a^2 = b^2 + c^2$.

$$6^2 = b^2 + 4^2$$

$$36 = b^2 + 16$$

$$b^2 = 36 - 16$$

$$b = \sqrt{20}$$

∴ The equation of the ellipse is $x^2/6^2 + y^2/(\sqrt{20})^2 = 1$ or $x^2/36 + y^2/20 = 1$

13. Ends of major axis (± 3, 0), ends of minor axis (0, ± 2)

Solution:

Given:

Ends of major axis $(\pm 3, 0)$ and ends of minor axis $(0, \pm 2)$

Here, the major axis is along the x-axis.

So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.

Then, $a = 3$ and $b = 2$.

\therefore The equation for the ellipse $x^2/3^2 + y^2/2^2 = 1$ or $x^2/9 + y^2/4 = 1$

14. Ends of major axis $(0, \pm\sqrt{5})$, ends of minor axis $(\pm 1, 0)$

Solution:

Given:

Ends of major axis $(0, \pm\sqrt{5})$ and ends of minor axis $(\pm 1, 0)$

Here, the major axis is along the y-axis.

So, the equation of the ellipse will be of the form $x^2/b^2 + y^2/a^2 = 1$, where 'a' is the semi-major axis.

Then, $a = \sqrt{5}$ and $b = 1$.

\therefore The equation for the ellipse $x^2/1^2 + y^2/(\sqrt{5})^2 = 1$ or $x^2/1 + y^2/5 = 1$

15. Length of major axis 26, foci $(\pm 5, 0)$

Solution:

Given:

Length of major axis is 26 and foci $(\pm 5, 0)$

Since the foci are on the y-axis, the major axis is along the x-axis.

So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.

Then, $2a = 26$

$a = 13$ and $c = 5$.

It is known that $a^2 = b^2 + c^2$.

$$13^2 = b^2 + 5^2$$

$$169 = b^2 + 25$$

$$b^2 = 169 - 25$$

$$b = \sqrt{144}$$

$$= 12$$

\therefore The equation of the ellipse is $x^2/13^2 + y^2/12^2 = 1$ or $x^2/169 + y^2/144 = 1$

16. Length of minor axis 16, foci $(0, \pm 6)$.

Solution:

Given:

Length of minor axis is 16 and foci $(0, \pm 6)$.

Since the foci are on the y-axis, the major axis is along the y-axis.

So, the equation of the ellipse will be of the form $x^2/b^2 + y^2/a^2 = 1$, where 'a' is the semi-major axis.

Then, $2b = 16$

$$b = 8 \text{ and } c = 6.$$

It is known that $a^2 = b^2 + c^2$.

$$\begin{aligned} a^2 &= 8^2 + 6^2 \\ &= 64 + 36 \\ &= 100 \end{aligned}$$

$$\begin{aligned} a &= \sqrt{100} \\ &= 10 \end{aligned}$$

∴ The equation of the ellipse is $x^2/8^2 + y^2/10^2 = 1$ or $x^2/64 + y^2/100 = 1$

17. Foci $(\pm 3, 0)$, $a = 4$

Solution:

Given:

Foci $(\pm 3, 0)$ and $a = 4$

Since the foci are on the x-axis, the major axis is along the x-axis.

So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.

Then, $c = 3$ and $a = 4$.

It is known that $a^2 = b^2 + c^2$.

$$\begin{aligned} a^2 &= 8^2 + 6^2 \\ &= 64 + 36 \\ &= 100 \end{aligned}$$

$$16 = b^2 + 9$$

$$\begin{aligned} b^2 &= 16 - 9 \\ &= 7 \end{aligned}$$

∴ The equation of the ellipse is $x^2/16 + y^2/7 = 1$

18. $b = 3$, $c = 4$, centre at the origin; foci on the x axis.

Solution:

Given:

$b = 3$, $c = 4$, centre at the origin and foci on the x axis.

Since the foci are on the x-axis, the major axis is along the x-axis.

So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.

Then, $b = 3$ and $c = 4$.

It is known that $a^2 = b^2 + c^2$.

$$a^2 = 3^2 + 4^2$$

$$\begin{aligned}
 &= 9 + 16 \\
 &= 25 \\
 a &= \sqrt{25} \\
 &= 5
 \end{aligned}$$

∴ The equation of the ellipse is $x^2/5^2 + y^2/3^2$ or $x^2/25 + y^2/9 = 1$

19. Centre at (0, 0), major axis on the y-axis and passes through the points (3, 2) and (1, 6).

Solution:

Given:

Centre at (0, 0), major axis on the y-axis and passes through the points (3, 2) and (1, 6).

Since the centre is at (0, 0) and the major axis is on the y-axis, the equation of the ellipse will be of the form $x^2/b^2 + y^2/a^2 = 1$, where 'a' is the semi-major axis.

The ellipse passes through points (3, 2) and (1, 6).

So, by putting the values $x = 3$ and $y = 2$, we get,

$$3^2/b^2 + 2^2/a^2 = 1$$

$$9/b^2 + 4/a^2 \dots (1)$$

And by putting the values $x = 1$ and $y = 6$, we get,

$$1^2/b^2 + 6^2/a^2 = 1$$

$$1/b^2 + 36/a^2 = 1 \dots (2)$$

On solving equation (1) and (2), we get

$$b^2 = 10 \text{ and } a^2 = 40.$$

∴ The equation of the ellipse is $x^2/10 + y^2/40 = 1$ or $4x^2 + y^2 = 40$

20. Major axis on the x-axis and passes through the points (4,3) and (6,2).

Solution:

Given:

Major axis on the x-axis and passes through the points (4, 3) and (6, 2).

Since the major axis is on the x-axis, the equation of the ellipse will be the form $x^2/a^2 + y^2/b^2 = 1 \dots (1)$ [Where 'a' is the semi-major axis.]

The ellipse passes through points (4, 3) and (6, 2).

So by putting the values $x = 4$ and $y = 3$ in equation (1), we get,

$$16/a^2 + 9/b^2 = 1 \dots (2)$$

Putting, $x = 6$ and $y = 2$ in equation (1), we get,

$$36/a^2 + 4/b^2 = 1 \dots (3)$$

From equation (2)

$$16/a^2 = 1 - 9/b^2$$

$$1/a^2 = (1/16 (1 - 9/b^2)) \dots (4)$$

Substituting the value of $1/a^2$ in equation (3) we get,

$$36/a^2 + 4/b^2 = 1$$

$$36(1/a^2) + 4/b^2 = 1$$

$$36[1/16(1 - 9/b^2)] + 4/b^2 = 1$$

$$36/16(1 - 9/b^2) + 4/b^2 = 1$$

$$9/4(1 - 9/b^2) + 4/b^2 = 1$$

$$9/4 - 81/4b^2 + 4/b^2 = 1$$

$$-81/4b^2 + 4/b^2 = 1 - 9/4$$

$$(-81+16)/4b^2 = (4-9)/4$$

$$-65/4b^2 = -5/4$$

$$-5/4(13/b^2) = -5/4$$

$$13/b^2 = 1$$

$$1/b^2 = 1/13$$

$$b^2 = 13$$

Now substitute the value of b^2 in equation (4) we get,

$$1/a^2 = 1/16(1 - 9/b^2)$$

$$= 1/16(1 - 9/13)$$

$$= 1/16((13-9)/13)$$

$$= 1/16(4/13)$$

$$= 1/52$$

$$a^2 = 52$$

Equation of ellipse is $x^2/a^2 + y^2/b^2 = 1$

By substituting the values of a^2 and b^2 in above equation we get,

$$x^2/52 + y^2/13 = 1$$