EXERCISE 11.4 PAGE NO: 262

In each of the Exercises 1 to 6, find the coordinates of the foci and the vertices, the eccentricity and the length of the latus rectum of the hyperbolas.

1. $x^2/16 – y^2/9 = 1$

Solution:

Given:

The equation is $x^2/16 – y^2/9 = 1$ or $x^2/4^2 – y^2/3^2 = 1$

On comparing this equation with the standard equation of hyperbola $x^2/a^2 – y^2/b^2 = 1$,

We get a = 4 and b = 3,

It is known that, $a^2 + b^2 = c^2$

So,

$c^2 = 4^2 + 3^2$

$= \sqrt{25}$

c = 5

Then,

The coordinates of the foci are (±5, 0).

The coordinates of the vertices are (±4, 0).

Eccentricity, e = c/a = 5/4

Length of latus rectum = $2b^2/a = (2 \times 3^2)/4 = (2\times9)/4 = 18/4 = 9/2$

2. $y^2/9 – x^2/27 = 1$

Solution:

Given:

The equation is $y^2/9 – x^2/27 = 1$ or $y^2/3^2 – x^2/27^2 = 1$

On comparing this equation with the standard equation of hyperbola $y^2/a^2 – x^2/b^2 = 1$,

We get a = 3 and b = $\sqrt{27}$,

It is known that, $a^2 + b^2 = c^2$

So,

$c^2 = 3^2 + (\sqrt{27})^2$

$= 9 + 27$

$c^2 = 36$

$c = \sqrt{36}$

$= 6$

Then,

The coordinates of the foci are (0, 6) and (0, -6).

The coordinates of the vertices are (0, 3) and (0, - 3).

Eccentricity, e = c/a = 6/3 = 2

Length of latus rectum = $2b^2/a = (2 \times 27)/3 = (54)/3 = 18$

3. $9y^2 - 4x^2 = 36$

Solution:

Given:

The equation is $9y^2 - 4x^2 = 36$ or $y^2/4 - x^2/9 = 1$ or $y^2/2^2 - x^2/3^2 = 1$

On comparing this equation with the standard equation of hyperbola $y^2/a^2 - x^2/b^2 = 1$,

We get a = 2 and b = 3,

It is known that, $a^2 + b^2 = c^2$

So,

$c^2 = 4 + 9$

$c^2 = 13$

$c = \sqrt{13}$

Then,

The coordinates of the foci are $(0, \sqrt{13})$ and $(0, -\sqrt{13})$.

The coordinates of the vertices are (0, 2) and (0, - 2).

Eccentricity, $e = c/a = \sqrt{13}/2$

Length of latus rectum = $2b^2/a = (2 \times 3^2)/2 = (2\times9)/2 = 18/2 = 9$

4. $16x^2 - 9y^2 = 576$

Solution:

Given:

The equation is $16x^2 - 9y^2 = 576$

Let us divide the whole equation by 576, we get

$16x^2/576 - 9y^2/576 = 576/576$

$x^2/36 - y^2/64 = 1$

On comparing this equation with the standard equation of hyperbola $x^2/a^2 - y^2/b^2 = 1$,

We get a = 6 and b = 8,

It is known that, $a^2 + b^2 = c^2$

So,

$c^2 = 36 + 64$

$c^2 = \sqrt{100}$

$c = 10$

Then,

The coordinates of the foci are (10, 0) and (-10, 0).

The coordinates of the vertices are (6, 0) and (-6, 0).

Eccentricity, $e = c/a = 10/6 = 5/3$

Length of latus rectum = $2b^2/a = (2 \times 8^2)/6 = (2\times64)/6 = 64/3$

5. $5y^2 - 9x^2 = 36$

Solution:

Given:

The equation is $5y^2 - 9x^2 = 36$

Let us divide the whole equation by 36, we get

$5y^2/36 - 9x^2/36 = 36/36$

$y^2/(36/5) - x^2/4 = 1$

On comparing this equation with the standard equation of hyperbola $y^2/a^2 - x^2/b^2 = 1$,

We get $a = 6/\sqrt{5}$ and $b = 2$,

It is known that, $a^2 + b^2 = c^2$

So,

$c^2 = 36/5 + 4$

$c^2 = 56/5$

$c = \sqrt{(56/5)}$

$= 2\sqrt{14}/\sqrt{5}$

Then,

The coordinates of the foci are $(0, 2\sqrt{14}/\sqrt{5})$ and $(0, - 2\sqrt{14}/\sqrt{5})$.

The coordinates of the vertices are $(0, 6/\sqrt{5})$ and $(0, -6/\sqrt{5})$.

Eccentricity, $e = c/a = (2\sqrt{14}/\sqrt{5}) / (6/\sqrt{5}) = \sqrt{14}/3$

Length of latus rectum $= 2b^2/a = (2 \times 2^2)/6/\sqrt{5} = (2\times4)/6/\sqrt{5} = 4/\sqrt{5}/3$

6. $49y^2 - 16x^2 = 784$.

Solution:

Given:

The equation is $49y^2 - 16x^2 = 784$.

Let us divide the whole equation by 784, we get

$49y^2/784 - 16x^2/784 = 784/784$

$y^2/16 - x^2/49 = 1$

On comparing this equation with the standard equation of hyperbola $y^2/a^2 - x^2/b^2 = 1$,

We get $a = 4$ and $b = 7$,

It is know that, $a^2 + b^2 = c^2$

So,

$c^2 = 16 + 49$

$c^2 = 65$

$c = \sqrt{65}$

Then,

The coordinates of the foci are $(0, \sqrt{65})$ and $(0, -\sqrt{65})$.

The coordinates of the vertices are $(0, 4)$ and $(0, -4)$.

Eccentricity, $e = c/a = \sqrt{65}/4$

Length of latus rectum = $2b^2/a = (2 \times 7^2)/4 = (2\times49)/4 = 49/2$

In each Exercises 7 to 15, find the equations of the hyperbola satisfying the given conditions

7. Vertices (±2, 0), foci (±3, 0)
Solution:
Given:
Vertices (±2, 0) and foci (±3, 0)
Here, the vertices are on the x-axis.
So, the equation of the hyperbola is of the form $x^2/a^2 – y^2/b^2 = 1$
Since, the vertices are (±2, 0), so, a = 2
Since, the foci are (±3, 0), so, c = 3
It is know that, $a^2 + b^2 = c^2$
So, $2^2 + b^2 = 3^2$
$b^2 = 9 – 4 = 5$
∴ The equation of the hyperbola is $x^2/4 – y^2/5 = 1$

8. Vertices (0, ± 5), foci (0, ± 8)
Solution:
Given:
Vertices (0, ± 5) and foci (0, ± 8)
Here, the vertices are on the y-axis.
So, the equation of the hyperbola is of the form $y^2/a^2 – x^2/b^2 = 1$
Since, the vertices are (0, ±5), so, a = 5
Since, the foci are (0, ±8), so, c = 8
It is know that, $a^2 + b^2 = c^2$
So, $5^2 + b^2 = 8^2$
$b^2 = 64 – 25 = 39$
∴ The equation of the hyperbola is $y^2/25 – x^2/39 = 1$

9. Vertices (0, ± 3), foci (0, ± 5)
Solution:
Given:
Vertices (0, ± 3) and foci (0, ± 5)
Here, the vertices are on the y-axis.
So, the equation of the hyperbola is of the form $y^2/a^2 – x^2/b^2 = 1$
Since, the vertices are (0, ±3), so, a = 3
Since, the foci are (0, ±5), so, c = 5

It is known that, $a^2 + b^2 = c^2$
So, $3^2 + b^2 = 5^2$
$b^2 = 25 – 9 = 16$
∴ The equation of the hyperbola is $y^2/9 – x^2/16 = 1$

10. Foci (±5, 0), the transverse axis is of length 8.
Solution:
Given:
Foci (±5, 0) and the transverse axis is of length 8.
Here, the foci are on x-axis.
The equation of the hyperbola is of the form $x^2/a^2 – y^2/b^2 = 1$
Since, the foci are (±5, 0), so, c = 5
Since, the length of the transverse axis is 8,
2a = 8
a = 8/2
= 4
It is known that, $a^2 + b^2 = c^2$
$4^2 + b^2 = 5^2$
$b^2 = 25 – 16$
= 9
∴ The equation of the hyperbola is $x^2/16 – y^2/9 = 1$

11. Foci (0, ±13), the conjugate axis is of length 24.
Solution:
Given:
Foci (0, ±13) and the conjugate axis is of length 24.
Here, the foci are on y-axis.
The equation of the hyperbola is of the form $y^2/a^2 – x^2/b^2 = 1$
Since, the foci are (0, ±13), so, c = 13
Since, the length of the conjugate axis is 24,
2b = 24
b = 24/2
= 12
It is known that, $a^2 + b^2 = c^2$
$a^2 + 12^2 = 13^2$
$a^2 = 169 – 144$
= 25
∴ The equation of the hyperbola is $y^2/25 – x^2/144 = 1$

12. Foci (± 3√5, 0), the latus rectum is of length 8.
Solution:
Given:
Foci (± 3√5, 0) and the latus rectum is of length 8.
Here, the foci are on x-axis.
The equation of the hyperbola is of the form $x^2/a^2 - y^2/b^2 = 1$
Since, the foci are (± 3√5, 0), so, c = ± 3√5
Length of latus rectum is 8
$2b^2/a = 8$
$2b^2 = 8a$
$b^2 = 8a/2$
$= 4a$
It is known that, $a^2 + b^2 = c^2$
$a^2 + 4a = 45$
$a^2 + 4a - 45 = 0$
$a^2 + 9a - 5a - 45 = 0$
$(a + 9) (a - 5) = 0$
a = -9 or 5
Since, a is non – negative, a = 5
So, $b^2 = 4a$
$= 4 \times 5$
$= 20$
∴ The equation of the hyperbola is $x^2/25 - y^2/20 = 1$

13. Foci (± 4, 0), the latus rectum is of length 12
Solution:
Given:
Foci (± 4, 0) and the latus rectum is of length 12
Here, the foci are on x-axis.
The equation of the hyperbola is of the form $x^2/a^2 - y^2/b^2 = 1$
Since, the foci are (± 4, 0), so, c = 4
Length of latus rectum is 12
$2b^2/a = 12$
$2b^2 = 12a$
$b^2 = 12a/2$
$= 6a$
It is known that, $a^2 + b^2 = c^2$
$a^2 + 6a = 16$
$a^2 + 6a - 16 = 0$

$a^2 + 8a – 2a - 16 = 0$
$(a + 8) (a - 2) = 0$
$a = -8$ or 2
Since, a is non – negative, $a = 2$
So, $b^2 = 6a$
$= 6 \times 2$
$= 12$
∴ The equation of the hyperbola is $x^2/4 – y^2/12 = 1$

14. Vertices (±7, 0), e = 4/3
Solution:
Given:
Vertices (±7, 0) and $e = 4/3$
Here, the vertices are on the x- axis
The equation of the hyperbola is of the form $x^2/a^2 – y^2/b^2 = 1$
Since, the vertices are (± 7, 0), so, $a = 7$
It is given that $e = 4/3$
$c/a = 4/3$
$3c = 4a$
Substitute the value of a, we get
$3c = 4(7)$
$c = 28/3$
It is known that, $a^2 + b^2 = c^2$
$7^2 + b^2 = (28/3)^2$
$b^2 = 784/9 - 49$
$= (784 - 441)/9$
$= 343/9$
∴ The equation of the hyperbola is $x^2/49 – 9y^2/343 = 1$

15. Foci (0, ±√10), passing through (2, 3)
Solution:
Given:
Foci (0, ±√10) and passing through (2, 3)
Here, the foci are on y-axis.
The equation of the hyperbola is of the form $y^2/a^2 – x^2/b^2 = 1$
Since, the foci are (±√10, 0), so, $c = \sqrt{10}$
It is known that, $a^2 + b^2 = c^2$
$b^2 = 10 – a^2$ ………….. (1)
It is given that the hyperbola passes through point (2, 3)

So, $9/a^2 - 4/b^2 = 1$ … (2)
From equations (1) and (2), we get,
$9/a^2 - 4/(10-a^2) = 1$
$9(10 - a^2) - 4a^2 = a^2(10 - a^2)$
$90 - 9a^2 - 4a^2 = 10a^2 - a^4$
$a^4 - 23a^2 + 90 = 0$
$a^4 - 18a^2 - 5a^2 + 90 = 0$
$a^2(a^2 - 18) - 5(a^2 - 18) = 0$
$(a^2 - 18)(a^2 - 5) = 0$
$a^2 = 18$ or 5
In hyperbola, $c > a$ i.e., $c^2 > a^2$
So, $a^2 = 5$
$b^2 = 10 - a^2$
$= 10 - 5$
$= 5$
∴ The equation of the hyperbola is $y^2/5 - x^2/5 = 1$