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1. Which of the following pairs of linear equations has unique solution, no solution, or infinitely many solutions. In case there is a unique solution, find it by using cross multiplication method.

(i) x – 3y – 3 = 0 and 3x – 9y – 2 = 0
(ii) 2x + y = 5 and 3x + 2y = 8
(iii) 3x – 5y = 20 and 6x – 10y = 40
(iv) x – 3y – 7 = 0 and 3x – 3y – 15 = 0

Solutions:

(i) Given, x – 3y – 3 =0 and 3x – 9y -2 =0

$a_1/a_2=1/3$, $b_1/b_2= -3/-9 =1/3$, $c_1/c_2=-3/-2 = 3/2$

$(a_1/a_2) = (b_1/b_2) \neq (c_1/c_2)$

Since, the given set of lines are parallel to each other they will not intersect each other and therefore there is no solution for these equations.

(ii) Given, 2x + y = 5 and 3x +2y = 8

$a_1/a_2 = 2/3$, $b_1/b_2 = 1/2$, $c_1/c_2 = -5/-8$

$(a_1/a_2) \neq (b_1/b_2)$

Since they intersect at a unique point these equations will have a unique solution by cross multiplication method:

$x/(b_1c_2-c_1b_2) = y/(c_1a_2 – c_2a=) = 1/(a_1b_2-a_2b_1)$

x/(-8-(-10)) = y/(15+16) = 1/(4-3)

x/2 = y/1 = 1

∴ x = 2 and y =1

(iii) Given, 3x – 5y = 20 and 6x – 10y = 40

$(a_1/a_2) = 3/6 = 1/2$
$(b_1/b_2) = -5/-10 = 1/2$
$(c_1/c_2) = 20/40 = 1/2$

$a_1/a_2 = b_1/b_2 = c_1/c_2$

Since the given sets of lines are overlapping each other there will be infinite number of solutions for this pair of equation.

(iv) Given, x – 3y – 7 = 0 and 3x – 3y – 15 = 0

$(a_1/a_2) = 1/3$
$(b_1/b_2) = -3/-3 = 1$
$(c_1/c_2) = -7/-15$

$a_1/a_2 \neq b_1/b_2$

Since this pair of lines are intersecting each other at a unique point, there will be a unique solution.

By cross multiplication,

x/(45-21) = y/(-21+15) = 1/(-3+9)

x/24 = y/ -6 = 1/6

x/24 = 1/6 and y/-6 = 1/6

∴ x = 4 and y = 1.

2. (i) For which values of a and b does the following pair of linear equations have an infinite number of solutions?

2x + 3y = 7

(a – b) x + (a + b) y = 3a + b – 2

(ii) For which value of k will the following pair of linear equations have no solution?

3x + y = 1

(2k – 1) x + (k – 1) y = 2k + 1

Solution:

(i) 3y + 2x -7 =0

(a + b)y + (a-b)y – (3a + b -2) = 0

$a_1/a_2 = 2/(a-b)$, $b_1/b_2 = 3/(a+b)$, $c_1/c_2 = -7/-(3a + b -2)$

For infinitely many solutions,

$a_1/a_2 = b_1/b_2 = c_1/c_2$

Thus 2/(a-b) = 7/(3a+b– 2)

6a + 2b – 4 = 7a – 7b

a – 9b = -4 ……(i)

2/(a-b) = 3/(a+b)

2a + 2b = 3a – 3b

a - 5b = 0 ……(ii)

Subtracting (i) from (ii), we get

4b = 4

b =1

Substituting this eq. in (ii), we get

a -5 x 1= 0

a = 5

Thus at a = 5 and b = 1 the given equations will have infinite solutions.

(ii) 3x + y -1 = 0

(2k -1)x + (k-1)y – 2k -1 = 0

$a_1/a_2 = 3/(2k -1)$, $b_1/b_2 = 1/(k-1)$, $c_1/c_2 = -1/(-2k -1) = 1/(2k +1)$

For no solutions

$a_1/a_2 = b_1/b_2 \neq c_1/c_2$

$3/(2k-1) = 1/(k -1) \neq 1/(2k +1)$

3/(2k –1) = 1/(k -1)

3k -3 = 2k -1

k =2

Therefore, for k = 2 the given pair of linear equations will have no solution.

3. Solve the following pair of linear equations by the substitution and cross-multiplication methods:

8x + 5y = 9

3x + 2y = 4

Solution:

8x + 5y = 9 ……..(1)

3x + 2y = 4 ……..(2)

From equation (2) we get

x = (4 – 2y)/ 3 …… (3)

Using this value in equation 1, we get

8(4-2y)/3 + 5y = 9

32 – 16y +15y = 27

-y = -5

y = 5 ……(4)

Using this value in equation (2), we get

3x + 10 = 4

x = -2

Thus, x = -2 and y = 5.

Now, Using Cross Multiplication method:

8x +5y – 9 = 0

3x + 2y – 4 = 0

x/(-20+18) = y/(-27 + 32) = 1/(16-15)

-x/2 = y/5 =1/1

∴ x = -2 and y =5.

4. Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method:

(i) A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs.1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs.1180 as hostel charges. Find the fixed charges and the cost of food per day.

(ii) A fraction becomes 1/3 when 1 is subtracted from the numerator and it becomes 1/4 when 8 is added to its denominator. Find the fraction.

(iii) Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

(iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

(v) The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Solutions:

(i) Let x be the fixed charge and y be the charge of food per day.

According to the question,

x + 20y = 1000…….. (i)

x + 26y = 1180……..(ii)

Subtracting (i) from (ii) we get

6y = 180

y = Rs.30

Using this value in equation (ii) we get

x = 1180 -26 x 30

x= Rs.400.

Therefore, fixed charges is Rs.400 and charge per day is Rs.30.

(ii) L*et* the fraction be x/y.

So, as per the question given,

(x -1)/y = 1/3 => 3x – y = 3……...(1)

x/(y + 8) = 1/4 => 4x –y =8 ……..(2)

Subtracting equation (1) from (2) , we get

x = 5 …….(3)

Using this value in equation (2), we get,

(4×5)– y = 8

y= 12

Therefore, the fraction is 5/12.

(iii) Let the number of right answers is x and number of wrong answers be y

According to the given question;

3x−y=40……..(1)

4x−2y=50

⇒2x−y=25…….(2)

Subtracting equation (2) from equation (1), we get;

x = 15 ……..(3)

Putting this in equation (2), we obtain;

30 - y = 25

Or y = 5

Therefore, number of right answers = 15 and number of wrong answers = 5

Hence, total number of questions = 20

(iv) Let x km/h be the speed of car from point A and y km/h be the speed of car from point B.

If the car travels in the same direction,
5x – 5y = 100
x – y = 20 ……(i)

If the car travels in the opposite direction,
x + y = 100……(ii)

Solving equation (i) and (ii), we get
x = 60 km/h……(iii)

Using this in equation (i), we get,
60 – y = 20
y = 40 km/h

Therefore, the speed of car from point A = 60 km/h

Speed of car from point B = 40 km/h.

(v) Let,

The length of rectangle = x unit

And breadth of the rectangle = y unit

Now, as per the question given,

(x – 5) (y + 3) = xy -9

3x – 5y – 6 = 0……(1)

(x + 3) (y + 2) = xy + 67

2x + 3y – 61 = 0……..(2)

Using cross multiplication method, we get,

x/(305 +18) = y/(-12+183) = 1/(9+10)

x/323 = y/171 = 1/19

Therefore, x = 17 and y = 9.

Hence, the length of rectangle = 17 units

And breadth of the rectangle = 9 units