EXERCISE 18.4

1. Find the absolute maximum and the absolute minimum values of the following functions in the given intervals:

(i) f (x) = 4x – x^2/2 in [–2, 9/2]

Solution:

Given function is f(x) = $4x - \frac{x^2}{2}$

On differentiation we get

∴f'(x) = 4 – x

Now, for local minima and local maxima we have f'(x) = 0

4 – x = 0

x = 4

Then, we evaluate of f at critical points x = 4 and at the interval $[-2, \frac{9}{2}]$

f (4) = $4(4) - \frac{(4)^2}{2}$ = 8

f (– 2) = $4(-2) - \frac{(-2)^2}{2} = -10$

f $(\frac{9}{2})$ = $4\left(\frac{9}{2}\right) - \frac{\left(\frac{9}{2}\right)^2}{2} = 18 - \frac{81}{8} = 7.875$

Hence, we can conclude that the absolute maximum value of f on [– 2, 9/2] is 8 occurring at x = 4 and the absolute minimum value of f on [– 2, 9/2] is – 10 occurring at x = – 2

(ii) f (x) = $(x – 1)^2$ + 3, x ∈ [–3, 1]

Solution:

Given function is f(x) = $(x – 1)^2$ + 3

On differentiation we get

⇒ f'(x) = 2(x – 1)

Now, for local minima and local maxima we have f'(x) = 0

2(x – 1) = 0

x = 1

Then, we evaluate of f at critical point x = 1 and at the interval [– 3, 1]

$f(1) = (1 - 1)^2 + 3 = 3$

$f(-3) = (-3 - 1)^2 + 3 = 19$

Hence, we can conclude that the absolute maximum value of f on [– 3, 1] is 19 occurring at x = – 3 and the minimum value of f on [– 3, 1] is 3 occurring at x = 1

(iii) $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the interval [0, 3]

Solution:

Given function is $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the interval [0, 3]

On differentiating we get

$f'(x) = 12x^3 - 24x^2 + 24x - 48$

$f'(x) = 12(x^3 - 2x^2 + 2x - 4)$

$f'(x) = 12(x - 2)(x^2 + 2)$

Now, for local minima and local maxima we have f'(x) = 0

x = 2 or $x^2 + 2 = 0$ for which there are no real roots.

Therefore, we consider only x = 2 ∈ [0, 3].

Then, we evaluate of f at critical point x = 2 and at the interval [0, 3]

$f(2) = 3(2)^4 - 8(2)^3 + 12(2)^2 - 48(2) + 25$

$f(2) = 48 - 64 + 48 - 96 + 25 = -39$

$f(0) = 3(0)^4 - 8(0)^3 + 12(0)^2 - 48(0) + 25 = 25$

$f(3) = 3(3)^4 - 8(3)^3 + 12(3)^2 - 48(3) + 25 = 16$

Hence, we can conclude that the absolute maximum value of f on [0, 3] is 25 occurring at x = 0 and the minimum value of f at[0, 3] is – 39 occurring at x = 2

(iv) $f(x) = (x - 2)\sqrt{x - 1}$ in [1, 9]

Solution:

Given $f(x) = (x - 2)\sqrt{x - 1}$

$f'(x) = \sqrt{x - 1} + \frac{(x-2)}{2\sqrt{x-1}}$

Put f'(x) = 0

$\Rightarrow \sqrt{x-1} + \frac{(x-2)}{2\sqrt{x-1}} = 0$

$\Rightarrow \frac{2(x-1)+(x-2)}{2\sqrt{x-1}} = 0$

$\Rightarrow \frac{3x-4}{2\sqrt{x-1}} = 0$

$\Rightarrow x = \frac{4}{3}$

Now, f (1) = 0

f (4/3) = $\left(\frac{4}{3} - 2\right)\sqrt{\frac{4}{3} - 1} = -\frac{2}{3\sqrt{3}} = -\frac{2\sqrt{3}}{9}$

And,
f(9) = (9 - 2) √(9 - 1) = 7√8 = 14√2
Hence, we can conclude that the absolute maximum value of f is 14√2 occurring at x = 9 and the minimum value of f is -2 √3/9 occurring at x = 4/3.

2. Find the maximum value of $2x^3 – 24x + 107$ in the interval [1, 3]. Find the maximum value of the same function in [–3, –1].

Solution:
Let $f(x) = 2x^3 – 24 x + 107$
∴ $f'(x) = 6 x^2 – 24 = 6(x^2 – 4)$
Now, for local maxima and local minima we have f'(x) = 0
⇒ $6(x^2 – 4) = 0$
⇒ $x^2 = 4$
⇒ x = ±2
We first consider the interval [1, 3].
Then, we evaluate the value of f at the critical point x = 2 ∈ [1, 3] and at the end points of the interval [1, 3].
$f(2) = 2(2^3) – 24(2) + 107 = 75$
$f(1) = 2(1)^3 – 24(1) + 107 = 85$
$f(3) = 2(3)^3 – 24(3) + 107 = 89$
Hence, the absolute maximum value of f(x) in the interval [1, 3] is 89 occurring at x = 3,
Next, we consider the interval [– 3, – 1].
Evaluate the value of f at the critical point x = – 2 ∈ [-3, -1]

f (-3) = 2 $(-3)^3$ – 24 (-3) + 107 = 125
f (-2) = 2 $(-2)^3$ – 24 (-3) + 107 = 139
f (-1) = 2 $(-1)^3$ – 24 (-2) + 107 = 129
Hence, the absolute maximum value of f is 139 and occurs when x = -2.