

EXERCISE 4.4**PAGE NO: 4.30****1. Find the cube roots of each of the following integers:****(i)-125 (ii) -5832****(iii)-2744000 (iv) -753571****(v) -32768****Solution:****(i) -125**

The cube root of -125 is

$$-125 = \sqrt[3]{-125} = -\sqrt[3]{125} = -\sqrt[3]{(5 \times 5 \times 5)} = -5$$

(ii) -5832

The cube root of -5832 is

$$-5832 = \sqrt[3]{-5832} = -\sqrt[3]{5832}$$

To find the cube root of 5832, we shall use the method of unit digits.

Let us consider the number 5832. Where, unit digit of 5832 = 2

Unit digit in the cube root of 5832 will be 8

After striking out the units, tens and hundreds digits of 5832,

Now we left with 5 only.

We know that 1 is the Largest number whose cube is less than or equal to 5.

So, the tens digit of the cube root of 5832 is 1.

$$\sqrt[3]{-5832} = -\sqrt[3]{5832} = -18$$

(iii) -2744000

$$\sqrt[3]{-2744000} = -\sqrt[3]{2744000}$$

We shall use the method of factorization to find the cube root of 2744000

So let's find the prime factors for 2744000

$$2744000 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 7 \times 7 \times 7$$

Now by grouping the factors into triples of equal factors, we get,

$$2744000 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (5 \times 5 \times 5) \times (7 \times 7 \times 7)$$

Since all the prime factors of 2744000 is grouped in to triples of equal factors and no factor is left over.

So now take one factor from each group and by multiplying we get,

$$2 \times 2 \times 5 \times 7 = 140$$

Thereby we can say that 2744000 is a cube of 140

$$\therefore \sqrt[3]{-2744000} = -\sqrt[3]{2744000} = -140$$

(iv) -753571

$$\sqrt[3]{-753571} = -\sqrt[3]{753571}$$

We shall use the unit digit method,

Let us consider the number 753571, where unit digit = 1

Unit digit in the cube root of 753571 will be 1

After striking out the units, tens and hundreds digits of 753571,

Now we left with 753.

We know that 9 is the Largest number whose cube is less than or equal to 753 ($9^3 < 753 < 10^3$).

So, the tens digit of the cube root of 753571 is 9.

$$\sqrt[3]{753571} = 91$$

$$\sqrt[3]{-753571} = -\sqrt[3]{753571} = -91$$

(v) -32768

$$\sqrt[3]{-32765} = -\sqrt[3]{32768}$$

We shall use the unit digit method,

Let us consider the Number = 32768, where unit digit = 8

Unit digit in the cube root of 32768 will be 2

After striking out the units, tens and hundreds digits of 32768,

Now we left with 32.

As we know that 3 is the Largest number whose cube is less than or equals to 32 ($3^3 < 32 < 4^3$).

So, the tens digit of the cube root of 32768 is 3.

$$\sqrt[3]{32765} = 32$$

$$\sqrt[3]{-32765} = -\sqrt[3]{32768} = -32$$

2. Show that:

(i) $\sqrt[3]{27} \times \sqrt[3]{64} = \sqrt[3]{(27 \times 64)}$

(ii) $\sqrt[3]{(64 \times 729)} = \sqrt[3]{64} \times \sqrt[3]{729}$

(iii) $\sqrt[3]{(-125 \times 216)} = \sqrt[3]{-125} \times \sqrt[3]{216}$

(iv) $\sqrt[3]{(-125 \times -1000)} = \sqrt[3]{-125} \times \sqrt[3]{-1000}$

Solution:

(i) $\sqrt[3]{27} \times \sqrt[3]{64} = \sqrt[3]{(27 \times 64)}$

Let us consider LHS $\sqrt[3]{27} \times \sqrt[3]{64}$

$$\sqrt[3]{27} \times \sqrt[3]{64} = \sqrt[3]{(3 \times 3 \times 3)} \times \sqrt[3]{(4 \times 4 \times 4)}$$

$$= 3 \times 4$$

$$= 12$$

Let us consider RHS $\sqrt[3]{(27 \times 64)}$

$$\sqrt[3]{(27 \times 64)} = \sqrt[3]{(3 \times 3 \times 3 \times 4 \times 4 \times 4)}$$

$$= 3 \times 4$$

$$= 12$$

∴ LHS = RHS, the given equation is verified.

$$(ii) \sqrt[3]{(64 \times 729)} = \sqrt[3]{64} \times \sqrt[3]{729}$$

Let us consider LHS $\sqrt[3]{(64 \times 729)}$

$$\begin{aligned} \sqrt[3]{(64 \times 729)} &= \sqrt[3]{(4 \times 4 \times 4 \times 9 \times 9 \times 9)} \\ &= 4 \times 9 \\ &= 36 \end{aligned}$$

Let us consider RHS $\sqrt[3]{64} \times \sqrt[3]{729}$

$$\begin{aligned} \sqrt[3]{64} \times \sqrt[3]{729} &= \sqrt[3]{(4 \times 4 \times 4)} \times \sqrt[3]{(9 \times 9 \times 9)} \\ &= 4 \times 9 \\ &= 36 \end{aligned}$$

∴ LHS = RHS, the given equation is verified.

$$(iii) \sqrt[3]{(-125 \times 216)} = \sqrt[3]{-125} \times \sqrt[3]{216}$$

Let us consider LHS $\sqrt[3]{(-125 \times 216)}$

$$\begin{aligned} \sqrt[3]{(-125 \times 216)} &= \sqrt[3]{(-5 \times -5 \times -5 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3)} \\ &= -5 \times 2 \times 3 \\ &= -30 \end{aligned}$$

Let us consider RHS $\sqrt[3]{-125} \times \sqrt[3]{216}$

$$\begin{aligned} \sqrt[3]{-125} \times \sqrt[3]{216} &= \sqrt[3]{(-5 \times -5 \times -5)} \times \sqrt[3]{(2 \times 2 \times 2 \times 3 \times 3 \times 3)} \\ &= -5 \times 2 \times 3 \\ &= -30 \end{aligned}$$

∴ LHS = RHS, the given equation is verified.

$$(iv) \sqrt[3]{(-125 \times -1000)} = \sqrt[3]{-125} \times \sqrt[3]{-1000}$$

Let us consider LHS $\sqrt[3]{(-125 \times -1000)}$

$$\begin{aligned} \sqrt[3]{(-125 \times -1000)} &= \sqrt[3]{(-5 \times -5 \times -5 \times -10 \times -10 \times -10)} \\ &= -5 \times -10 \\ &= 50 \end{aligned}$$

Let us consider RHS $\sqrt[3]{-125} \times \sqrt[3]{-1000}$

$$\begin{aligned} \sqrt[3]{-125} \times \sqrt[3]{-1000} &= \sqrt[3]{(-5 \times -5 \times -5)} \times \sqrt[3]{(-10 \times -10 \times -10)} \\ &= -5 \times -10 \\ &= 50 \end{aligned}$$

∴ LHS = RHS, the given equation is verified.

3. Find the cube root of each of the following numbers:

(i) 8×125

(ii) -1728×216

(iii) -27×2744

(iv) -729×-15625

Solution:

(i) 8×125

We know that for any two integers a and b, $\sqrt[3]{(a \times b)} = \sqrt[3]{a} \times \sqrt[3]{b}$

By using the property

$$\begin{aligned}\sqrt[3]{(8 \times 125)} &= \sqrt[3]{8} \times \sqrt[3]{125} \\ &= \sqrt[3]{(2 \times 2 \times 2)} \times \sqrt[3]{(5 \times 5 \times 5)} \\ &= 2 \times 5 \\ &= 10\end{aligned}$$

(ii) -1728×216

We know that for any two integers a and b, $\sqrt[3]{(a \times b)} = \sqrt[3]{a} \times \sqrt[3]{b}$

By using the property

$$\sqrt[3]{(-1728 \times 216)} = \sqrt[3]{-1728} \times \sqrt[3]{216}$$

We shall use the unit digit method

Let us consider the number 1728, where Unit digit = 8

The unit digit in the cube root of 1728 will be 2

After striking out the units, tens and hundreds digits of the given number, we are left with the 1.

We know 1 is the largest number whose cube is less than or equal to 1.

So, the tens digit of the cube root of 1728 = 1

$$\sqrt[3]{1728} = 12$$

Now, let's find the prime factors for $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

By grouping the factors in triples of equal factor, we get,

$$216 = (2 \times 2 \times 2) \times (3 \times 3 \times 3)$$

By taking one factor from each group we get,

$$\sqrt[3]{216} = 2 \times 3 = 6$$

\therefore by equating the values in the given equation we get,

$$\begin{aligned}\sqrt[3]{(-1728 \times 216)} &= \sqrt[3]{-1728} \times \sqrt[3]{216} \\ &= -12 \times 6 \\ &= -72\end{aligned}$$

(iii) -27×2744

We know that for any two integers a and b, $\sqrt[3]{(a \times b)} = \sqrt[3]{a} \times \sqrt[3]{b}$

By using the property

$$\sqrt[3]{(-27 \times 2744)} = \sqrt[3]{-27} \times \sqrt[3]{2744}$$

We shall use the unit digit method

Let us consider the number 2744, where Unit digit = 4

The unit digit in the cube root of 2744 will be 4

After striking out the units, tens and hundreds digits of the given number, we are left with the 2.

We know 2 is the largest number whose cube is less than or equal to 2.

So, the tens digit of the cube root of 2744 = 2

$$\sqrt[3]{2744} = 14$$

Now, let's find the prime factors for $27 = 3 \times 3 \times 3$

By grouping the factors in triples of equal factor, we get,

$$27 = (3 \times 3 \times 3)$$

Cube root of 27 is

$$\sqrt[3]{27} = 3$$

∴ by equating the values in the given equation we get,

$$\begin{aligned}\sqrt[3]{(-27 \times 2744)} &= \sqrt[3]{-27} \times \sqrt[3]{2744} \\ &= -3 \times 14 \\ &= -42\end{aligned}$$

(iv) -729×-15625

We know that for any two integers a and b, $\sqrt[3]{(a \times b)} = \sqrt[3]{a} \times \sqrt[3]{b}$

By using the property

$$\sqrt[3]{(-729 \times -15625)} = \sqrt[3]{-729} \times \sqrt[3]{-15625}$$

We shall use the unit digit method

Let us consider the number 15625, where Unit digit = 5

The unit digit in the cube root of 15625 will be 5

After striking out the units, tens and hundreds digits of the given number, we are left with the 15.

We know 15 is the largest number whose cube is less than or equal to $15(2^3 < 15 < 3^3)$.

So, the tens digit of the cube root of 15625 = 2

$$\sqrt[3]{15625} = 25$$

Now, let's find the prime factors for $729 = 9 \times 9 \times 9$

By grouping the factors in triples of equal factor, we get,

$$729 = (9 \times 9 \times 9)$$

Cube root of 729 is

$$\sqrt[3]{729} = 9$$

∴ by equating the values in the given equation we get,

$$\begin{aligned}\sqrt[3]{(-729 \times -15625)} &= \sqrt[3]{-729} \times \sqrt[3]{-15625} \\ &= -9 \times -25 \\ &= 225\end{aligned}$$

4. Evaluate:

(i) $\sqrt[3]{4^3 \times 6^3}$

(ii) $\sqrt[3]{8 \times 17 \times 17 \times 17}$

(iii) $\sqrt[3]{700 \times 2 \times 49 \times 5}$

(iv) $125 \sqrt[3]{a^6} - \sqrt[3]{125a^6}$

Solution:

(i) $\sqrt[3]{4^3 \times 6^3}$

We know that for any two integers a and b, $\sqrt[3]{(a \times b)} = \sqrt[3]{a} \times \sqrt[3]{b}$

By using the property

$$\begin{aligned}\sqrt[3]{4^3 \times 6^3} &= \sqrt[3]{4^3} \times \sqrt[3]{6^3} \\ &= 4 \times 6 \\ &= 24\end{aligned}$$

(ii) $\sqrt[3]{8 \times 17 \times 17 \times 17}$

We know that for any two integers a and b, $\sqrt[3]{(a \times b)} = \sqrt[3]{a} \times \sqrt[3]{b}$

By using the property

$$\begin{aligned}\sqrt[3]{8 \times 17 \times 17 \times 17} &= \sqrt[3]{8} \times \sqrt[3]{17 \times 17 \times 17} \\ &= \sqrt[3]{2^3} \times \sqrt[3]{17^3} \\ &= 2 \times 17 \\ &= 34\end{aligned}$$

(iii) $\sqrt[3]{700 \times 2 \times 49 \times 5}$

Firstly let us find the prime factors for the above numbers

$$\begin{aligned}\sqrt[3]{700 \times 2 \times 49 \times 5} &= \sqrt[3]{(2 \times 2 \times 5 \times 5 \times 7 \times 2 \times 7 \times 7 \times 5)} \\ &= \sqrt[3]{(2^3 \times 5^3 \times 7^3)} \\ &= 2 \times 5 \times 7 \\ &= 70\end{aligned}$$

(iv) $125 \sqrt[3]{a^6} - \sqrt[3]{125a^6}$

$$\begin{aligned}125 \sqrt[3]{a^6} - \sqrt[3]{125a^6} &= 125 \sqrt[3]{(a^2)^3} - \sqrt[3]{5^3(a^2)^3} \\ &= 125a^2 - 5a^2 \\ &= 120a^2\end{aligned}$$

5. Find the cube root of each of the following rational numbers:

(i) $-125/729$

(ii) $10648/12167$

(iii) $-19683/24389$

(iv) $686/-3456$

(v) $-39304/-42875$

Solution:**(i)** $-125/729$

Let us find the prime factors of 125 and 729

$$\begin{aligned} -125/729 &= -(\sqrt[3]{(5 \times 5 \times 5)}) / (\sqrt[3]{(9 \times 9 \times 9)}) \\ &= -(\sqrt[3]{(5^3)}) / (\sqrt[3]{(9^3)}) \\ &= -5/9 \end{aligned}$$

(ii) $10648/12167$

Let us find the prime factors of 10648 and 12167

$$\begin{aligned} 10648/12167 &= (\sqrt[3]{(2 \times 2 \times 2 \times 11 \times 11 \times 11)}) / (\sqrt[3]{(23 \times 23 \times 23)}) \\ &= (\sqrt[3]{(2^3 \times 11^3)}) / (\sqrt[3]{(23^3)}) \\ &= (2 \times 11)/23 \\ &= 22/23 \end{aligned}$$

(iii) $-19683/24389$

Let us find the prime factors of 19683 and 24389

$$\begin{aligned} -19683/24389 &= -(\sqrt[3]{(3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3)}) / (\sqrt[3]{(29 \times 29 \times 29)}) \\ &= -(\sqrt[3]{(3^3 \times 3^3 \times 3^3)}) / (\sqrt[3]{(29^3)}) \\ &= -(3 \times 3 \times 3)/29 \\ &= -27/29 \end{aligned}$$

(iv) $686/-3456$

Let us find the prime factors of 686 and -3456

$$\begin{aligned} 686/-3456 &= -(\sqrt[3]{(2 \times 7 \times 7 \times 7)}) / (\sqrt[3]{(2^7 \times 3^3)}) \\ &= -(\sqrt[3]{(2 \times 7^3)}) / (\sqrt[3]{(2^7 \times 3^3)}) \\ &= -(\sqrt[3]{(7^3)}) / (\sqrt[3]{(2^6 \times 3^3)}) \\ &= -7/(2 \times 2 \times 3) \\ &= -7/12 \end{aligned}$$

(v) $-39304/-42875$

Let us find the prime factors of -39304 and -42875

$$\begin{aligned} -39304/-42875 &= -(\sqrt[3]{(2 \times 2 \times 2 \times 17 \times 17 \times 17)}) / -(\sqrt[3]{(5 \times 5 \times 5 \times 7 \times 7 \times 7)}) \\ &= -(\sqrt[3]{(2^3 \times 17^3)}) / -(\sqrt[3]{(5^3 \times 7^3)}) \\ &= -(2 \times 17)/-(5 \times 7) \\ &= -34/-35 \\ &= 34/35 \end{aligned}$$

6. Find the cube root of each of the following rational numbers:**(i)** 0.001728

(ii) 0.003375

(iii) 0.001

(iv) 1.331

Solution:

(i) 0.001728

$$0.001728 = 1728/1000000$$

$$\sqrt[3]{(0.001728)} = \sqrt[3]{1728} / \sqrt[3]{1000000}$$

Let us find the prime factors of 1728 and 1000000

$$\sqrt[3]{(0.001728)} = \sqrt[3]{(2^3 \times 2^3 \times 3^3)} / \sqrt[3]{(100^3)}$$

$$= (2 \times 2 \times 3) / 100$$

$$= 12/100$$

$$= 0.12$$

(ii) 0.003375

$$0.003375 = 3375/1000000$$

$$\sqrt[3]{(0.003375)} = \sqrt[3]{3375} / \sqrt[3]{1000000}$$

Let us find the prime factors of 3375 and 1000000

$$\sqrt[3]{(0.003375)} = \sqrt[3]{(3^3 \times 5^3)} / \sqrt[3]{(100^3)}$$

$$= (3 \times 5) / 100$$

$$= 15/100$$

$$= 0.15$$

(iii) 0.001

$$0.001 = 1/1000$$

$$\sqrt[3]{(0.001)} = \sqrt[3]{1} / \sqrt[3]{1000}$$

$$= 1 / \sqrt[3]{10^3}$$

$$= 1/10$$

$$= 0.1$$

(iv) 1.331

$$1.331 = 1331/1000$$

$$\sqrt[3]{(1.331)} = \sqrt[3]{1331} / \sqrt[3]{1000}$$

Let us find the prime factors of 1331 and 1000

$$\sqrt[3]{(1.331)} = \sqrt[3]{(11^3)} / \sqrt[3]{(10^3)}$$

$$= 11/10$$

$$= 1.1$$

7. Evaluate each of the following:

(i) $\sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064}$

$$(ii) \sqrt[3]{1000} + \sqrt[3]{0.008} - \sqrt[3]{0.125}$$

$$(iii) \sqrt[3]{(729/216)} \times 6/9$$

$$(iv) \sqrt[3]{(0.027/0.008)} \div \sqrt[3]{(0.09/0.04)} - 1$$

$$(v) \sqrt[3]{(0.1 \times 0.1 \times 0.1 \times 13 \times 13 \times 13)}$$

Solution:

$$(i) \sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064}$$

Let us simplify

$$\sqrt[3]{(3 \times 3 \times 3)} + \sqrt[3]{(0.2 \times 0.2 \times 0.2)} + \sqrt[3]{(0.4 \times 0.4 \times 0.4)}$$

$$\sqrt[3]{(3)^3} + \sqrt[3]{(0.2)^3} + \sqrt[3]{(0.4)^3}$$

$$3 + 0.2 + 0.4$$

$$3.6$$

$$(ii) \sqrt[3]{1000} + \sqrt[3]{0.008} - \sqrt[3]{0.125}$$

Let us simplify

$$\sqrt[3]{(10 \times 10 \times 10)} + \sqrt[3]{(0.2 \times 0.2 \times 0.2)} - \sqrt[3]{(0.5 \times 0.5 \times 0.5)}$$

$$\sqrt[3]{(10)^3} + \sqrt[3]{(0.2)^3} - \sqrt[3]{(0.5)^3}$$

$$10 + 0.2 - 0.5$$

$$9.7$$

$$(iii) \sqrt[3]{(729/216)} \times 6/9$$

Let us simplify

$$\sqrt[3]{(9 \times 9 \times 9 / 6 \times 6 \times 6)} \times 6/9$$

$$(\sqrt[3]{(9)^3} / \sqrt[3]{(6)^3}) \times 6/9$$

$$9/6 \times 6/9$$

$$1$$

$$(iv) \sqrt[3]{(0.027/0.008)} \div \sqrt[3]{(0.09/0.04)} - 1$$

Let us simplify $\sqrt[3]{(0.027/0.008)} \div \sqrt[3]{(0.09/0.04)}$

$$\sqrt[3]{(0.3 \times 0.3 \times 0.3 / 0.2 \times 0.2 \times 0.2)} \div \sqrt[3]{(0.3 \times 0.3 / 0.2 \times 0.2)}$$

$$(\sqrt[3]{(0.3)^3} / \sqrt[3]{(0.2)^3}) \div (\sqrt[3]{(0.3)^2} / \sqrt[3]{(0.2)^2})$$

$$(0.3/0.2) \div (0.3/0.2) - 1$$

$$(0.3/0.2 \times 0.2/0.3) - 1$$

$$1 - 1$$

$$0$$

$$(v) \sqrt[3]{(0.1 \times 0.1 \times 0.1 \times 13 \times 13 \times 13)}$$

$$\sqrt[3]{(0.1^3 \times 13^3)}$$

$$0.1 \times 13 = 1.3$$

8. Show that:

(i) $\sqrt[3]{729}/\sqrt[3]{1000} = \sqrt[3]{729/1000}$

(ii) $\sqrt[3]{-512}/\sqrt[3]{343} = \sqrt[3]{-512/343}$

Solution:

(i) $\sqrt[3]{729}/\sqrt[3]{1000} = \sqrt[3]{729/1000}$

 Let us consider LHS $\sqrt[3]{729}/\sqrt[3]{1000}$

$$\begin{aligned}\sqrt[3]{729}/\sqrt[3]{1000} &= \sqrt[3]{(9 \times 9 \times 9)}/\sqrt[3]{(10 \times 10 \times 10)} \\ &= \sqrt[3]{(9^3/10^3)} \\ &= 9/10\end{aligned}$$

 Let us consider RHS $\sqrt[3]{729/1000}$

$$\begin{aligned}\sqrt[3]{729/1000} &= \sqrt[3]{(9 \times 9 \times 9/10 \times 10 \times 10)} \\ &= \sqrt[3]{(9^3/10^3)} \\ &= 9/10\end{aligned}$$

 \therefore LHS = RHS

(ii) $\sqrt[3]{-512}/\sqrt[3]{343} = \sqrt[3]{-512/343}$

 Let us consider LHS $\sqrt[3]{-512}/\sqrt[3]{343}$

$$\begin{aligned}\sqrt[3]{-512}/\sqrt[3]{343} &= \sqrt[3]{-(8 \times 8 \times 8)}/\sqrt[3]{(7 \times 7 \times 7)} \\ &= \sqrt[3]{-(8^3/7^3)} \\ &= -8/7\end{aligned}$$

 Let us consider RHS $\sqrt[3]{-512/343}$

$$\begin{aligned}\sqrt[3]{-512/343} &= \sqrt[3]{-(8 \times 8 \times 8/7 \times 7 \times 7)} \\ &= \sqrt[3]{-(8^3/7^3)} \\ &= -8/7\end{aligned}$$

 \therefore LHS = RHS

9. Fill in the blanks:

(i) $\sqrt[3]{125 \times 27} = 3 \times \dots$

(ii) $\sqrt[3]{8 \times \dots} = 8$

(iii) $\sqrt[3]{1728} = 4 \times \dots$

(iv) $\sqrt[3]{480} = \sqrt[3]{3 \times 2 \times \dots}$

(v) $\sqrt[3]{\dots} = \sqrt[3]{7 \times \sqrt[3]{8}}$

(vi) $\sqrt[3]{\dots} = \sqrt[3]{4 \times \sqrt[3]{5 \times \sqrt[3]{6}}}$

(vii) $\sqrt[3]{27/125} = \dots/5$

(viii) $\sqrt[3]{729/1331} = 9/\dots$

(ix) $\sqrt[3]{512/\dots} = 8/13$

Solution:

(i) $\sqrt[3]{125 \times 27} = 3 \times \dots$

 Let us consider LHS $\sqrt[3]{125 \times 27}$

$$\begin{aligned}\sqrt[3]{(125 \times 27)} &= \sqrt[3]{(5 \times 5 \times 5 \times 3 \times 3 \times 3)} \\ &= \sqrt[3]{(5^3 \times 3^3)} \\ &= 5 \times 3 \text{ or } 3 \times 5\end{aligned}$$

(ii) $\sqrt[3]{(8 \times \dots)} = 8$

Let us consider LHS $\sqrt[3]{(8 \times \dots)}$

$$\sqrt[3]{(8 \times 8 \times 8)} = \sqrt[3]{8^3} = 8$$

(iii) $\sqrt[3]{1728} = 4 \times \dots$

Let us consider LHS

$$\begin{aligned}\sqrt[3]{1728} &= \sqrt[3]{(2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3)} \\ &= \sqrt[3]{(2^3 \times 2^3 \times 3^3)} \\ &= 2 \times 2 \times 3 \\ &= 4 \times 3\end{aligned}$$

(iv) $\sqrt[3]{480} = \sqrt[3]{3} \times 2 \times \sqrt[3]{\dots}$

Let us consider LHS

$$\begin{aligned}\sqrt[3]{480} &= \sqrt[3]{(2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5)} \\ &= \sqrt[3]{(2^3 \times 2^2 \times 3 \times 5)} \\ &= \sqrt[3]{2^3} \times \sqrt[3]{3} \times \sqrt[3]{2 \times 2 \times 5} \\ &= 2 \times \sqrt[3]{3} \times \sqrt[3]{20}\end{aligned}$$

(v) $\sqrt[3]{\dots} = \sqrt[3]{7} \times \sqrt[3]{8}$

Let us consider RHS

$$\begin{aligned}\sqrt[3]{7} \times \sqrt[3]{8} &= \sqrt[3]{(7 \times 8)} \\ &= \sqrt[3]{56}\end{aligned}$$

(vi) $\sqrt[3]{\dots} = \sqrt[3]{4} \times \sqrt[3]{5} \times \sqrt[3]{6}$

Let us consider RHS

$$\begin{aligned}\sqrt[3]{4} \times \sqrt[3]{5} \times \sqrt[3]{6} &= \sqrt[3]{(4 \times 5 \times 6)} \\ &= \sqrt[3]{120}\end{aligned}$$

(vii) $\sqrt[3]{(27/125)} = \dots/5$

Let us consider LHS

$$\begin{aligned}\sqrt[3]{(27/125)} &= \sqrt[3]{(3 \times 3 \times 3)/(5 \times 5 \times 5)} \\ &= \sqrt[3]{(3^3)/(5^3)} \\ &= 3/5\end{aligned}$$

(viii) $\sqrt[3]{(729/1331)} = 9/\dots$

Let us consider LHS

$$\begin{aligned}\sqrt[3]{(729/1331)} &= \sqrt[3]{(9 \times 9 \times 9)/(11 \times 11 \times 11)} \\ &= \sqrt[3]{(9^3)/(11^3)} \\ &= 9/11\end{aligned}$$

(ix) $\sqrt[3]{(512/\dots)} = 8/13$

Let us consider LHS

$$\begin{aligned}\sqrt[3]{(512/\dots)} &= \sqrt[3]{(2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2)} \\ &= \sqrt[3]{(2^3 \times 2^3 \times 2^3)} \\ &= 2 \times 2 \times 2 \\ &= 8\end{aligned}$$

So, $8/\sqrt[3]{\dots} = 8/13$

when numerators are same the denominators will also become equal.

$$8 \times 13 = 8 \times \sqrt[3]{\dots}$$

$$\sqrt[3]{\dots} = 13$$

$$\begin{aligned}\dots &= (13)^3 \\ &= 2197\end{aligned}$$

10. The volume of a cubical box is 474. 552 cubic metres. Find the length of each side of the box.

Solution:

Volume of a cubical box is 474.552 cubic metres

$$V = 8^3,$$

Let 'S' be the side of the cube

$$8^3 = 474.552 \text{ cubic metres}$$

$$\begin{aligned}8 &= \sqrt[3]{474.552} \\ &= \sqrt[3]{(474552/1000)}\end{aligned}$$

Let us factorise 474552 into prime factors, we get:

$$474552 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 13 \times 13 \times 13$$

By grouping the factors in triples of equal factors, we get:

$$474552 = (2 \times 2 \times 2) \times (3 \times 3 \times 3) \times (13 \times 13 \times 13)$$

$$\begin{aligned}\text{Now, } \sqrt[3]{474.552} &= \sqrt[3]{((2 \times 2 \times 2) \times (3 \times 3 \times 3) \times (13 \times 13 \times 13))} \\ &= 2 \times 3 \times 13 \\ &= 78\end{aligned}$$

Also,

$$\begin{aligned}\sqrt[3]{1000} &= \sqrt[3]{(10 \times 10 \times 10)} \\ &= \sqrt[3]{(10)^3} \\ &= 10\end{aligned}$$

So now let us equate in the above equation we get,

$$\begin{aligned}8 &= \sqrt[3]{(474552/1000)} \\ &= 78/10 \\ &= 7.8\end{aligned}$$

∴ length of the side is 7.8m.

11. Three numbers are to one another 2:3:4. The sum of their cubes is 0.334125. Find the numbers.

Solution:

Let us consider the ratio 2:3:4 be 2a, 3a, and 4a.

So according to the question:

$$(2a)^3 + (3a)^3 + (4a)^3 = 0.334125$$

$$8a^3 + 27a^3 + 64a^3 = 0.334125$$

$$99a^3 = 0.334125$$

$$a^3 = 334125/1000000 \times 99$$

$$= 3375/1000000$$

$$a = \sqrt[3]{(3375/1000000)}$$

$$= \sqrt[3]{((15 \times 15 \times 15) / 100 \times 100 \times 100)}$$

$$= 15/100$$

$$= 0.15$$

∴ The numbers are:

$$2 \times 0.15 = 0.30$$

$$3 \times 0.15 = 0.45$$

$$4 \times 0.15 = 0.6$$

12. Find the side of a cube whose volume is 24389/216m³.

Solution:

Volume of the side s = 24389/216 = v

$$V = s^3$$

$$s = \sqrt[3]{v}$$

$$= \sqrt[3]{(24389/216)}$$

By performing factorisation we get,

$$= \sqrt[3]{(29 \times 29 \times 29 / 2 \times 2 \times 2 \times 3 \times 3 \times 3)}$$

$$= 29 / (2 \times 3)$$

$$= 29/6$$

∴ The length of the side is 29/6.

13. Evaluate:

(i) $\sqrt[3]{36} \times \sqrt[3]{384}$

(ii) $\sqrt[3]{96} \times \sqrt[3]{144}$

(iii) $\sqrt[3]{100} \times \sqrt[3]{270}$

(iv) $\sqrt[3]{121} \times \sqrt[3]{297}$

Solution:

(i) $\sqrt[3]{36} \times \sqrt[3]{384}$

We know that $\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{(a \times b)}$

By using the above formula let us simplify

$$\sqrt[3]{36} \times \sqrt[3]{384} = \sqrt[3]{(36 \times 384)}$$

The prime factors of 36 and 384 are

$$\begin{aligned} &= \sqrt[3]{(2 \times 2 \times 3 \times 3) \times (2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3)} \\ &= \sqrt[3]{(2^3 \times 2^3 \times 2^3 \times 3^3)} \\ &= 2 \times 2 \times 2 \times 3 \\ &= 24 \end{aligned}$$

(ii) $\sqrt[3]{96} \times \sqrt[3]{144}$

We know that $\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{(a \times b)}$

By using the above formula let us simplify

$$\sqrt[3]{96} \times \sqrt[3]{144} = \sqrt[3]{(96 \times 144)}$$

The prime factors of 96 and 144 are

$$\begin{aligned} &= \sqrt[3]{(2 \times 2 \times 2 \times 2 \times 2 \times 3) \times (2 \times 2 \times 2 \times 2 \times 3 \times 3)} \\ &= \sqrt[3]{(2^3 \times 2^3 \times 2^3 \times 3^3)} \\ &= 2 \times 2 \times 2 \times 3 \\ &= 24 \end{aligned}$$

(iii) $\sqrt[3]{100} \times \sqrt[3]{270}$

We know that $\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{(a \times b)}$

By using the above formula let us simplify

$$\sqrt[3]{100} \times \sqrt[3]{270} = \sqrt[3]{(100 \times 270)}$$

The prime factors of 100 and 270 are

$$\begin{aligned} &= \sqrt[3]{(2 \times 2 \times 5 \times 5) \times (2 \times 3 \times 3 \times 3 \times 5)} \\ &= \sqrt[3]{(2^3 \times 3^3 \times 5^3)} \\ &= 2 \times 3 \times 5 \\ &= 30 \end{aligned}$$

(iv) $\sqrt[3]{121} \times \sqrt[3]{297}$

We know that $\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{(a \times b)}$

By using the above formula let us simplify

$$\sqrt[3]{121} \times \sqrt[3]{297} = \sqrt[3]{(121 \times 297)}$$

The prime factors of 121 and 297 are

$$= \sqrt[3]{(11 \times 11) \times (3 \times 3 \times 3 \times 11)}$$

$$\begin{aligned} &= \sqrt[3]{(11^3 \times 3^3)} \\ &= 11 \times 3 \\ &= 33 \end{aligned}$$

14. Find the cube roots of the numbers 3048625, 20346417, 210644875, 57066625 using the fact that

(i) $2460375 = 3375 \times 729$

(ii) $20346417 = 9261 \times 2197$

(iii) $210644875 = 42875 \times 4913$

(iv) $57066625 = 166375 \times 343$

Solution:

(i) $2460375 = 3375 \times 729$

By taking the cube root for the whole we get,

$$\sqrt[3]{3048625} = \sqrt[3]{3375} \times \sqrt[3]{729}$$

Now perform factorization

$$\begin{aligned} &= \sqrt[3]{3 \times 3 \times 3 \times 5 \times 5 \times 5} \times \sqrt[3]{9 \times 9 \times 9} \\ &= \sqrt[3]{3^3 \times 5^3} \times \sqrt[3]{9^3} \\ &= 3 \times 5 \times 9 \\ &= 135 \end{aligned}$$

(ii) $20346417 = 9261 \times 2197$

By taking the cube root for the whole we get,

$$\sqrt[3]{20346417} = \sqrt[3]{9261} \times \sqrt[3]{2197}$$

Now perform factorization

$$\begin{aligned} &= \sqrt[3]{3 \times 3 \times 3 \times 7 \times 7 \times 7} \times \sqrt[3]{13 \times 13 \times 13} \\ &= \sqrt[3]{3^3 \times 7^3} \times \sqrt[3]{13^3} \\ &= 3 \times 7 \times 13 \\ &= 273 \end{aligned}$$

(iii) $210644875 = 42875 \times 4913$

By taking the cube root for the whole we get,

$$\sqrt[3]{210644875} = \sqrt[3]{42875} \times \sqrt[3]{4913}$$

Now perform factorization

$$\begin{aligned} &= \sqrt[3]{5 \times 5 \times 5 \times 7 \times 7 \times 7} \times \sqrt[3]{17 \times 17 \times 17} \\ &= \sqrt[3]{5^3 \times 7^3} \times \sqrt[3]{17^3} \\ &= 5 \times 7 \times 17 \\ &= 595 \end{aligned}$$

(iv) $57066625 = 166375 \times 343$

By taking the cube root for the whole we get,

$$\sqrt[3]{57066625} = \sqrt[3]{166375} \times \sqrt[3]{343}$$

Now perform factorization

$$\begin{aligned} &= \sqrt[3]{5 \times 5 \times 5 \times 11 \times 11 \times 11} \times \sqrt[3]{7 \times 7 \times 7} \\ &= \sqrt[3]{5^3 \times 11^3} \times \sqrt[3]{7^3} \\ &= 5 \times 11 \times 7 \\ &= 385 \end{aligned}$$

15. Find the unit of the cube root of the following numbers:

(i) **226981**

(ii) **13824**

(iii) **571787**

(iv) **175616**

Solution:

(i) 226981

The given number is 226981.

Unit digit of 226981 = 1

The unit digit of the cube root of 226981 = 1

(ii) 13824

The given number is 13824.

Unit digit of 13824 = 4

The unit digit of the cube root of 13824 = 4

(iii) 571787

The given number is 571787.

Unit digit of 571787 = 7

The unit digit of the cube root of 571787 = 7

(iv) 175616

The given number is 175616.

Unit digit of 175616 = 6

The unit digit of the cube root of 175616 = 6

16. Find the tens digit of the cube root of each of the numbers in Q.No.15.

(i) **226981**

(ii) **13824**

(iii) **571787**

(iv) **175616**

Solution:**(i)** 226981

The given number is 226981.

Unit digit of 226981 = 1

The unit digit in the cube root of 226981 = 1

After striking out the units, tens and hundreds digits of 226981, now we left with 226 only.

We know that 6 is the Largest number whose cube root is less than or equal to 226($6^3 < 226 < 7^3$). \therefore The tens digit of the cube root of 226981 is 6.**(ii)** 13824

The given number is 13824.

Unit digit of 13824 = 4

The unit digit in the cube root of 13824 = 4

After striking out the units, tens and hundreds digits of 13824, now we left with 13 only.

We know that 2 is the Largest number whose cube root is less than or equal to 13($2^3 < 13 < 3^3$). \therefore The tens digit of the cube root of 13824 is 2.**(iii)** 571787

The given number is 571787.

Unit digit of 571787 = 7

The unit digit in the cube root of 571787 = 3

After striking out the units, tens and hundreds digits of 571787, now we left with 571 only.

We know that 8 is the Largest number whose cube root is less than or equals to 571($8^3 < 571 < 9^3$). \therefore The tens digit of the cube root of 571787 is 8.**(iv)** 175616

The given number is 175616.

Unit digit of 175616 = 6

The unit digit in the cube root of 175616 = 6

After striking out the units, tens and hundreds digits of 175616, now we left with 175 only.

We know that 5 is the Largest number whose cube root is less than or equals to 175($5^3 < 175 < 6^3$). \therefore The tens digit of the cube root of 175616 is 5.