EXERCISE 7.1 PAGE NO: 7.3

Find the greatest common factor (GCF/HCF) of the following polynomials: (1-14)

1. $2x^2$ and $12x^2$

Solution:

We know that the numerical coefficients of given numerical are 2 and 12
The greatest common factor of 2 and 12 is 2
The common literals appearing in given monomial is x
The smallest power of x in two monomials is 2
The monomial of common literals with smallest power is x^2
∴ The greatest common factor = $2x^2$

2. $6x^3y$ and $18x^2y^3$

Solution:

We know that the numerical coefficients of given numerical are 6 and18
The greatest common factor of 6 and 18 is 6
Common literals appearing in given numerical are x and y
Smallest power of x in three monomial is 2
Smallest power of y in three monomial is 1
Monomial of common literals with smallest power is x^2y
∴ The greatest common factor = $6x^2y$

3. $7x$, $21x^2$ and $14xy^2$

Solution:

We know that the numerical coefficients of given numerical are 7, 21 and 14
Greatest common factor of 7, 21 and 14 is 7
Common literals appearing in given numerical are x and y
Smallest power of x in three monomials is 1
Smallest power of y in three monomials is 0
Monomials of common literals with smallest power is x
∴ The greatest common factor = $7x$

4. $42x^2yz$ and $63x^3y^2z^3$

Solution:

We know that the numerical coefficients of given numerical are 42 and 63.
Greatest common factor of 42, 63 is 21.
Common literals appearing in given numerical are x, y and z
Smallest power of x in two monomials is 2

Smallest power of y in two monomials is 1
Smallest power of z in two monomials is 1
Monomials of common literals with smallest power is x^2yz
∴ The greatest common factor = $21x^2yz$

5. $12ax^2$, $6a^2x^3$ and $2a^3x^5$
Solution:
We know that the numerical coefficients of given numerical are 12, 6 and 2
Greatest common factor of 12, 6 and 2 is 2.
Common literals appearing in given numerical are a and x
Smallest power of x in three monomials is 2
Smallest power of a in three monomials is 1
Monomials of common literals with smallest power is ax^2
∴ The greatest common factor = $2ax^2$

6. $9x^2$, $15x^2y^3$, $6xy^2$ and $21x^2y^2$
Solution:
We know that the numerical coefficients of given numerical are 9, 15, 16 and 21
Greatest common factor of 9, 15, 16 and 21 is 3.
Common literals appearing in given numerical are x and y
Smallest power of x in four monomials is 1
Smallest power of y in four monomials is 0
Monomials of common literals with smallest power is x
∴ The greatest common factor = 3x

7. $4a^2b^3$, $-12a^3b$, $18a^4b^3$
Solution:
We know that the numerical coefficients of given numerical are 4, -12 and 18.
Greatest common factor of 4, -12 and 18 is 2.
Common literals appearing in given numerical are a and b
Smallest power of a in three monomials is 2
Smallest power of b in three monomials is 1
Monomials of common literals with smallest power is a^2b
∴ The greatest common factor = $2a^2b$

8. $6x^2y^2$, $9xy^3$, $3x^3y^2$
Solution:
We know that the numerical coefficients of given numerical are 6, 9 and 3
Greatest common factor of 6, 9 and 3 is 3.

Common literals appearing in given numerical are x and y
Smallest power of x in three monomials is 1
Smallest power of y in three monomials is 2
Monomials of common literals with smallest power is xy^2
∴ The greatest common factor = $3xy^2$

9. a^2b^3, a^3b^2
Solution:
We know that the numerical coefficients of given numerical are 0
Common literals appearing in given numerical are a and b
Smallest power of a in two monomials = 2
Smallest power of b in two monomials = 2
Monomials of common literals with smallest power is a^2b^2
∴ The greatest common factor = a^2b^2

10. $36a^2b^2c^4$, $54a^5c^2$, $90a^4b^2c^2$
Solution:
We know that the numerical coefficients of given numerical are 36, 54 and 90
Greatest common factor of 36, 54 and 90 is 18.
Common literals appearing in given numerical are a, b and c
Smallest power of a in three monomials is 2
Smallest power of b in three monomials is 0
Smallest power of c in three monomials is 2
Monomials of common literals with smallest power is a^2c^2
∴ The greatest common factor = $18a^2c^2$

11. x^3, $-yx^2$
Solution:
We know that the numerical coefficients of given numerical are 0
Common literals appearing in given numerical are x and y
Smallest power of x in two monomials is 2
Smallest power of y in two monomials is 0
Monomials of common literals with smallest power is x^2
∴ The greatest common factor = x^2

12. $15a^3$, $-45a^2$, $-150a$
Solution:
We know that the numerical coefficients of given numerical are 15, -45 and 150
Greatest common factor of 15, -45 and 150 is 15.

Common literals appearing in given numerical is a
Smallest power of a in three monomials is 1
Monomials of common literals with smallest power is a
∴ The greatest common factor = 15a

13. $2x^3y^2$, $10x^2y^3$, 14xy
Solution:
We know that the numerical coefficients of given numerical are 2, 10 and 14.
Greatest common factor of 2, 10 and 14 is 2.
Common literals appearing in given numerical are x and y
Smallest power of x in three monomials is 1
Smallest power of y in three monomials is 1
Monomials of common literals with smallest power is xy
∴ The greatest common factor = 2xy

14. $14x^3y^5$, $10x^5y^3$, $2x^2y^2$
Solution:
We know that the numerical coefficients of given numerical are 14, 10 and 2.
Greatest common factor of 14, 10 and 2 is 2.
Common literals appearing in given numerical are x and y
Smallest power of x in three monomials is 2
Smallest power of y in three monomials is 2
Monomials of common literals with smallest power is x^2y^2
∴ The greatest common factor = $2x^2y^2$

Find the greatest common factor of the terms in each of the following expressions:
15. $5a^4 + 10a^3 - 15a^2$
Solution:
The greatest common factor of the three terms is $5a^2$

16. $2xyz + 3x^2y + 4y^2$
Solution:
The greatest common factor of the three terms is y

17. $3a^2b^2 + 4b^2c^2 + 12a^2b^2c^2$
Solution:
The greatest common factor of the three terms is b^2.

EXERCISE 7.2 PAGE NO: 7.5

Factorize the following:

1. 3x – 9

Solution:

The greatest common factor in the given two terms is 3

3x – 9

3 (x - 3)

2. $5x - 15x^2$

Solution:

The greatest common factor in the given two terms is 5x

$5x – 15x^2$

5x (1 – 3x)

3. $20a^{12}b^2 – 15a^8b^4$

Solution:

Greatest common factor in the given two terms is $5a^8b^2$

$20a^{12}b^2 – 15a^8b^4$

$5a^8b^2 (4a^4 - 3b^2)$

4. $72x^6y^7 - 96x^7y^6$

Solution:

Greatest common factor in the given two terms is $24x^6y^6$

$72x^6y^7 - 96x^7y^6$

$24x^6y^6 (3y – 4x)$

5. $20x^3 - 40x^2 + 80x$

Solution:

Greatest common factor in the given three terms is 20x

$20x^3 - 40x^2 + 80x$

$20x (x^2 - 2x +4)$

6. $2x^3y^2 - 4x^2y^3 + 8xy^4$

Solution:

Greatest common factor in the given three terms is $2xy^2$

$2x^3y^2 - 4x^2y^3 + 8xy^4$

$2xy^2 (x^2 - 2xy + 4y^2)$

7. $10m^3n^2 + 15m^4n - 20m^2n^3$

Solution:

Greatest common factor in the given three terms is $5mn^2$

$10m^3n^2 + 15m^4n - 20m^2n^3$

$5m^2n\ (2mn + 3m^2 - 4n^2)$

8. $2a^4b^4 - 3a^3b^5 + 4a^2b^5$

Solution:

Greatest common factor in the given three terms is a^2b^4

$2a^4b^4 - 3a^3b^5 + 4a^2b^5$

$a^2b^4\ (2a^2 - 3ab + 4b)$

9. $28a^2 + 14a^2b^2 - 21a^4$

Solution:

Greatest common factor in the given three terms is $7a^2$

$28a^2 + 14a^2b^2 - 21a^4$

$7a^2\ (4a + 2b^2 - 3a^2)$

10. $a^4b - 3a^2b^2 - 6ab^3$

Solution:

Greatest common factor in the given three terms is ab

$a^4b - 3a^2b^2 - 6ab^3$

$ab\ (a^3 - 3ab - 6b^2)$

11. $2l^2mn - 3lm^2n + 4lmn^2$

Solution:

Greatest common factor in the given three terms is lmn

$2l^2mn - 3lm^2n + 4lmn^2$

$lmn\ (2l - 3m + 4n)$

12. $x^4y^2 - x^2y^4 - x^4y^4$

Solution:

Greatest common factor in the given three terms is x^2y^2

$x^4y^2 - x^2y^4 - x^4y^4$

$x^2y^2\ (x^2 - y^2 - x^2y^2)$

13. $9x^2y + 3axy$

Solution:

Greatest common factor in the given three terms is 3xy

$9x^2y + 3axy$
$3xy (3x + a)$

14. $16m - 4m^2$
Solution:
Greatest common factor in the given two terms is 4m
$16m - 4m^2$
$4m (4 - m)$

15. $-4a^2 + 4ab - 4ca$
Solution:
Greatest common factor in the given three terms is - 4a
$-4a^2 + 4ab - 4ca$
$-4a (a - b + c)$

16. $x^2yz + xy^2z + xyz^2$
Solution:
Greatest common factor in the given three terms is xyz
$x^2yz + xy^2z + xyz^2$
$xyz (x + y + z)$

17. $ax^2y + bxy^2 + cxyz$
Solution:
Greatest common factor in the given three terms is xy
$ax^2y + bxy^2 + cxyz$
$xy (ax + by + cz)$

EXERCISE 7.3 PAGE NO: 7.7

Factorize each of the following algebraic expressions:

1. 6x (2x - y) + 7y (2x - y)

Solution:

We have,

6x (2x - y) + 7y (2x - y)

By taking (2x – y) as common we get,

(6x + 7y) (2x – y)

2. 2r (y - x) + s (x - y)

Solution:

We have,

2r (y - x) + s (x - y)

By taking (-1) as common we get,

-2r (x – y) + s (x – y)

By taking (x - y) as common we get,

(x – y) (-2r + s)

(x – y) (s – 2r)

3. 7a (2x - 3) + 3b (2x - 3)

Solution:

We have,

7a (2x - 3) + 3b (2x - 3)

By taking (2x - 3) as common we get,

(7a + 3b) (2x – 3)

4. 9a (6a – 5b) – 12a^2 (6a – 5b)

Solution:

We have,

9a (6a – 5b) – 12a^2 (6a – 5b)

By taking (6a – 5b) as common we get,

(9a – 12a^2) (6a – 5b)

3a(3 – 4a) (6a – 5b)

5. 5 (x – 2y)^2 + 3 (x – 2y)

Solution:

We have,

5 (x – 2y)^2 + 3 (x – 2y)

By taking (x – 2y) as common we get,
(x – 2y) [5 (x – 2y) + 3]
(x – 2y) (5x – 10y + 3)

6. $16 (2l – 3m)^2 - 12 (3m – 2l)$
Solution:
We have,
$16 (2l – 3m)^2 - 12 (3m – 2l)$
By taking (-1) as common we get,
$16 (2l – 3m)^2 + 12 (2l – 3m)$
By taking 4(2l – 3m) as common we get,
4(2l – 3m) [4 (2l – 3m) + 3]
4(2l – 3m) (8l – 12m + 3)

7. 3a (x – 2y) – b (x – 2y)
Solution:
We have,
3a (x – 2y) – b (x – 2y)
By taking (x – 2y) as common we get,
(3a – b) (x – 2y)

8. $a^2 (x + y) + b^2 (x + y) + c^2 (x + y)$
Solution:
We have,
$a^2 (x + y) + b^2 (x + y) + c^2 (x + y)$
By taking (x + y) as common we get,
$(a^2 + b^2 + c^2) (x + y)$

9. $(x - y)^2 + (x - y)$
Solution:
We have,
$(x - y)^2 + (x - y)$
By taking (x - y) as common we get,
(x – y) (x – y + 1)

10. $6 (a + 2b) – 4 (a + 2b)^2$
Solution:
We have,
$6 (a + 2b) – 4 (a + 2b)^2$

By taking (a + 2b) as common we get,
[6 – 4 (a + 2b)] (a + 2b)
(6 – 4a – 8b) (a + 2b)
2(3 – 2a – 4b) (a + 2b)

11. a (x - y) + 2b (y - x) + c $(x – y)^2$
Solution:
We have,
a (x - y) + 2b (y - x) + c $(x – y)^2$
By taking (-1) as common we get,
a (x - y) - 2b (x - y) + c $(x – y)^2$
By taking (x - y) as common we get,
[a – 2b + c(x - y)] (x - y)
(x - y) (a – 2b + cx - cy)

12. -4 $(x – 2y)^2$ + 8 (x – 2y)
Solution:
We have,
-4 $(x – 2y)^2$ + 8 (x – 2y)
By taking 4(x - 2y) as common we get,
[-(x – 2y) + 2] 4(x - 2y)
4(x - 2y) (-x + 2y + 2)

13. x^3 (a – 2b) + x^2 (a – 2b)
Solution:
We have,
x^3 (a – 2b) + x^2 (a – 2b)
By taking x^2 (a – 2b) as common we get,
(x + 1) [x^2 (a – 2b)]
x^2 (a – 2b) (x + 1)

14. (2x – 3y) (a + b) + (3x – 2y) (a + b)
Solution:
We have,
(2x – 3y) (a + b) + (3x – 2y) (a + b)
By taking (a + b) as common we get,
(a + b) [(2x – 3y) + (3x – 2y)]
(a + b) [2x -3y + 3x – 2y]
(a + b) [5x – 5y]

(a + b) 5(x - y)

15. 4(x + y) (3a - b) + 6(x + y) (2b – 3a)
Solution:
We have,
4(x + y) (3a - b) + 6(x + y) (2b – 3a)
By taking (x + y) as common we get,
(x + y) [4(3a - b) + 6(2b – 3a)]
(x + y) [12a – 4b + 12b – 18a]
(x + y) [-6a + 8b]
(x + y) 2(-3a + 4b)
(x + y) 2(4b – 3a)

EXERCISE 7.4 PAGE NO: 7.12

Factorize each of the following expressions:

1. qr – pr + qs - ps

Solution:

We have,

qr – pr + qs - ps

By grouping similar terms we get,

qr + qs – pr – ps

q(r + s) –p (r + s)

(q - p) (r + s)

2. $p^2q – pr^2 – pq + r^2$

Solution:

We have,

$p^2q – pr^2 – pq + r^2$

By grouping similar terms we get,

$p^2q – pq – pr^2 + r^2$

$pq(p - 1) –r^2(p - 1)$

$(p - 1) (pq – r^2)$

3. $1 + x + xy + x^2y$

Solution:

We have,

$1 + x + xy + x^2y$

$1 (1 + x) + xy(1 + x)$

$(1 + x) (1 + xy)$

4. ax + ay – bx – by

Solution:

We have,

ax + ay – bx – by

a(x + y) –b (x + y)

(a - b) (x + y)

5. $xa^2 + xb^2 – ya^2 – yb^2$

Solution:

We have,

$xa^2 + xb^2 – ya^2 – yb^2$

$x(a^2 + b^2) – y (a^2 + b^2)$
$(x - y) (a^2 + b^2)$

6. $x^2 + xy + xz + yz$
Solution:
We have,
$x^2 + xy + xz + yz$
$x (x + y) + z (x + y)$
$(x + y) (x + z)$

7. $2ax + bx + 2ay + by$
Solution:
We have,
$2ax + bx + 2ay + by$
By grouping similar terms we get,
$2ax + 2ay + bx + by$
$2a (x + y) + b (x + y)$
$(2a + b) (x + y)$

8. $ab – by – ay + y^2$
Solution:
We have,
$ab – by – ay + y^2$
By grouping similar terms we get,
$Ab – ay – by + y^2$
$a (b – y) – y (b – y)$
$(a – y) (b – y)$

9. $axy + bcxy – az – bcz$
Solution:
We have,
$axy + bcxy – az – bcz$
By grouping similar terms we get,
$axy – az + bcxy – bcz$
$a (xy – z) + bc (xy – z)$
$(a + bc) (xy – z)$

10. $lm^2 – mn^2 – lm + n^2$
Solution:

We have,
$lm^2 – mn^2 – lm + n^2$
By grouping similar terms we get,
$lm^2 – lm – mn^2 + n^2$
$lm (m – 1) – n^2 (m – 1)$
$(lm – n^2) (m – 1)$

11. $x^3 – y^2 + x – x^2y^2$
Solution:
We have,
$x^3 – y^2 + x – x^2y^2$
By grouping similar terms we get,
$x + x^3 – y^2 – x^2y^2$
$x (1 + x^2) - y^2 (1 + x^2)$
$(x – y^2) (1 + x^2)$

12. $6xy + 6 – 9y – 4x$
Solution:
We have,
$6xy + 6 – 9y – 4x$
By grouping similar terms we get,
$6xy – 4x – 9y + 6$
$2x (3y – 2) – 3 (3y – 2)$
$(2x – 3) (3y – 2)$

13. $x^2 – 2ax – 2ab + bx$
Solution:
We have,
$x^2 – 2ax – 2ab + bx$
By grouping similar terms we get,
$x^2 + bx – 2ax – 2ab$
$x (x + b) – 2a (x + b)$
$(x – 2a) (x + b)$

14. $x^3 – 2x^2y + 3xy^2 – 6y^3$
Solution:
We have,
$x^3 – 2x^2y + 3xy^2 – 6y^3$
By grouping similar terms we get,

$x^3 + 3xy^2 – 2x^2y – 6y^3$
$x (x^2 + 3y^2) – 2y (x^2 + 3y^2)$
$(x – 2y) (x^2 + 3y^2)$

15. $abx^2 + (ay - b) x – y$
Solution:
We have,
$abx^2 + (ay - b) x – y$
$abx^2 + ayx – bx – y$
By grouping similar terms we get,
$abx^2 – bx + ayx – y$
$bx (ax – 1) + y (ax – 1)$
$(bx + y) (ax – 1)$

16. $(ax + by)^2 + (bx - ay)^2$
Solution:
We have,
$(ax + by)^2 + (bx - ay)^2$
$a^2x^2 + b^2y^2 + 2axby + b^2x^2 + a^2y^2 – 2axby$
$a^2x^2 + b^2y^2 + b^2x^2 + a^2y^2$
By grouping similar terms we get,
$a^2x^2 + a^2y^2 + b^2y^2 + b^2x^2$
$a^2 (x^2 + y^2) + b^2 (x^2 + y^2)$
$(a^2 + b^2) (x^2 + y^2)$

17. $16 (a - b)^3 – 24 (a - b)^2$
Solution:
We have,
$16(a - b)^3 – 24(a - b)^2$
$8 (a – b)^2 [2 (a – b) – 3]$
$8 (a – b)^2 (2a – 2b – 3)$

18. $ab (x^2 + 1) + x (a^2 + b^2)$
Solution:
We have,
$ab(x^2 + 1) + x(a^2 + b^2)$
$abx^2 + ab + xa^2 + xb^2$
By grouping similar terms we get,
$abx^2 + xa^2 + xb^2 + ab$

ax (bx + a) + b (bx + a)
(ax + b) (bx + a)

19. $a^2x^2 + (ax^2 + 1)x + a$
Solution:
We have,
$a^2x^2 + (ax^2 + 1)x + a$
$a^2x^2 + ax^3 + x + a$
$ax^2 (a + x) + 1 (x + a)$
$(x + a) (ax^2 + 1)$

20. a (a – 2b – c) + 2bc
Solution:
We have,
a (a – 2b – c) + 2bc
a^2 – 2ab – ac + 2bc
a (a – 2b) – c (a – 2b)
(a – 2b) (a – c)

21. a (a + b – c) – bc
Solution:
We have,
a (a + b – c) – bc
a^2 + ab – ac – bc
a (a + b) – c (a + b)
(a + b) (a – c)

22. x^2 – 11xy – x + 11y
Solution:
We have,
x^2 – 11xy – x + 11y
By grouping similar terms we get,
x^2 – x – 11xy + 11y
x (x – 1) – 11y (x – 1)
(x – 11y) (x – 1)

23. ab – a – b + 1
Solution:
We have,

ab – a – b + 1
a (b – 1) – 1 (b – 1)
(a – 1) (b – 1)

24. $x^2 + y – xy – x$
Solution:
We have,
$x^2 + y – xy – x$
By grouping similar terms we get,
$x^2 – x + y – xy$
x (x – 1) – y (x – 1)
(x – y) (x – 1)

EXERCISE 7.5 PAGE NO: 7.17

Factorize each of the following expressions:

1. $16x^2 – 25y^2$

Solution:

We have,

$16x^2 – 25y^2$

$(4x)^2 – (5y)^2$

By using the formula $(a^2 – b^2) = (a + b) (a - b)$ we get,

$(4x + 5y) (4x – 5y)$

2. $27x^2 – 12y^2$

Solution:

We have,

$27x^2 – 12y^2$

By taking 3 as common we get,

$3 [(3x)^2 – (2y)^2]$

By using the formula $(a^2 - b^2) = (a-b) (a+b)$

$3 (3x + 2y) (3x – 2y)$

3. $144a^2 – 289b^2$

Solution:

We have,

$144a^2 – 289b^2$

$(12a)^2 – (17b)^2$

By using the formula $(a^2 - b^2) = (a-b) (a+b)$

$(12a + 17b) (12a – 17b)$

4. $12m^2 – 27$

Solution:

We have,

$12m^2 – 27$

By taking 3 as common we get,

$3 (4m^2 – 9)$

$3 [(2m)^2 – 3^2]$

By using the formula $(a^2 - b^2) = (a-b) (a+b)$

$3 (2m + 3) (2m – 3)$

5. $125x^2 – 45y^2$

Solution:
We have,
$125x^2 – 45y^2$
By taking 5 as common we get,
$5 (25x^2 – 9y^2)$
$5 [(5x)^2 – (3y)^2]$
By using the formula $(a^2 - b^2) = (a-b) (a+b)$
$5 (5x + 3y) (5x – 3y)$

6. $144a^2 – 169b^2$
Solution:
We have,
$144a^2 – 169b^2$
$(12a)^2 – (13b)^2$
By using the formula $(a^2 - b^2) = (a-b) (a+b)$
$(12a + 13b) (12a – 13b)$

7. $(2a - b)^2 – 16c^2$
Solution:
We have,
$(2a - b)^2 – 16c^2$
$(2a – b)^2 – (4c)^2$
By using the formula $(a^2 - b^2) = (a-b) (a+b)$
$(2a – b + 4c) (2a – b – 4c)$

8. $(x + 2y)^2 – 4 (2x - y)^2$
Solution:
We have,
$(x + 2y)^2 – 4 (2x - y)^2$
$(x + 2y)^2 – [2 (2x – y)]^2$
By using the formula $(a^2 – b^2)= (a + b) (a - b)$ we get,
$[(x + 2y) + 2 (2x – y)] [x + 2y – 2 (2x – y)]$
$(x + 4x + 2y – 2y) (x – 4x + 2y + 2y)$
$(5x) (4y – 3x)$

9. $3a^5 – 48a^3$
Solution:
We have,
$3a^5 – 48a^3$

By taking 3 as common we get,
$3a^3 (a^2 – 16)$
$3a^3 (a^2 – 4^2)$
By using the formula $(a^2 - b^2) = (a-b) (a+b)$
$3a^3 (a + 4) (a – 4)$

10. $a^4 – 16b^4$
Solution:
We have,
$a^4 – 16b^4$
$(a^2)^2 – (4b^2)^2$
By using the formula $(a^2 - b^2) = (a-b) (a+b)$
$(a^2 + 4b^2) (a^2 – 4b^2)$
By using the formula $(a^2 - b^2) = (a-b) (a+b)$
$(a^2 + 4b^2) (a + 2b) (a – 2b)$

11. $x^8 – 1$
Solution:
We have,
$x^8 – 1$
$(x^4)^2–(1)^2$
By using the formula $(a^2 - b^2) = (a-b) (a+b)$
$(x^4 + 1) (x^4 – 1)$
By using the formula $(a^2 - b^2) = (a-b) (a+b)$
$(x^4 + 1) (x^2 + 1) (x – 1) (x + 1)$

12. $64 – (a + 1)^2$
Solution:
We have,
$64 – (a + 1)^2$
$8^2 – (a + 1)^2$
By using the formula $(a^2 - b^2) = (a-b) (a+b)$
$[8 + (a + 1)] [8 – (a + 1)]$
$(a + 9) (7 – a)$

13. $36l^2 – (m + n)^2$
Solution:
We have,
$36l^2 – (m + n)^2$

$(6l)^2 – (m + n)^2$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(6l + m + n)(6l – m – n)$

14. $25x^4y^4 – 1$
Solution:
We have,
$25x^4y^4 – 1$
$(5x^2y^2)^2 – (1)^2$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(5x^2y^2 – 1)(5x^2y^2 + 1)$

15. $a^4 – 1/b^4$
Solution:
We have,
$a^4 – 1/b^4$
$(a^2)^2 – (1/b^2)^2$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(a^2 + 1/b^2)(a^2 – 1/b^2)$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(a^2 + 1/b^2)(a – 1/b)(a + 1/b)$

16. $x^3 – 144x$
Solution:
We have,
$x^3 – 144x$
$x [x^2 – (12)^2]$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$x (x + 12)(x – 12)$

17. $(x – 4y)^2 – 625$
Solution:
We have,
$(x – 4y)^2 – 625$
$(x – 4y)^2 – (25)^2$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(x – 4y + 25)(x – 4y – 25)$

18. $9 (a – b)^2 – 100 (x – y)^2$

Solution:
We have,
$9 (a – b)^2 – 100 (x – y)^2$
$[3 (a – b)]^2 – [10 (x – y)]^2$
By using the formula $(a^2 - b^2) = (a-b) (a+b)$
$[3 (a – b) + 10 (x + y)] [3 (a – b) – 10 (x – y)]$
$[3a – 3b + 10x – 10y] [3a – 3b – 10x + 10y]$

19. $(3 + 2a)^2 – 25a^2$
Solution:
We have,
$(3 + 2a)^2 – 25a^2$
$(3 + 2a)^2 – (5a)^2$
By using the formula $(a^2 - b^2) = (a-b) (a+b)$
$(3 + 2a + 5a) (3 + 2a – 5a)$
$(3 + 7a) (3 – 3a)$
$(3 + 7a) 3(1 – a)$

20. $(x + y)^2 – (a – b)^2$
Solution:
We have,
$(x + y)^2 – (a – b)^2$
By using the formula $(a^2 - b^2) = (a-b) (a+b)$
$[(x + y) + (a – b)] [(x + y) – (a – b)]$
$(x + y + a – b) (x + y – a + b)$

21. $1/16x^2y^2 – 4/49y^2z^2$
Solution:
We have,
$1/16x^2y^2 – 4/49y^2z^2$
$(1/4xy)^2 – (2/7yz)^2$
By using the formula $(a^2 - b^2) = (a-b) (a+b)$
$(xy/4 + 2yz/7) (xy/4 - 2yz/7)$
$y^2 (x/4 + 2/7z) (x/4 – 2/7z)$

22. $75a^3b^2 – 108ab^4$
Solution:
We have,
$75a^3b^2 – 108ab^4$

$3ab^2 (25a^2 – 36b^2)$
$3ab^2 [(5a)^2 – (6b)^2]$
By using the formula $(a^2 - b^2) = (a-b) (a+b)$
$3ab^2 (5a + 6b) (5a – 6b)$

23. $x^5 – 16x^3$
Solution:
We have,
$x^5 – 16x^3$
$x^3 (x^2 – 16)$
$x^3 (x^2 – 4^2)$
By using the formula $(a^2 - b^2) = (a-b) (a+b)$
$x^3 (x + 4) (x – 4)$

24. $50/x^2 – 2x^2/81$
Solution:
We have,
$50/x^2 – 2x^2/81$
$2 (25/x^2 – x^2/81)$
$2 [(5/x)^2 – (x/9)^2]$
By using the formula $(a^2 - b^2) = (a-b) (a+b)$
$2 (5/x+ x/9) (5/x – x/9)$

25. $256x^3 – 81x$
Solution:
We have,
$256x^3 – 81x$
$x (256x^4 – 81)$
$x [(16x^2)^2 – 9^2]$
By using the formula $(a^2 - b^2) = (a-b) (a+b)$
$x (4x + 3) (4x - 3) (16x^2 + 9)$

26. $a^4 – (2b + c)^4$
Solution:
We have,
$a^4 – (2b + c)^4$
$(a^2)^2 – [(2b + c)^2]^2$
By using the formula $(a^2 - b^2) = (a-b) (a+b)$
$[a^2 + (2b + c)^2] [a^2 – (2b + c)^2]$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$[a^2 + (2b + c)^2] [a + 2b + c] [a – 2b – c]$

27. $(3x + 4y)^4 – x^4$
Solution:
We have,
$(3x + 4y)^4 – x^4$
$[(3x + 4y)^2]^2 – (x^2)^2$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$[(3x + 4y)^2 + x^2] [(3x + 4y)^2 – x^2]$
$[(3x + 4y)^2 + x^2] [3x + 4y + x] [3x + 4y – x]$
$[(3x + 4y)^2 + x^2] [4x + 4y] [2x + 4y]$
$[(3x + 4y)^2 + x^2] 8[x + 2y] [x + y]$

28. $p^2q^2 – p^4q^4$
Solution:
We have,
$p^2q^2 – p^4q^4$
$(pq)^2 – (p^2q^2)^2$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(pq + p^2q^2) (pq – p^2q^2)$
$p^2q^2 (1 + pq) (1 – pq)$

29. $3x^3y – 243xy^3$
Solution:
We have,
$3x^3y – 243xy^3$
$3xy (x^2 – 81y^2)$
$3xy [x^2 – (9y)^2]$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(3xy) (x + 9y) (x – 9y)$

30. $a^4b^4 – 16c^4$
Solution:
We have,
$a^4b^4 – 16c^4$
$(a^2b^2)^2 – (4c^2)^2$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(a^2b^2 + 4c^2) (a^2b^2 – 4c^2)$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(a^2b^2 + 4c^2)(ab + 2c)(ab – 2c)$

31. $x^4 – 625$
Solution:
We have,
$x^4 – 625$
$(x^2)^2 – (25)^2$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(x^2 + 25)(x^2 – 25)$
$(x^2 + 25)(x^2 – 5^2)$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(x^2 + 25)(x + 5)(x – 5)$

32. $x^4 – 1$
Solution:
We have,
$x^4 – 1$
$(x^2)^2 – (1)^2$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(x^2 + 1)(x^2 – 1)$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(x^2 + 1)(x + 1)(x – 1)$

33. $49(a – b)^2 – 25(a + b)^2$
Solution:
We have,
$49(a – b)^2 – 25(a + b)^2$
$[7(a – b)]^2 – [5(a + b)]^2$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$[7(a – b) + 5(a + b)][7(a – b) – 5(a + b)]$
$(7a – 7b + 5a + 5b)(7a – 7b – 5a – 5b)$
$(12a – 2b)(2a – 12b)$
$2(6a – b)\ 2(a – 6b)$
$4(6a – b)(a – 6b)$

34. $x – y - x^2 + y^2$
Solution:
We have,

$x – y - x^2 + y^2$
$x – y – (x^2 – y^2)$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$x – y – (x + y)(x – y)$
$(x – y)(1 – x – y)$

35. $16(2x – 1)^2 – 25y^2$
Solution:
We have,
$16(2x – 1)^2 – 25y^2$
$[4(2x – 1)]^2 – (5y)^2$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(8x + 5y – 4)(8x – 5y – 4)$

36. $4(xy + 1)^2 – 9(x – 1)^2$
Solution:
We have,
$4(xy + 1)^2 – 9(x – 1)^2$
$[2(xy + 1)]^2 – [3(x – 1)]^2$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(2xy + 2 + 3x – 3)(2xy + 2 – 3x + 3)$
$(2xy + 3x – 1)(2xy – 3x + 5)$

37. $(2x + 1)^2 – 9x^4$
Solution:
We have,
$(2x + 1)^2 – 9x^4$
$(2x + 1)^2 – (3x^2)^2$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(2x + 1 + 3x^2)(2x + 1 – 3x^2)$
$(3x^2 + 2x + 1)(-3x^2 + 2x + 1)$

38. $x^4 – (2y – 3z)^2$
Solution:
We have,
$x^4 – (2y – 3z)^2$
$(x^2)^2 – (2y – 3z)^2$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(x^2 + 2y – 3z)(x^2 – 2y + 3z)$

39. $a^2 – b^2 + a – b$

Solution:

We have,

$a^2 – b^2 + a – b$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$(a + b)(a – b) + (a – b)$

$(a – b)(a + b + 1)$

40. $16a^4 – b^4$

Solution:

We have,

$16a^4 – b^4$

$(4a^2)^2 – (b^2)^2$

$(4a^2 + b^2)(4a^2 – b^2)$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$(4a^2 + b^2)(2a + b)(2a – b)$

41. $a^4 – 16(b – c)^4$

Solution:

We have,

$a^4 – 16(b – c)^4$

$(a^2)^2 – [4(b – c)^2]$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$[a^2 + 4(b – c)^2][a^2 – 4(b – c)^2]$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$[a^2 + 4(b – c)^2][(a + 2b – 2c)(a – 2b + 2c)]$

42. $2a^5 – 32a$

Solution:

We have,

$2a^5 – 32a$

$2a(a^4 – 16)$

$2a[(a^2)^2 – (4)^2]$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$2a(a^2 + 4)(a^2 – 4)$

$2a(a^2 + 4)(a^2 – 2^2)$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$2a(a^2 + 4)(a + 2)(a – 2)$

43. $a^4b^4 – 81c^4$

Solution:

We have,

$a^4b^4 – 81c^4$

$(a^2b^2)^2 – (9c^2)^2$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$(a^2b^2 + 9c^2)(a^2b^2 – 9c^2)$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$(a^2b^2 + 9c^2)(ab + 3c)(ab – 3c)$

44. $xy^9 – yx^9$

Solution:

We have,

$xy^9 – yx^9$

$-xy(x^8 – y^8)$

$-xy[(x^4)^2 – (y^4)^2]$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$-xy(x^4 + y^4)(x^4 – y^4)$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$-xy(x^4 + y^4)(x^2 + y^2)(x^2 – y^2)$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$-xy(x^4 + y^4)(x^2 + y^2)(x + y)(x – y)$

45. $x^3 – x$

Solution:

We have,

$x^3 – x$

$x(x^2 – 1)$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$x(x + 1)(x – 1)$

46. $18a^2x^2 – 32$

Solution:

We have,

$18a^2x^2 – 32$

$2[(3ax)^2 – (4)^2]$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$2(3ax + 4)(3ax – 4)$

EXERCISE 7.6 PAGE NO: 7.22

Factorize each of the following algebraic expressions:

1. $4x^2 + 12xy + 9y^2$

Solution:

We have,

$4x^2 + 12xy + 9y^2$

By using the formula $(x + y)^2 = x^2 + y^2 + 2xy$

$(2x)^2 + (3y)^2 + 2\ (2x)\ (3y)$

$(2x + 3y)^2$

$(2x + 3y)\ (2x + 3y)$

2. $9a^2 – 24ab + 16b^2$

Solution:

We have,

$9a^2 – 24ab + 16b^2$

By using the formula $(x - y)^2 = x^2 + y^2 - 2xy$

Here $x = 3a$, $y = 4b$ So,

$(3a)^2 + (4b)^2 – 2\ (3a)\ (4b)$

$(3a – 4b)^2$

$(3a – 4b)\ (3a – 4b)$

3. $p^2q^2 – 6pqr + 9r^2$

Solution:

We have,

$p^2q^2 – 6pqr + 9r^2$

By using the formula $(a – b)^2 = a^2 + b^2 – 2ab$

$(pq)^2 + (3r)^2 – 2\ (pq)\ (3r)$

$(pq – 3r)^2$

$(pq – 3r)\ (pq – 3r)$

4. $36a^2 + 36a + 9$

Solution:

We have,

$36a^2 + 36a + 9$

$(6a)^2 + 2 \times 6a \times 3 + 3^2$

$(6a + 3)^2$

5. $a^2 + 2ab + b^2 - 16$

Solution:

We have,

$a^2 + 2ab + b^2 – 16$

By using the formula $(a – b)^2 = a^2 + b^2 – 2ab$

$(a + b)^2 - 4^2$

By using the formula $(a^2 – b^2) = (a+b) (a-b)$

$(a + b + 4) (a + b – 4)$

6. $9z^2 – x^2 + 4xy – 4y^2$

Solution:

We have,

$9z^2 – x^2 + 4xy – 4y^2$

$(3z)^2 – [x^2 – 2 (x) (2y) + (2y)^2]$

By using the formula $(a – b)^2 = a^2 + b^2 – 2ab$

$(3z)^2 – (x – 2y)^2$

By using the formula $(a^2 – b^2) = (a+b) (a-b)$

$[(x – 2y) + 3z] [–x + 2y + 3z)]$

7. $9a^4 – 24a^2b^2 + 16b^4 – 256$

Solution:

We have,

$9a^4 – 24a^2b^2 + 16b^4 – 256$

$(3a^2)^2 – 2 (4a^2) (3b^2) + (4b^2)^2 – (16)^2$

By using the formula $(a – b)^2 = a^2 + b^2 – 2ab$

$(3a^2 – 4b^2)^2 – (16)^2$

By using the formula $(a^2 – b^2) = (a+b) (a-b)$

$(3a^2 – 4b^2 + 16) (3a^2 – 4b^2 – 16)$

8. $16 – a^6 + 4a^3b^3 - 4b^6$

Solution:

We have,

$16 – a^6 + 4a^3b^3 – 4b^6$

$4^2 – [(a^3)^2 – 2 (a^3) (2b^3) + (2b^3)^2]$

By using the formula $(a – b)^2 = a^2 + b^2 – 2ab$

$4^2 – (a^3 – 2b^3)^2$

By using the formula $(a^2 – b^2) = (a+b) (a-b)$

$[4 + (a^3 – 2b^3)] [4 – (a^3 – 2b^3)]$

9. $a^2 - 2ab + b^2 – c^2$

Solution:
We have,
$a^2 - 2ab + b^2 – c^2$
By using the formula $(a – b)^2 = a^2 + b^2 – 2ab$
$(a – b)^2 – c^2$
By using the formula $(a^2 – b^2) = (a+b) (a-b)$
(a – b + c) (a – b – c)

10. $x^2 + 2x + 1 – 9y^2$
Solution:
We have,
$x^2 + 2x + 1 – 9y^2$
By using the formula $(a – b)^2 = a^2 + b^2 – 2ab$
$(x + 1)^2 – (3y)^2$
By using the formula $(a^2 – b^2) = (a+b) (a-b)$
(x + 3y + 1) (x – 3y + 1)

11. $a^2 + 4ab + 3b^2$
Solution:
We have,
$a^2 + 4ab + 3b^2$
By using factors for 3 i.e., 3 and 1
$a^2 + ab + 3ab + 3b^2$
By grouping we get,
a (a + b) + 3b (a + b)
(a + 3b) (a + b)

12. $96 – 4x - x^2$
Solution:
We have,
$96 – 4x - x^2$
$-x^2 – 4x + 96$
By using factors for 96 i.e., 12 and 8
$-x^2 – 12x + 8x + 96$
By grouping we get,
-x (x + 12) + 8 (x + 12)
(x + 12) (-x + 8)

13. $a^4 + 3a^2 + 4$

Solution:
We have,
$a^4 + 3a^2 + 4$
$(a^2)^2 + (a^2)^2 + 2(2a^2) + 4 - a^2$
$(a^2 + 2)^2 + (-a^2)$
By using the formula $(a^2 - b^2) = (a+b)(a-b)$
$(a^2 + 2 + a)(a^2 + 2 - a)$
$(a^2 + a + 2)(a^2 - a + 2)$

14. $4x^4 + 1$
Solution:
We have,
$4x^4 + 1$
$(2x^2)^2 + 1 + 4x^2 - 4x^2$
$(2x^2 + 1)^2 - 4x^2$
By using the formula $(a^2 - b^2) = (a+b)(a-b)$
$(2x^2 + 1 + 2x)(2x^2 + 1 - 2x)$
$(2x^2 + 2x + 1)(2x^2 - 2x + 1)$

15. $4x^4 + y^4$
Solution:
We have,
$4x^4 + y^4$
$(2x^2)^2 + (y^2)^2 + 4x^2y^2 - 4x^2y^2$
$(2x^2 + y^2)^2 - 4x^2y^2$
By using the formula $(a^2 - b^2) = (a+b)(a-b)$
$(2x^2 + y^2 + 2xy)(2x^2 + y^2 - 2xy)$

16. $(x + 2)^4 - 6(x + 2) + 9$
Solution:
We have,
$(x + 2)^4 - 6(x + 2) + 9$
$(x^2 + 2^2)^2 - 6x - 12 + 9$
$(x^2 + 2^2 + 2(2)(x)) - 6x - 12 + 9$
$x^2 + 4 + 4x - 6x - 12 + 9$
$x^2 - 2x + 1$
By using the formula $(a - b)^2 = a^2 + b^2 - 2ab$
$(x - 1)^2$

17. $25 – p^2 – q^2 – 2pq$

Solution:

We have,

$25 – p^2 – q^2 – 2pq$

$25 – (p^2 + q^2 + 2pq)$

$(5)^2 – (p + q)^2$

By using the formula $(a^2 – b^2) = (a+b) (a-b)$

$(5 + p + q) (5 –p – q)$

$-(p + q + 5) (p + q – 5)$

18. $x^2 + 9y^2 – 6xy – 25a^2$

Solution:

We have,

$x^2 + 9y^2 – 6xy – 25a^2$

$(x – 3y)^2 – (5a)^2$

By using the formula $(a^2 – b^2) = (a+b) (a-b)$

$(x – 3y + 5a) (x – 3y – 5a)$

19. $49 - a^2 + 8ab – 16b^2$

Solution:

We have,

$49 - a^2 + 8ab – 16b^2$

$49 – (a^2 – 8ab + 16b^2)$

$49 – (a – 4b)^2$

By using the formula $(a^2 – b^2) = (a + b) (a - b)$

$(7 + a – 4b) (7 – a + 4b)$

$-(a – 4b + 7) (a – 4b – 7)$

20. $a^2 - 8ab + 16b^2 – 25c^2$

Solution:

We have,

$a^2 - 8ab + 16b^2 – 25c^2$

$(a – 4b)^2 - (5c)^2$

By using the formula $(a^2 – b^2) = (a+b) (a-b)$

$(a – 4b + 5c) (a – 4b – 5c)$

21. $x^2 - y^2 + 6y - 9$

Solution:

We have,

$x^2 - y^2 + 6y – 9$
$x^2 + 6y – (y^2 – 6y + 9)$
$x^2 – (y – 3)^2$
By using the formula $(a^2 – b^2) = (a+b) (a-b)$
$(x + y – 3) (x – y + 3)$

22. $25x^2 – 10x + 1 – 36y^2$
Solution:
We have,
$25x^2 – 10x + 1 – 36y^2$
$(5x)^2 – 2 (5x) + 1 – (6y)^2$
$(5x – 1)^2 – (6y)^2$
By using the formula $(a^2 – b^2) = (a+b) (a-b)$
$(5x – 6y – 1) (5x + 6y – 1)$

23. $a^2 – b^2 + 2bc – c^2$
Solution:
We have,
$a^2 – b^2 + 2bc – c^2$
$a^2 – (b^2 – 2bc + c^2)$
$a^2 – (b – c)^2$
By using the formula $(a^2 – b^2) = (a+b) (a-b)$
$(a + b – c) (a – b + c)$

24. $a^2 + 2ab + b^2 – c^2$
Solution:
We have,
$a^2 + 2ab + b^2 – c^2$
$(a + b)^2 – c^2$
By using the formula $(a^2 – b^2) = (a+b) (a-b)$
$(a + b + c) (a + b – c)$

25. $49 – x^2 – y^2 + 2xy$
Solution:
We have,
$49 – x^2 – y^2 + 2xy$
$49 – (x^2 + y^2 – 2xy)$
$7^2 – (x – y)^2$
By using the formula $(a^2 – b^2) = (a+b) (a-b)$

$[7 + (x – y)] [7 – x + y]$
$(x – y + 7) (y – x + 7)$

26. $a^2 + 4b^2 – 4ab – 4c^2$
Solution:
We have,
$a^2 + 4b^2 – 4ab – 4c^2$
$a^2 – 2 (a) (2b) + (2b)^2 – (2c)^2$
$(a – 2b)^2 – (2c)^2$
By using the formula $(a^2 – b^2) = (a+b) (a-b)$
$(a – 2b + 2c) (a – 2b – 2c)$

27. $x^2 – y^2 – 4xz + 4z^2$
Solution:
We have,
$x^2 – y^2 – 4xz + 4z^2$
$x^2 – 2 (x) (2z) + (2z)^2 – y^2$
As $(a-b)^2 = a^2 + b^2 – 2ab$
$(x – 2z)^2 – y^2$
By using the formula $(a^2 – b^2) = (a+b) (a-b)$
$(x + y – 2z) (x – y – 2z)$

EXERCISE 7.7 PAGE NO: 7.27

Factorize each of the following algebraic expressions:

1. $x^2 + 12x – 45$

Solution:

We have,

$x^2 + 12x – 45$

To factorize the given expression we have to find two numbers p and q such that p+q = 12 and pq = -45

So we can replace 12x by 15x – 3x

-45 by 15 × 3

$x^2 + 12x – 45 = x^2 + 15x – 3x – 45$

$= x (x + 15) – 3 (x + 15)$

$= (x – 3) (x + 15)$

2. $40 + 3x – x^2$

Solution:

We have,

$40 + 3x – x^2$

$-(x^2 – 3x – 40)$

By considering, p+q = -3 and pq = -40

So we can replace -3x by 5x – 8x

-40 by 5 × -8

$-(x^2 – 3x – 40) = x^2 + 5x – 8x – 40$

$= -x (x + 5) – 8 (x + 5)$

$= -(x - 8) (x + 5)$

$= (-x + 8) (x + 5)$

3. $a^2 + 3a – 88$

Solution:

We have,

$a^2 + 3a – 88$

By considering, p+q = 3 and pq = -88

So we can replace 3a by 11a – 8a

-40 by -11 × 8

$a^2 + 3a – 88 = a^2 + 11a – 8a – 88$

$= a (a + 11) – 8 (a + 11)$

$= (a – 8) (a + 11)$

4. $a^2 – 14a – 51$

Solution:

We have,

$a^2 – 14a – 51$

By considering, p+q = -14 and pq = -51

So we can replace -14a by 3a – 17a

-51 by -17 × 3

$a^2 – 14a – 51 = a^2 + 3a – 17a – 51$

$= a (a + 3) – 17 (a + 3)$

$= (a – 17) (a + 3)$

5. $x^2 + 14x + 45$

Solution:

We have,

$x^2 + 14x + 45$

By considering, p+q = 14 and pq = 45

So we can replace 14x by 5x + 9x

45 by 5 × 9

$x^2 + 14x + 45 = x^2 + 5x + 9x + 45$

$= x (x + 5) – 9 (x + 5)$

$= (x + 9) (x + 5)$

6. $x^2 – 22x + 120$

Solution:

We have,

$x^2 – 22x + 120$

By considering, p+q = -22 and pq = 120

So we can replace -22x by -12x -10x

120 by -12 × -10

$x^2 - 22x + 120 = x^2 - 12x – 10x + 120$

$= x (x - 12) – 10 (x - 12)$

$= (x – 10) (x - 12)$

7. $x^2 – 11x – 42$

Solution:

We have,

$x^2 – 11x – 42$

By considering, p+q = -11 and pq = -42

So we can replace -11x by 3x -14x

-42 by 3 × -14

$x^2 - 11x – 42 = x^2 + 3x – 14x – 42$

$= x (x + 3) – 14 (x + 3)$

$= (x – 14) (x + 3)$

8. $a^2 + 2a – 3$

Solution:

We have,

$a^2 + 2a – 3$

By considering, p+q = 2 and pq = -3

So we can replace 2a by 3a -a

-3 by 3 × -1

$a^2 + 2a – 3 = a^2 + 3a – a – 3$

$= a (a + 3) – 1 (a + 3)$

$= (a – 1) (a + 3)$

9. $a^2 + 14a + 48$

Solution:

We have,

$a^2 + 14a + 48$

By considering, p+q = 14 and pq = 48

So we can replace 14a by 8a + 6a

48 by 8 × 6

$a^2 + 14a + 48 = a^2 + 8a + 6a + 48$

$= a (a + 8) + 6 (a + 8)$

$= (a + 6) (a + 8)$

10. $x^2 – 4x – 21$

Solution:

We have,

$x^2 – 4x – 21$

By considering, p+q = -4 and pq = -21

So we can replace -4x by 3x – 7x

-21 by 3 × -7

$x^2 + 4x – 21 = x^2 + 3x – 7x – 21$

$= x (x + 3) – 7 (x + 3)$

$= (x – 7) (x + 3)$

11. $y^2 + 5y – 36$

Solution:
We have,
$y^2 + 5y – 36$
By considering, p+q = 5 and pq = -36
So we can replace 5y by 9y – 4y
-36 by 9 × -4
$y^2 + 5y – 36 = y^2 + 9y – 4y – 36$
$= y (y + 9) – 4 (y + 9)$
$= (y – 4) (y + 9)$

12. $(a^2 - 5a)^2 - 36$
Solution:
We have,
$(a^2 - 5a)^2 – 36$
$(a^2 - 5a)^2 - 6^2$
By using the formula $(a^2 – b^2) = (a+b) (a-b)$
$(a^2 – 5a)^2 – 6^2 = (a^2 – 5a + 6) (a^2 – 5a – 6)$
So now we shall factorize the expression $(a^2 – 5a + 6)$
By considering, p+q = -5 and pq = 6
So we can replace -5a by a -6a
6 by 1 × -6
$a^2 -5a – 6 = a^2 + a – 6a – 6$
$= a (a + 1) -6(a + 1)$
$= (a – 6) (a + 1)$

So now we shall factorize the expression $(a^2 – 5a + 6)$
By considering, p+q = -5 and pq = -6
So we can replace -5a by -2a -3a
6 by -2 × -3
$a^2 -5a + 6 = a^2 – 2a – 3a + 6$
$= a (a – 2) -3 (a - 2)$
$= (a – 3) (a – 2)$

$\therefore (a^2 - 5a)^2 – 36 = (a^2 – 5a + 6) (a^2 – 5a – 6)$
$= (a + 1) (a – 6) (a – 2) (a – 3)$

13. (a + 7) (a – 10) + 16
Solution:
We have,

$(a + 7)(a – 10) + 16$
$a^2 – 10a + 7a – 70 + 16$
$a^2 – 3a – 54$
By considering, $p+q = -3$ and $pq = -54$
So we can replace $-3a$ by $6a – 9a$
-54 by 6×-9
$a^2 – 3a – 54 = a^2 + 6a – 9a – 54$
$= a(a + 6) -9(a + 6)$
$= (a - 9)(a + 6)$

EXERCISE 7.8 PAGE NO: 7.30

Resolve each of the following quadratic trinomials into factors:

1. $2x^2 + 5x + 3$

Solution:

We have,

$2x^2 + 5x + 3$

The coefficient of x^2 is 2

The coefficient of x is 5

Constant term is 3

We shall split up the center term i.e., 5 into two parts such that their sum p+q is 5 and product pq = 2 × 3 is 6

So, we express the middle term 5x as 2x + 3x

$2x^2 + 5x + 3 = 2x^2 + 2x + 3x + 3$

$= 2x (x + 1) + 3 (x + 1)$

$= (2x + 3) (x + 1)$

2. $2x^2 - 3x - 2$

Solution:

We have,

$2x^2 - 3x – 2$

The coefficient of x^2 is 2

The coefficient of x is -3

Constant term is -2

So, we express the middle term -3x as -4x + x

$2x^2 - 3x – 2 = 2x^2 - 4x + x – 2$

$= 2x (x – 2) + 1 (x – 2)$

$= (x – 2) (2x + 1)$

3. $3x^2 + 10x + 3$

Solution:

We have,

$3x^2 + 10x + 3$

The coefficient of x^2 is 3

The coefficient of x is 10

Constant term is 3

So, we express the middle term 10x as 9x + x

$3x^2 + 10x + 3 = 3x^2 + 9x + x + 3$

$= 3x (x + 3) + 1 (x + 3)$

= (3x + 1) (x + 3)

4. $7x – 6 – 2x^2$

Solution:

We have,

$7x – 6 – 2x^2$

$– 2x^2 + 7x – 6$

$2x^2 - 7x + 6$

The coefficient of x^2 is 2

The coefficient of x is -7

Constant term is 6

So, we express the middle term -7x as -4x - 3x

$2x^2 - 7x + 6 = 2x^2 - 4x - 3x + 6$

$= 2x (x – 2) - 3 (x – 2)$

$= (x – 2) (2x – 3)$

5. $7x^2 - 19x - 6$

Solution:

We have,

$7x^2 - 19x – 6$

The coefficient of x^2 is 7

The coefficient of x is -19

Constant term is -6

So, we express the middle term -19x as 2x – 21x

$7x^2 - 19x – 6 = 7x^2 + 2x - 21x – 6$

$= x (7x + 2) - 3 (7x + 2)$

$= (7x + 2) (x – 3)$

6. $28 – 31x – 5x^2$

Solution:

We have,

$28 – 31x – 5x^2$

$– 5x^2 -31x + 28$

$5x^2 + 31x - 28$

The coefficient of x^2 is 5

The coefficient of x is 31

Constant term is -28

So, we express the middle term 31x as -4x + 35x

$5x^2 + 31x - 28 = 5x^2 - 4x + 35x - 28$

$= x(5x - 4) + 7(5x - 4)$
$= (x + 7)(5x - 4)$

7. $3 + 23y - 8y^2$
Solution:
We have,
$3 + 23y - 8y^2$
$-8y^2 + 23y + 3$
$8y^2 - 23y - 3$
The coefficient of y^2 is 8
The coefficient of y is -23
Constant term is -3
So, we express the middle term -23y as -24y + y
$8y^2 - 23y - 3 = 8y^2 - 24y + y - 3$
$= 8y(y - 3) + 1(y - 3)$
$= (8y + 1)(y - 3)$

8. $11x^2 - 54x + 63$
Solution:
We have,
$11x^2 - 54x + 63$
The coefficient of x^2 is 11
The coefficient of x is -54
Constant term is 63
So, we express the middle term -54x as -33x – 21x
$11x^2 - 54x + 63 = 11x^2 - 33x - 21x + 63$
$= 11x(x - 3) - 21(x - 3)$
$= (11x - 21)(x - 3)$

9. $7x - 6x^2 + 20$
Solution:
We have,
$7x - 6x^2 + 20$
$-6x^2 + 7x + 20$
$6x^2 - 7x - 20$
The coefficient of x^2 is 6
The coefficient of x is -7
Constant term is -20
So, we express the middle term -7x as -15x + 8x

$6x^2 - 7x - 20 = 6x^2 - 15x + 8x - 20$
$= 3x (2x – 5) + 4 (2x – 5)$
$= (3x + 4) (2x - 5)$

10. $3x^2 + 22x + 35$
Solution:
We have,
$3x^2 + 22x + 35$
The coefficient of x^2 is 3
The coefficient of x is 22
Constant term is 35
So, we express the middle term 22x as 15x + 7x
$3x^2 + 22x + 35 = 3x^2 + 15x + 7x + 35$
$= 3x (x + 5) + 7 (x + 5)$
$= (3x + 7) (x+ 5)$

11. $12x^2 - 17xy + 6y^2$
Solution:
We have,
$12x^2 - 17xy + 6y^2$
The coefficient of x^2 is 12
The coefficient of x is -17y
Constant term is $6y^2$
So, we express the middle term -17xy as -9xy - 8xy
$12x^2 -17xy+ 6y^2 = 12x^2 - 9xy - 8xy + 6y^2$
$= 3x (4x – 3y) – 2y (4x – 3y)$
$= (3x – 2y) (4x – 3y)$

12. $6x^2 - 5xy - 6y^2$
Solution:
We have,
$6x^2 - 5xy - 6y^2$
The coefficient of x^2 is 6
The coefficient of x is -5y
Constant term is $-6y^2$
So, we express the middle term -5xy as 4xy - 9xy
$6x^2 -5xy- 6y^2 = 6x^2 + 4xy - 9xy - 6y^2$
$= 2x (3x + 2y) -3y (3x + 2y)$
$= (2x – 3y) (3x + 2y)$

13. $6x^2 - 13xy + 2y^2$

Solution:

We have,

$6x^2 - 13xy + 2y^2$

The coefficient of x^2 is 6

The coefficient of x is -13y

Constant term is $2y^2$

So, we express the middle term -13xy as -12xy – xy

$6x^2 - 13xy + 2y^2 = 6x^2 - 12xy - xy + 2y^2$

$= 6x (x – 2y) – y (x – 2y)$

$= (6x – y) (x - 2y)$

14. $14x^2 + 11xy - 15y^2$

Solution:

We have,

$14x^2 + 11xy - 15y^2$

The coefficient of x^2 is 14

The coefficient of x is 11y

Constant term is $-15y^2$

So, we express the middle term 11xy as 21xy – 10xy

$14x^2 + 11xy - 15y^2 = 14x^2 + 21xy - 10xy - 15y^2$

$= 2x (7x – 5y) + 3y (7x – 5y)$

$= (2x + 3y) (7x - 5y)$

15. $6a^2 + 17ab – 3b^2$

Solution:

We have,

$6a^2 + 17ab – 3b^2$

The coefficient of a^2 is 6

The coefficient of a is 17b

Constant term is $-3b^2$

So, we express the middle term 17ab as 18ab – ab

$6a^2 + 17ab – 3b^2 = 6a^2 + 18ab - ab – 3b^2$

$= 6a (a + 3b) – b (a + 3b)$

$= (6a – b) (a + 3b)$

16. $36a^2 + 12abc – 15b^2c^2$

Solution:

We have,
$36a^2 + 12abc – 15b^2c^2$
The coefficient of a^2 is 36
The coefficient of a is 12bc
Constant term is $-15b^2c^2$
So, we express the middle term 12abc as 30abc – 18abc
$36a^2 –12abc– 15b^2c^2 = 36a^2 + 30abc – 18abc – 15b^2c^2$
$= 6a (6a + 5bc) – 3bc (6a + 5bc)$
$= (6a + 5bc) (6a – 3bc)$
$= (6a + 5bc) 3(2a - bc)$

17. $15x^2 – 16xyz – 15y^2z^2$
Solution:
We have,
$15x^2 – 16xyz – 15y^2z^2$
The coefficient of x^2 is 15
The coefficient of x is -16yz
Constant term is $-15y^2z^2$
So, we express the middle term -16xyz as -25xyz + 9xyz
$15x^2 -16xyz- 15y^2z^2 = 15x^2 - 25yz + 9yz - 15y^2z^2$
$= 5x (3x – 5yz) + 3yz (3x – 5yz)$
$= (5x + 3yz) (3x - 5yz)$

18. $(x – 2y)^2 – 5 (x – 2y) + 6$
Solution:
We have,
$(x – 2y)^2 – 5 (x – 2y) + 6$
The coefficient of $(x-2y)^2$ is 1
The coefficient of (x-2y) is -5
Constant term is 6
So, we express the middle term -5(x – 2y) as -2(x – 2y) -3(x – 2y)
$(x – 2y)^2 – 5 (x – 2y) + 6 = (x – 2y)^2 – 2 (x – 2y) – 3 (x – 2y) + 6$
$= (x – 2y - 2) (x - 2y - 3)$

19. $(2a – b)^2 + 2 (2a – b) – 8$
Solution:
We have,
$(2a – b)^2 + 2 (2a – b) – 8$
The coefficient of $(2a-b)^2$ is 1

The coefficient of (2a-b) is 2
Constant term is -8
So, we express the middle term 2(2a – b) as 4 (2a –b) – 2 (2a – b)

$(2a – b)^2 + 2 (2a – b) – 8 = (2a – b)^2 + 4 (2a – b) – 2 (2a – b) - 8$

$= (2a – b) (2a – b + 4) – 2 (2a – b + 4)$

$= (2a – b + 4) (2a – b – 2)$

EXERCISE 7.9 PAGE NO: 7.32

Factorize each of the following quadratic polynomials by using the method of completing the square:

1. $p^2 + 6p + 8$

Solution:

We have,

$p^2 + 6p + 8$

Coefficient of p^2 is unity. So, we add and subtract square of half of coefficient of p.

$p^2 + 6p + 8 = p^2 + 6p + 3^2 – 3^2 + 8$ (Adding and subtracting 3^2)

$= (p + 3)^2 – 1^2$ (By completing the square)

By using the formula $(a^2 – b^2) = (a+b)(a-b)$

$= (p + 3 – 1)(p + 3 + 1)$

$= (p + 2)(p + 4)$

2. $q^2 – 10q + 21$

Solution:

We have,

$q^2 – 10q + 21$

Coefficient of q^2 is unity. So, we add and subtract square of half of coefficient of q.

$q^2 – 10q + 21 = q^2 – 10q + 5^2 – 5^2 + 21$ (Adding and subtracting 5^2)

$= (q – 5)^2 – 2^2$ (By completing the square)

By using the formula $(a^2 – b^2) = (a+b)(a-b)$

$= (q – 5 – 2)(q – 5 + 2)$

$= (q – 3)(q – 7)$

3. $4y^2 + 12y + 5$

Solution:

We have,

$4y^2 + 12y + 5$

$4(y^2 + 3y + 5/4)$

Coefficient of y^2 is unity. So, we add and subtract square of half of coefficient of y.

$4(y^2 + 3y + 5/4) = 4[y^2 + 3y + (3/2)^2 – (3/2)^2 + 5/4]$ (Adding and subtracting $(3/2)^2$)

$= 4[(y + 3/2)^2 – 1^2]$ (Completing the square)

By using the formula $(a^2 – b^2) = (a+b)(a-b)$

$= 4(y + 3/2 + 1)(y + 3/2 – 1)$

$= 4(y + 1/2)(y + 5/2)$ (by taking LCM)

$= 4[(2y + 1)/2][(2y + 5)/2]$

$= (2y + 1)(2y + 5)$

4. $p^2 + 6p - 16$

Solution:

We have,

$p^2 + 6p - 16$

Coefficient of p^2 is unity. So, we add and subtract square of half of coefficient of p.

$p^2 + 6p – 16 = p^2 + 6p + 3^2 – 3^2 – 16$ (Adding and subtracting 3^2)

$= (p + 3)^2 – 5^2$ (Completing the square)

By using the formula $(a^2 – b^2) = (a+b) (a-b)$

$= (p + 3 + 5) (p + 3 – 5)$

$= (p + 8) (p – 2)$

5. $x^2 + 12x + 20$

Solution:

We have,

$x^2 + 12x + 20$

Coefficient of x^2 is unity. So, we add and subtract square of half of coefficient of x.

$x^2 + 12x + 20 = x^2 + 12x + 6^2 – 6^2 + 20$ (Adding and subtracting 6^2)

$= (x + 6)^2 – 4^2$ (Completing the square)

By using the formula $(a^2 – b^2) = (a+b) (a-b)$

$= (x + 6 + 4) (x + 6 – 4)$

$= (x + 2) (x + 10)$

6. $a^2 – 14a – 51$

Solution:

We have,

$a^2 – 14a – 51$

Coefficient of a^2 is unity. So, we add and subtract square of half of coefficient of a.

$a^2 – 14a – 51 = a^2 – 14a + 7^2 – 7^2 – 51$ (Adding and subtracting 7^2)

$= (a – 7)^2 – 10^2$ (Completing the square)

By using the formula $(a^2 – b^2) = (a+b) (a-b)$

$= (a – 7 + 10) (9 – 7 – 10)$

$= (a – 17) (a + 3)$

7. $a^2 + 2a – 3$

Solution:

We have,

$a^2 + 2a – 3$

Coefficient of a^2 is unity. So, we add and subtract square of half of coefficient of a.

$a^2 + 2a – 3 = a^2 + 2a + 1^2 – 1^2 – 3$ (Adding and subtracting 1^2)
$= (a + 1)^2 – 2^2$ (Completing the square)
By using the formula $(a^2 – b^2) = (a+b) (a-b)$
$= (a + 1 + 2) (a + 1 – 2)$
$= (a + 3) (a – 1)$

8. $4x^2 - 12x + 5$
Solution:
We have,
$4x^2 - 12x + 5$
$4(x^2 - 3x + 5/4)$
Coefficient of x^2 is unity. So, we add and subtract square of half of coefficient of x.
$4(x^2 – 3x + 5/4) = 4 [x^2 – 3x + (3/2)^2 – (3/2)^2 + 5/4]$ (Adding and subtracting $(3/2)^2$)
$= 4 [(x - 3/2)^2 – 1^2]$ (Completing the square)
By using the formula $(a^2 – b^2) = (a+b) (a-b)$
$= 4 (x - 3/2 + 1) (x - 3/2 – 1)$
$= 4 (x – 1/2) (x – 5/2)$ (by taking LCM)
$= 4 [(2x-1)/2] [(2x - 5)/2]$
$= (2x - 5) (2x - 1)$

9. $y^2 - 7y + 12$
Solution:
We have,
$y^2 - 7y + 12$
Coefficient of y^2 is unity. So, we add and subtract square of half of coefficient of y.
$y^2 - 7y + 12 = y^2 – 7y + (7/2)^2 – (7/2)^2 + 12$ [Adding and subtracting $(7/2)^2$]
$= (y – 7/2)^2 – (7/2)^2$ (Completing the square)
By using the formula $(a^2 – b^2) = (a+b) (a-b)$
$= (y – (7/2- 1/2)) (y – (7/2 + 1/2))$
$= (y – 3) (y – 4)$

10. $z^2 – 4z - 12$
Solution:
We have,
$z^2 – 4z – 12$
Coefficient of z^2 is unity. So, we add and subtract square of half of coefficient of z.
$z^2 – 4z – 12 = z^2 – 4z + 2^2 – 2^2 – 12$ [Adding and subtracting 2^2]
$= (z – 2)^2 – 4^2$ (Completing the square)
By using the formula $(a^2 – b^2) = (a+b) (a-b)$

= (z – 2 + 4) (z – 2 – 4)
= (z – 6) (z + 2)