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Question 1: Find the following products:

(i) $(3x + 2y)(9x^2 – 6xy + 4y^2)$

(ii) $(4x – 5y)(16x^2 + 20xy + 25y^2)$

(iii) $(7p^4 + q)(49p^8 – 7p^4q + q^2)$

(iv) $(x/2 + 2y)(x^2/4 – xy + 4y^2)$

(v) $(3/x – 5/y)(9/x^2 + 25/y^2 + 15/xy)$

(vi) $(3 + 5/x)(9 – 15/x + 25/x^2)$

(vii) $(2/x + 3x)(4/x^2 + 9x^2 – 6)$

(viii) $(3/x – 2x^2)(9/x^2 + 4x^4 – 6x)$

(ix) $(1 – x)(1 + x + x^2)$

(x) $(1 + x)(1 – x + x^2)$

(xi) $(x^2 – 1)(x^4 + x^2 +1)$

(xii) $(x^3 + 1)(x^6 – x^3 + 1)$

Solution:

(i) $(3x + 2y)(9x^2 – 6xy + 4y^2)$

$= (3x + 2y)[(3x)^2 – (3x)(2y) + (2y)^2)]$

We know, a^3 + b^3 = (a + b)(a^2 + b^2 – ab)

$= (3x)^3 + (2y)^3$

$= 27x^3 + 8y^3$

(ii) $(4x - 5y)(16x^2 + 20xy + 25y^2)$

$= (4x - 5y)[(4x)^2 + (4x)(5y) + (5y)^2)]$

We know, a^3 - b^3 = (a - b)(a^2 + b^2 + ab)

$= (4x)^3 - (5y)^3$

$= 64x^3 - 125y^3$

(iii) $(7p^4 + q)(49p^8 - 7p^4q + q^2)$

$= (7p^4 + q)[(7p^4)^2 - (7p^4)(q) + (q)^2)]$

We know, a^3 + b^3 = (a + b)(a^2 + b^2 – ab)

$= (7p^4)^3 + (q)^3$

$= 343\ p^{12} + q^3$

(iv) $(x/2 + 2y)(x^2/4 - xy + 4y^2)$

We know, a^3 - b^3 = (a - b)(a^2 + b^2 + ab)

$(x/2 + 2y)(x^2/4 - xy + 4y^2)$

$=(\frac{x}{2} + 2y)[(\frac{x}{2})^2 - \frac{x}{2}(2y) + (2y)^2]$

$=(\frac{x}{2})^3 + (2y)^3$

$= \frac{x^3}{8} + 8y^3$

(v) $(3/x - 5/y)(9/x^2 + 25/y^2 + 15/xy)$

$=(\frac{3}{x} - \frac{5}{y})(\frac{3}{x})^2 + (\frac{5}{y})^2 + (\frac{3}{x})(\frac{5}{y})$

$=(\frac{3}{x})^3 - (\frac{5}{y})^3$

$= (\frac{27}{x^3}) - (\frac{125}{y^3})$

[Using a^3 - b^3 = (a - b)(a^2 + b^2 + ab)]

(vi) (3 + 5/x)(9 – 15/x + 25/x^2)

= $(3 + \frac{5}{x})[(3^2) - 3(\frac{5}{x}) + (\frac{5}{x})^2]$

= $(3)^3 + (\frac{5}{x})^3$

= $27 + \frac{125}{x^3}$

[Using: a^3 + b^3 = (a + b)(a^2 + b^2 – ab)]

(vii) (2/x + 3x)(4/x^2 + 9x^2 – 6)

= $(\frac{2}{x} + 3x)[(\frac{2}{x})^2 + (3x)^2 - (\frac{2}{x})(3x)]$

= $(\frac{2}{x})^3 + (3x)^3$

= $\frac{8}{x^3} + 27x^3$

[Using: a^3 + b^3 = (a + b)(a^2 + b^2 – ab)]

(viii) (3/x – 2x^2)(9/x^2 + 4x^4 – 6x)

= $(\frac{3}{x} - 2x^2)[(\frac{3}{x})^2 + (2x^2)^2 - (\frac{3}{x})(2x^2)]$

= $(\frac{3}{x} - 2x^2)[(\frac{9}{x^2}) + 4x^4 - (\frac{3}{x})(2x^2)]$

= $(\frac{3}{x})^3 - (2x^2)^3$

= $\frac{27}{x^3} - 8x^6$

[Using : a^3 - b^3 = (a - b)(a^2 + b^2 + ab)]

(ix) (1 – x)(1 + x + x^2)

And we know, a^3 - b^3 = (a - b)(a^2 + b^2 + ab)

(1 – x)(1 + x + x^2) can be written as

$(1 – x)[(1^2 + (1)(x)+ x^2)]$

$= (1)^3 – (x)^3$

$= 1 – x^3$

(x) $(1 + x)(1 – x + x^2)$
And we know, a^3 + b^3 = (a + b)(a^2 + b^2 – ab)]

(1 + x)(1 – x + x^2) can be written as,

$(1 + x)[(1^2 – (1)(x) + x^2)]$

$= (1)^3 + (x)^3$

$= 1 + x^3$

(xi) $(x^2 – 1)(x^4 + x^2 +1)$ can be written as,

$(x^2 – 1)[(x^2)^2 – 1^2 + (x^2)(1)]$

$= (x^2)^3 – 1^3$

$= x^6 – 1$

[using a^3 - b^3 = (a - b)(a^2 + b^2 + ab)]

(xii) $(x^3 + 1)(x^6 – x^3 + 1)$ can be written as,

$(x^3 + 1)[(x^3)^2 – (x^3)(1) + 1^2]$

$= (x^3)^3 + 1^3$

$= x^9 + 1$

[using a^3 + b^3 = (a + b)(a^2 + b^2 - ab)]

Question 2: If x = 3 and y = -1, find the values of each of the following using in identity:

(i) $(9y^2 – 4x^2)(81y^4 + 36x^2y^2 + 16x^4)$

(ii) $(3/x - x/3)(x^2/9 + 9/x^2 + 1)$
(iii) $(x/7 + y/3)(x^2/49 + y^2/9 - xy/21)$
(iv) $(x/4 - y/3)(x^2/16 + xy/12 + y^2/9)$
(v) $(5/x + 5x)(25/x^2 - 25 + 25x^2)$

Solution:

(i) $(9y^2 - 4x^2)(81y^4 + 36x^2y^2 + 16x^4)$

$= (9y^2 - 4x^2)\,[(9y^2)^2 + 9y^2 \times 4x^2 + (4x^2)^2]$

$= (9y^2)^3 - (4x^2)^3$

$= 729y^6 - 64x^6$

Put x = 3 and y = -1

= 729 – 46656

= - 45927

(ii) Put x = 3 and y = -1

$(3/x - x/3)(x^2/9 + 9/x^2 + 1)$

$$= \left(\frac{3}{x}-\frac{x}{3}\right)\left[\left(\frac{x}{3}\right)^2+\frac{x}{3}\times\frac{3}{x}+\left(\frac{3}{x}\right)^2\right]$$

$$= \left(\frac{3}{x}\right)^3-\left(\frac{x}{3}\right)^3$$

$$= \left(\frac{3}{3}\right)^3-\left(\frac{3}{3}\right)^3$$

$$= 1^3 - 1^3 = 0$$

(iii) Put x = 3 and y = -1

$(x/7 + y/3)(x^2/49 + y^2/9 - xy/21)$

$= \left(\frac{x}{7}+\frac{y}{3}\right)\left[\left(\frac{x}{7}\right)^2 - \frac{x}{7}\times\frac{y}{3} - \left(\frac{y}{3}\right)^2\right]$

$= \left(\frac{x}{7}\right)^3 + \left(\frac{y}{3}\right)^3$

$= \frac{x^3}{343} + \frac{y^3}{27}$

$= \frac{(3)^3}{343} + \frac{(-1)^3}{27}$

$= \frac{27}{343} - \frac{1}{27} = \frac{729-343}{9261} = \frac{386}{9261}$

(iv) Put x = 3 and y = -1

$(x/4 – y/3)(x^2/16 + xy/12 + y^2/9)$

$= \left(\frac{x}{4}-\frac{y}{3}\right)\left[\left(\frac{x}{4}\right)^2 + \frac{x}{4}\times\frac{y}{3} + \left(\frac{y}{3}\right)^2\right]$

$= \left(\frac{x}{4}\right)^3 - \left(\frac{y}{3}\right)^3 = \frac{x^3}{64} - \frac{y^3}{27}$

$= \frac{(3)^3}{64} - \frac{(-1)^3}{27} = \frac{27}{64} + \frac{1}{27}$

$= \frac{793}{1728}$

(v) Put x = 3 and y = -1

$(5/x + 5x)(25/x^2 – 25 + 25x^2)$

$= \left(\frac{5}{x}+5x\right)\left[\left(\frac{5}{x}\right)^2 - \frac{5}{x}\times 5x + (5x)^2\right]$

$= \left(\frac{5}{x}\right)^3 + (5x)^3 = \frac{125}{x^3} + 125x^3$

$= \frac{125}{(3)^3} + 125 \times (3)^3 = \frac{125}{27} + 125 \times 27$

$= \frac{125}{27} + 3375$

$= \frac{91250}{27}$

Question 3: If a + b = 10 and ab = 16, find the value of a^2 – ab + b^2 and a^2 + ab + b^2.
Solution:
a + b = 10, ab = 16
Squaring, a + b = 10, both sides
(a + b)^2 = (10)^2
a^2 + b^2 + 2ab = 100
a^2 + b^2 + 2 x 16 = 100
a^2 + b^2 + 32 = 100
a^2 + b^2 = 100 – 32 = 68

a^2 + b^2 = 68

Again, a^2 – ab + b^2 = a^2 + b^2 – ab = 68 – 16 = 52 and
a^2 + ab + b^2 = a^2 + b^2 + ab = 68 + 16 = 84

Question 4: If a + b = 8 and ab = 6, find the value of a^3 + b^3.
Solution:
a + b = 8, ab = 6
Cubing, a + b = 8, both sides, we get
(a + b)^3 = (8)^3
a^3 + b^3 + 3ab(a + b) = 512
a^3 + b^3 + 3 x 6 x 8 = 512
a^3 + b^3 + 144 = 512
a^3 + b^3 = 512 – 144 = 368
a^3 + b^3 = 368