Exercise 4.5 Page No: 4.28

Question 1: Find the following products:
(i) $(3x + 2y + 2z)(9x^2 + 4y^2 + 4z^2 – 6xy – 4yz – 6zx)$
(ii) $(4x - 3y + 2z)(16x^2 + 9y^2 + 4z^2 + 12xy + 6yz – 8zx)$
(iii) $(2a – 3b – 2c)(4a^2 + 9b^2 + 4c^2 + 6ab – 6bc + 4ca)$
(iv) $(3x -4y + 5z)(9x^2 + 16y^2 + 25z^2 + 12xy- 15zx + 20yz)$

Solution:
(i) $(3x + 2y + 2z)(9x^2 + 4y^2 + 4z^2 – 6xy – 4yz – 6zx)$
$= (3x + 2y + 2z)[(3x)^2 + (2y)^2 + (2z)^2 – 3x \times 2y - 2y \times 2z - 2z \times 3x]$
$= (3x)^3 + (2y)^3 + (2z)^3 – 3 \times 3x \times 2y \times 2z$
$= 27x^3 + 8y^3 + 8Z^3 – 36xyz$

(ii) $(4x – 3y + 2z)(16x^2 + 9y^2 + 4z^2 + 12xy + 6yz – 8zx)$
$= (4x -3y + 2z)[(4x)^2 + (-3y)^2 + (2z)^2 – 4x \times (-3y) - (-3y) \times (2z) – (2z \times 4x)]$ =
$(4x)^3 + (-3y)^3 + (2z)^3 – 3 \times 4x \times (-3y) \times (2z)$
$= 64x^3 – 27y^3 + 8z^3 + 72xyz$

(iii) $(2a -3b- 2c)(4a^2 + 9b^2 + 4c^2 + 6ab – 6bc + 4ca)$
$= (2a -3b- 2c)[(2a)^2 + (-3b)^2 + (-2c)^2 – 2a \times (-3b) – (-3b) \times (-2c) – (-2c) \times 2a]$
$= (2a)^3 + (3b)^3 + (-2c)^3 -3x\ 2a \times (-3\ b) (-2c)$
$= 8a^3 – 21b^3 - 8c^3 – 36abc$

(iv) $(3x – 4y + 5z)(9x^2 + 16y^2 + 25z^2 + 12xy – 15zx + 20yz)$
$= [3x + (-4y) + 5z][(3x)^2 + (-4y)^2 + (5z)^2 – 3x \times (-4y) -(-4y)(5z) – 5z \times 3x]$
$= (3x)^3 + (-4y)^3 + (5z)^3 – 3 \times 3x \times (-4y)(5z)$
$= 27x^3 – 64y^3 + 125z^3 + 180xyz$

Question 2: If $x + y + z = 8$ and $xy + yz + zx = 20$, find the value of $x^3 + y^3 + z^3 – 3xyz$.
Solution:
We know, $x^3 + y^3 + z^3 – 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

Squaring, $x + y + z = 8$ both sides, we get

$(x + y + z)^2 = (8)^2$
$x^2 + y^2 + z^2 + 2(xy + yz + zx) = 64$
$x^2 + y^2 + z^2 + 2 \times 20 = 64$
$x^2 + y^2 + z^2 + 40 = 64$
$x^2 + y^2 + z^2 = 24$

Now,

$x^3 + y^3 + z^3 – 3xyz = (x + y + z) [x^2 + y^2 + z^2 – (xy + yz + zx)]$
$= 8(24 – 20)$
$= 8 \times 4$
$= 32$
$\Rightarrow x^3 + y^3 + z^3 – 3xyz = 32$

Question 3: If a +b + c = 9 and ab + bc + ca = 26, find the value of $a^3 + b^3 + c^3 – 3abc$.
Solution:
a + b + c = 9, ab + bc + ca = 26
Squaring, a + b + c = 9 both sides, we get
$(a + b + c)^2 = (9)^2$
$a^2 + b^2 + c^2 + 2 (ab + bc + ca) = 81$
$a^2 + b^2 + c^2 + 2 \times 26 = 81$
$a^2 + b^2 + c^2 + 52 = 81$
$a^2 + b^2 + c^2 = 29$

Now, $a^3 + b^3 + c^3 – 3abc = (a + b + c) [(a^2 + b^2 + c^2 – (ab + bc + ca)]$
$= 9[29 – 26]$
$= 9 \times 3$
$= 27$
$\Rightarrow a^3 + b^3 + c^3 – 3abc = 27$